

Advances in Electronics and Electron Physics

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**EDITED BY
L. MARTON AND C. MARTON**



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Advances in Electronics and Electron Physics

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*Smithsonian Institution
Washington, D.C.*



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FOREWORD

The contributions in this volume serve as testimonials to the interdisciplinary nature of applied physics and engineering. The range of industrial applications of microwave radiation is covered by W. Schilz and B. Schiek. Radar today has value well beyond ranging and contour measurements.

With optical communications becoming more prominent, the need to integrate diverse technologies into a single system is well demonstrated. The solid-state technology of photodiodes in this context is dealt with by J. Müller.

A vital phase of solid-state work for electronic applications, heavy doping effects, is the subject of the chapter by R. P. Mertens, R. J. Van Overstraeten, and H. J. De Man.

Finally, two chapters are concerned with basic science, more so than the others presented here. However, the subject matter of these chapters is important for applied science. T. M. Miller presents the result of some of the studies using tunable lasers in atomic and molecular physics, and R. S. Symons and H. R. Jory discuss cyclotron resonance devices.

As is our custom, we present a list of articles to appear in future volumes of *Advances in Electronics and Electron Physics*.

Critical Reviews:

Large Molecules in Space	M. and G. Winnewisser
The Impact of Integrated Electronics in Medicine	J. D. Meindl
Electron Storage Rings	D. Trines
Radiation Damage in Semiconductors	N. D. Wilsey and J. W. Corbett
Spectroscopy of Electrons from High-Energy Atomic Collisions	D. Berényi
Solid Surfaces Analysis	M. H. Hgatsberger
Surface Analysis Using Charged Particle Beams	F. P. Viehböck, F. Rüdénauer, and P. Braun
Photovoltaic Effect	R. H. Bube
Electron Irradiation Effect in MOS Systems	J. N. Churchill, F. E. Holmstrom, and T. W. Collins
Light Valve Technology	J. Grinberg
High-Power Lasers	V. N. Smiley
Visualization of Single Heavy Atoms with the Electron Microscope	J. S. Wall
Spin-Polarized Low-Energy Electron Scattering	D. T. Pierce and R. J. Celotta
Defect Centers in III-IV Semiconductors	J. Schneider and V. Kaufmann
Atomic Frequency Standards	C. Audouin
Electron Scattering and Nuclear Structure	G. A. Peterson

Electrical Structure of the Middle Atmosphere	L. C. Hale
Microwave Superconducting Electronics	R. Adde
Diagnosis and Therapy Using Microwaves	M. Gautherie and A. Priou
Computer Microscopy Image Analysis of Biological Tissues Seen in the Light Microscope	E. M. Glaser
Collisional Detachment of Negative Ions	R. L. Champion
International Landing Systems for Aircraft	H. W. Redlien and R. J. Kelly
Ultrasensitive Detection	K. H. Purser
Radioastronomy in Millimeter Wavelengths	E. J. Blum
Low-Energy Atomic Beam Spectroscopy	E. M. Hörl and E. Semerad
History of Photoemission	W. E. Spicer
Power Switching Transistors	P. L. Hower
Radiation Technology	L. S. Birks
Diffraction of Neutral Atoms and Molecules from Crystalline Surfaces	G. Boato and P. Cantini
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Potential Calculation in Hall Plates	G. DeMey
Gamma-Ray Internal Conversion	O. Dragoun
CW Beam Annealing Process and Application for Superconducting Alloy Fabrication	J. F. Gibbons
Polarized Ion Sources	H. F. Glavish
Gunn-Hilsum Effect Electronics	H. L. Grubin and P. R. Solomon
Fiber Optics for Local Data Communications Applications	D. Hanson
High Field Effects in Semiconductor Devices	K. Hess
Digital Image Processing and Analysis	B. R. Hunt
Infrared Detector Arrays	D. Long and W. Scott
Energy Levels in Gallium Arsenide	A. G. Milnes
Polarized Electrons in Solid-State Physics	H. C. Siegmann, M. Erbudak, M. Landolt, and F. Meier
The Technical Development of the Shortwave Radio	E. Sivowitch
Chemical Trends of Deep Traps in Semiconductors	P. Vogl
Stimulated Cerenkov Radiation	J. W. Walsh
The Interactions of Measurement Principles, Interfaces and Microcomputers in Intelligent Instruments	W. G. Wolber
<i>Supplementary Volumes:</i>	
Nonsinusoidal Waves in Radio and Radar Communications	H. F. Harmuth
Microwave Field-Effect Transistors	J. Frey

Our sincere thanks to all of the authors for such splendid and valuable reviews.

