A clear and logical development of the basic fundamentals of fluid mechanics, unique in its development and application of the general equations employed in the field

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Foundations of Fluid Mechanics

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PREFACE

This book is intended primarily for engineering and applied science students as an introductory and/or a secondary course in fluid mechanics. However, it may also serve professional engineers who need a basic understanding of fluid mechanics for their current work, as well as preparation for advanced work. The book is further intended for those who have not had adequate foundations in fluid mechanics during their college work and who are independent and unassisted readers.

Fluid mechanics originally evolved from the combined results of the highly-developed classical hydrodynamics on one hand and the purely empirical hydraulics on the other. It was the Prandtl school that first developed the synthetic process between classical hydrodynamics and practical hydraulics. During the past two decades there has been a rapidly increasing improvement in modern technology and design. The trend is ever increasing. These advances demand a reassessment of current course content at college level, and fluid mechanics is no exception. We can no longer imagine that students can still be trained for the specific tasks they will have to perform in the course of their careers in engineering or applied science. But their training should have provided them with the ability to formulate their problems based on the fundamental laws of nature, to recognize the type of methods which are applicable, to read with understanding the relevant work and to come up with a usable answer. The need for a book on fluid mechanics which can realize this aim has long been felt.

In this textbook I have attempted to present, in a logical pattern of development, a unified comprehensive coverage of the basic foundations of the subject matter. It emphasizes the concepts and basic principles of fluid mechanics that are common to application in most branches of the engineering profession. The presentation demonstrates the universality of mathematical expressions portraying physical phenomena. It introduces a high level of general theoretical treatment of fundamentals, which are presented at an elementary level. A typical point showing the concept of generality is the treatment of the governing equations for a viscous compressible flow; first the equations are derived, and then the general realtions are reduced in accordance with the limitations of the specific problem under investigation. In this way, the student gets a much clearer understanding and better appreciation of the underlying physical assumptions and limitations of the solution obtained when he sees what terms in the governing equations are neglected. Problems are formulated whenever possible, on the basis of fundamental physical principles rather than the utilization of arbitrary assumptions. Many illustrative examples are included to enhance the reader's thorough understanding of the subject, and develop his ability to analyze new and challenging problems in this field.

The order of the contents of this text is believed to present a rational classification of the topics treated. The introductory chapter begins with a discussion of the general aspects of fluid mechanics and the physical properties of fluids. The regimes of the mechanics of fluids are presented and categorized here so that the reader can grasp a general picture of fluid mechanics and its related fields before attempting to study any particular branch of the subject. It is vitally important that the reader recognizes the limitations in the application of the theory to any specific problem. Chapter 2 deals with fluid statics and relative motion of liquids. The fundamental differential equation of fluid statics is obtained in general vector form. The kinematical foundations of fluid motion are discussed in Chapter 3. It has been my observation that one of the fundamental sources of difficulty for students in fluid mechanics is the method of describing fluid motion. I have attempted to give a clear mathematical derivation and simple mathematical and numerical examples to illustrate the relation between the Lagrange method and the Euler method. The discussion of topics such as conservation of mass, circulation, and velocity potential are deferred to later chapters for obvious reasons.

Chapters 4 and 5 are devoted to the governing equations of fluid dynamics which form the backbone of the remaining chapters of the text. Chapter 4 gives the concepts and relationships of the stresses and rates of strain which are essential in the development of the governing equations of a viscous compressible fluid. Unless the materials presented in Chapter 4 are well understood, the derivation of the governing equations in Chapter 5 cannot be made meaningful. In today's environment we encounter many structures which have cylindrical and spherical configurations. The readers may find the comprehensive presentation of the basic equations of fluid dynamics in cylindrical and spherical coordinates in this chapter very helpful.

The problems of one- and multidimensional flows of an inviscid incompressible fluid are given in Chapters 6 and 7, respectively. To illustrate the conciseness of the analysis in the presentation I shall use the two-dimensional source and sink flow as an example. The discussion begins with the physical definition of a source; the stream function and the velocity potential for source flow are obtained and discussed only after the flow satisfying the continuity equation and the condition of irrotationality are shown. I believe that only through this type of presentation can the student obtain a clear understanding of the basic physical problem which will affeçt his comprehension of future problems.

In Chapters 8, 9, and 10, a systematic development of the flow of a viscous incompressible fluid is presented. Chapter 8 begins with the discussion of the significance of the Reynolds number which was obtained from the Navier-Stokes equation (derived in Chapter 5) by means of the law of similarity. Topics such as measurement of viscosity and hydrodynamics of bearing lubrication are introduced only after the explanations of the significance of the Reynolds number, viscosity and parallel flow theory are made. The laminar boundary layer theory is presented in Chapter 9 which begins with the discussion of the properties of the Navier-Stokes equations and the boundary layer concept. Chapter 10 presents an introduction to turbulent flow in which the Reynolds stress has been clearly explained with an example of the rate of momentum transport of fluid masses. The analogy between the mean free path in kinetic theory of gases (given in Chapter 8) and the mixing length in turbulence is clearly shown.

