

**Operations Research
Techniques for
Management**

Herbert Moskowitz and Gordon P. Wright

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Operations Research Techniques for Management

by Herbert Moskowitz and Gordon P. Wright

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—*Herbert Moskowitz and Gordon P. Wright*

Preface

Our goal, as both teachers and authors, is to provide our readers with a good understanding of basic operations research methodology. We have written this book for the reader who has only an algebra background.

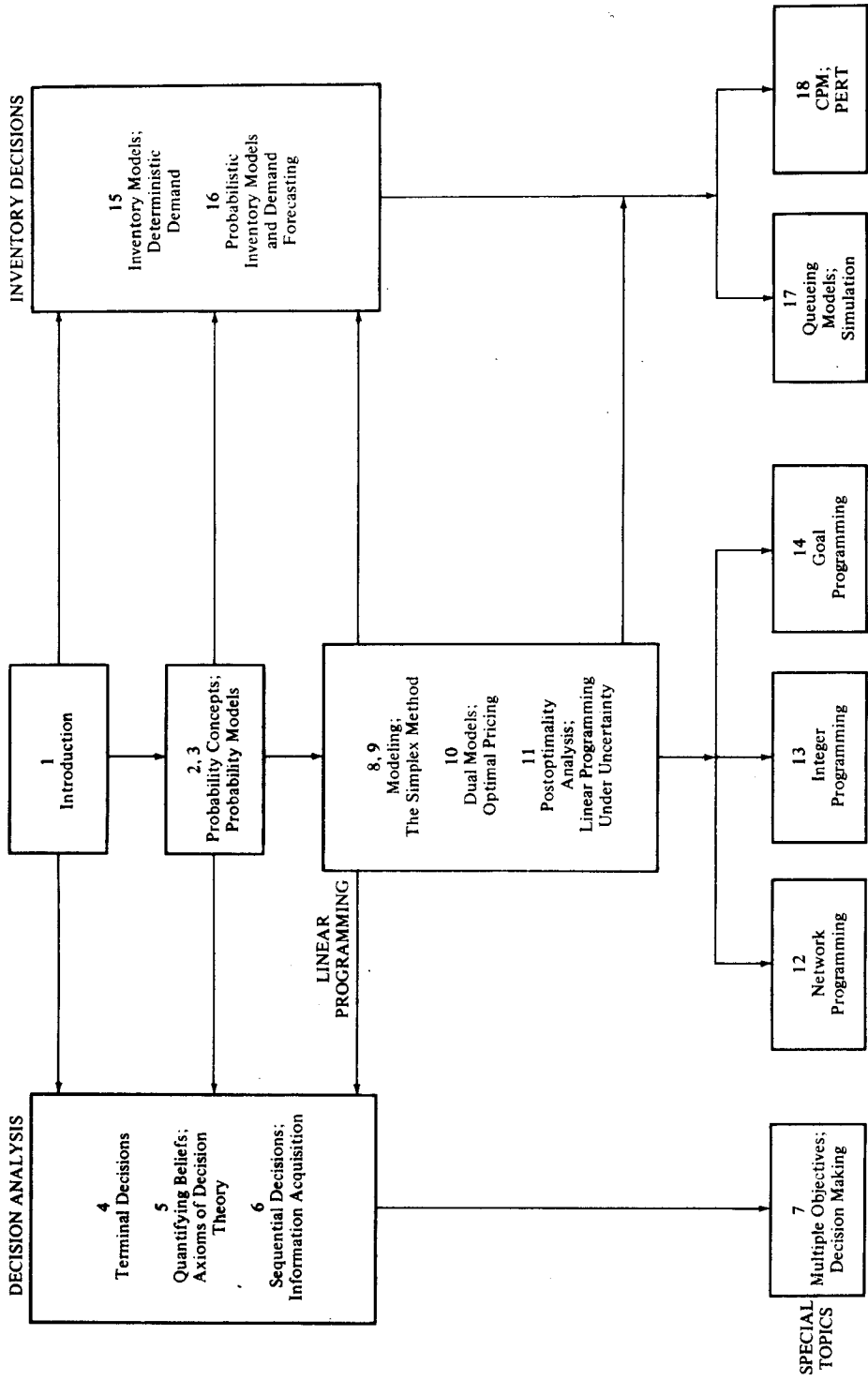
We have used three basic strategies to achieve our goal. First, to motivate the reader, we have tried as much as possible to make the material relevant to business problems; almost every chapter begins by citing activities or examples that relate to situations that are encountered by people in business. Then, as the chapter proceeds, numerous illustrations are included to maintain this relationship.

Our strategy has been to organize the book and individual chapters according to a simple instructional model. This model has the following components:

1. Specify what you want the student to understand.
2. Design instructional strategies to meet these goals.
3. Evaluate the degree of success achieved in meeting the objectives.

At the beginning of each chapter we have specified objectives to be attained by the readers. We believe that if the readers are successful in achieving these goals, they will obtain an important and adequate understanding of the material in a chapter. This is our third strategy.

To help the reader meet the goals that we have established, we have supplemented the text of each chapter with review questions and sometimes with review problems (most of which are analytical), with exercises, and with mini-cases. Each



section of review questions and review problems is followed by an answer section that allows our readers to check their work and get immediate feedback. The mini-cases will offer our readers various perspectives and will provide them with an opportunity for analysis and criticism.

The overall design of the book is modular as shown in the figure on page xiv. This allows for maximum flexibility in adapting the book to the objectives of a particular course in operations research. For example, Chapters 2 and 3 may be bypassed by readers who have had previous experience in statistics. Also, Chapters 2 and 3 may be omitted in a course that, for the most part, teaches the deterministic models of operations research.—*Herbert Moskowitz and Gordon P. Wright*

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PART I *Probabilty and
Decision Making*

CHAPTER 1

Introduction to Operations Research

OBJECTIVES. Upon completion of this chapter, you should be able to:

1. Have a perspective of what operations research is about and how it can improve managerial decision making.
2. Understand the concept of modeling, the different types of models that can be constructed, and the nature and development of a mathematical model.
3. Understand the five major steps in operations research: (a) problem definition, (b) model construction, (c) model solution, (d) model validation, and (e) implementation.
4. See how a linear programming and decision analysis model are constructed in two managerial applications.

1.1. MANAGERIAL DECISION MAKING AND THE NEED FOR OPERATIONS RESEARCH

This is a book about managerial decision making and how to make better managerial decisions. The decisions we are concerned with are complex, important decisions: those that require careful thought and deliberation in exercising the management function and those that would benefit from the use of mathematical models to aid the decision-making process.

We consider managerial decision making to be a process whereby a manager when confronted with a problem chooses a specific alternative course of action from a set of possible available courses of action. In most cases there is some uncertainty about the future, so that we cannot be sure of the ultimate consequences of the decision that is chosen. How do such (or any) decision situations arise? Decision making is a response to a decision problem, which generally arises as a result of a discrepancy between existing conditions and the manager's (and hopefully the organization's) goals or objectives.

1.1.1. Types of Decisions and Their Constituent Elements

Figure 1.1 illustrates several general types of decision situations that a manager might encounter. These situations include:

1. Decisions under *certainty* (where all the facts are known for sure) versus *uncertainty* [where the event that will occur (state of nature) is not known for sure but a probability or odds can be assigned to its possible occurrence].
2. *Static* decisions (decisions made once and only once) versus *dynamic* decisions (where a sequence of interrelated decisions are made either simultaneously or over several time periods).
3. Decisions where the opponent is *nature* (e.g., the weather, state of the economy) or a *thinking (rational)* opponent (developing a national energy policy where we have to consider the actions of OPEC).

Considering all possible combinations of these factors, there are eight general types of decision situations that a manager might face. For example, point 7 in Figure 1.1 represents a dynamic decision situation where there exists uncertainty, and nature is the opponent. Scheduling production and employment levels each month in the face of uncertain future sales is an illustration of such a decision situation.

The elements of any decision consist of:

1. A decision-making unit (individual, group, organization, or society).
2. A set of possible actions that may be taken to solve the decision problem.
3. A set of possible states that may occur.
4. A set of consequences associated with each possible action and state that may occur.
5. The relationship between the consequences and values of the decision-making unit.

Of course, in a real decision-making situation, definition and generation of the alternatives, states, and consequences are perhaps the most difficult, but nevertheless crucial, aspects of the decision problem.

1.1.2. What Makes Decision Making Complex?

Why do complex decision-making situations arise in the first place? One reason is that in our contemporary society, economic, political, technological, environmental, and competitive factors interact in a highly complicated fashion. For example, the selection of a site for a new plant must be based on the availability of the labor force, access to raw materials and markets, the attitude of the community, environmental and ecological considerations, the possibility of wage increases by indigenous companies in the area to avoid drainage of their work force, and so forth.