The last two chapters are devoted to the flow of compressible fluids. Chapter 11 deals with an inviscid compressible flow. The discussion begins with the four controlling parameters in compressible flow which are obtained from the energy equation (derived in Chapter 5) by means of the law of similarity. Chapter 12 is concerned with the flow of a viscous fluid. The governing equations of viscous compressible fluids (derived in Chapter 5) have been clearly applied here. The topic on steady flow through a constant-area pipe with friction is included in this chapter. The presentation of this topic in this chapter is believed to be both logical and orderly.

It is my conviction that topics such as dimensions and unit systems and dimensional analysis should be presented in a unified fashion. If this chapter were placed at the beginning of the text it could easily overwhelm most students by the strange parameters or numbers which actually belong to the latter part of the text. This is the very reason that the chapter on dimensional method in fluid mechanics is presented in an appendix so that it can be referred to whenever required. Both British and metric units are discussed in detail here and the significant "numbers" of different regimes in the fluid mechanics are obtained by dimensional analysis and are in complete agreement with results obtained from the similarity principle. A review of thermodynamic relations is also presented in an appendix to supplement the discussion of compressible flow theory given in Chapters 5, 11, and 12.

As the first course in solid mechanics is now being taught with vector methods and notations in many schools, these methods and notations therefore have been used consistently throughout the text. It is my belief that the vector analysis presentation gives a simple, clear, and suggestive exposition of the physical idea involved. A summary of vector analysis is given in Appendix A which may be helpful for those who have been little exposed to the subject as well as for those who wish to refer to vector identities and to the transformation formulae for orthogonal curvilinear coordinates. After a student has gained some basic knowledge of strength of material he should be able to follow the discussion in Chapter 4 without difficulty.

The material contained in this text can be covered in about six semester hours at junior-senior level or three semester hours in the first graduate course. A three-semester-hour terminal course could normally include most of the first four chapters plus portions of Chapters 5 through 7, with selected topics in Chapters 8 through 11. For those who are interested only in inviscid flow, Appendix D is specially prepared to replace Chapters 4 and 5. The instructor should select those topics in accordance with requirements of his class.

In preparing the manuscript, I have relied heavily on the works of authors who have written before me, particularly those authors whose writings are considered classics in this field. A list of selected references is given at the end of the text and I am indebted to many of these studies.

A number of my colleagues and friends have been patient enough to read portions of the manuscript and to offer valuable comments. In particular, I wish to take this opportunity to express my sincerest appreciation to Dr. J. O. Hinze of the Technological University, Delft, Dr. A. Miele of Rice University, Dr. M. Morduchow of the Polytechnic Institute of Brooklyn, Dr. P. H. Miller of the University of Missouri, Dr. R. W. Courter of the University of Wyoming, and Drs. L. R. Mack, D. G. Hull, J. W. Porter, and F. D. Masch of The University of Texas. I wish to heartily thank Drs. N. W. Newmark and W. J. Hall of the University of Illinois for their valuable comments and suggestions during the preparation of the manuscript. Mr. A. M. Bloom assisted the author with great devotion during the final preparation of the manuscript; his assistance in revising the manuscript and in proofreading was an essential help for the completion of the book. Finally, I greatly appreciate the efforts of Mrs. Georgia Courter, Mrs. Marie Kaak, and

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INTRODUCTION

1.1. General Description of Fluid Mechanics

Fluid mechanics, one of the oldest branches of physics and the foundation for the understanding of many other aspects of the applied sciences and engineering, concerns itself with the investigation of the motion and equilibrium of fluids. It is a subject of widespread interest in almost all fields of engineering as well as in astrophysics, biology, biomedicine, meteorology, physical chemistry, plasma physics, and geophysics. Since the nineteenth century when the study of hydraulics as a science was associated with the growth of the fields of civil engineering and naval architecture, the scope of fluid mechanics has steadily broadened. The development of aeronautical, chemical, and mechanical engineering during the past few decades on the one hand and the exploration of space in the past few years on the other, have given added stimuli to the study of fluid mechanics so that it now ranks as one of the most important basic subjects in engineering science.

The frontier of fluid dynamic research has been extended into the exotic regimes of hypervelocity flight and flow of electrically conducting fluids. This has introduced new fields of interest such as hypersonic flow and magnetofluidynamics. In this connection it has become necessary to combine a knowledge of thermodynamics, mass transfer, heat transfer, electromagnetic theory, and fluid mechanics to fully understand the physical phenomena involved.