L. MARTON
C. MARTON

CONTENTS

CONTRIBUTORS TO VOLUME 55	vii
FOREWORD	ix

Cyclotron Resonance Devices R. S. SYMONS AND H. R. JORY

I. Introduction	2
II. Gyro Device Theory	7
III. Gyrotron Oscillator Theory and Experiment	14
IV. Gyroklystron Theory and Experiment	39
V. Gyro-TWT Theory and Experiment	45
VI. Mode Competition, Space-Charge, and Velocity-Spread Effects	61
References	71

Heavy Doping Effects in Silicon ROBERT P. MERTENS, ROGER J. VAN OVERSTRAETEN, AND HUGO J. DE MAN

I. Introduction	77
II. Changes in the Electronic and Impurity Energy Levels for High Doping.	80
III. Measurement of the Band-Gap Narrowing in Heavily Doped Silicon	87
IV. Recombination and Minority Carrier Lifetime in Heavily Doped Silicon	95
V. Transport Equations in Heavily Doped Silicon	102
VI. Device Applications	106
VII. Conclusions.	115
References	117

Photodetachment and Photodissociation of Ions THOMAS M. MILLER

I. Introduction	119
II. Experimental Techniques	123
III. Photodetachment Results	146
IV. Photodissociation Results	169
V. Future Work	182
References	183

Photodiodes for Optical Communication
J. MÜLLER

I. Introduction	189
II. Physics of Photodiodes	192
III. Semiconductor Materials and Diode Designs	245
IV. Miscellaneous Photodiodes for Special Applications	287
V. Summary and Conclusions	302
References	304

Microwave Systems for Industrial Measurements
W. SCHILZ AND B. SCHIEK

I. Introduction	309
II. Physical Basis of Microwave Measuring Systems	312
III. Discussion of Selected Systems	331
IV. Conclusion	375
List of Symbols	376
References	377

AUTHOR INDEX.	383
SUBJECT INDEX.	394

Cyclotron Resonance Devices

R. S. SYMONS AND H. R. JORY

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I. Introduction	2
II. Gyro Device Theory	7
A. General	7
B. The Vlasov Equation	8
C. Guiding Center Equations of Motion	9
D. Motion in Fundamental Fields	12
E. Motion in Harmonic Fields	13
III. Gyrotron Oscillator Theory and Experiment	14
A. The Equivalent Circuit	14
B. Cavity Representation	14
C. Linear Theory of Gyrotrons	21
D. Nonlinear Gyrotron Theory	25
E. Oscillator Experiments	29
IV. Gyroklystron Theory and Experiment	39
A. General	39
B. Analytic Gyroklystron Theory	40
C. Nonlinear Gyroklystron Analysis	42
D. Gyroklystron Experiments	43
V. Gyro-TWT Theory and Experiment	45
A. Fast-Wave Interaction—General	45
B. Linear Theory of the Gyro TWT	46
C. Nonlinear Gyro-TWT Theory	50
D. Gyro-TWT Experiments	56
VI. Mode Competition, Space-Charge, and Velocity-Spread Effects	61
A. General	61
B. Mode Competition	62
C. The Weibel Instability	66
D. Space-Charge Effects	67
E. Velocity Spread	70
References	71

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I. INTRODUCTION

In the past twenty years, a new class of microwave tubes has emerged based upon the cyclotron resonance maser (CRM) instability. This class includes gyrotrons, gyroklystrons, and gyro traveling wave tubes. Their importance lies in their ability to produce more power at higher frequencies than other microwave tubes. Figure 1 shows a comparison of the maximum output powers available at various frequencies for the common types of microwave tubes and for gyro devices.

In a gyrotron, gyroklystron or gyro traveling wave tube (TWT) a beam of electrons having high transverse energy is formed in a magnetic field which produces a relativistic electron cyclotron frequency with a harmonic near the operating frequency of the tube. The beam then passes through one or more cavity resonators or a waveguide which will support a mode with electric field transverse to the beam. When the operating frequency of the device is near the electron cyclotron frequency, the orbits of the electrons are usually small enough so the oscillating electric field is essentially parallel and uniform over the orbit.

Figure 2a shows a number of electrons following a single helical path. Electrons which are traveling normal to the field when it is maximum and

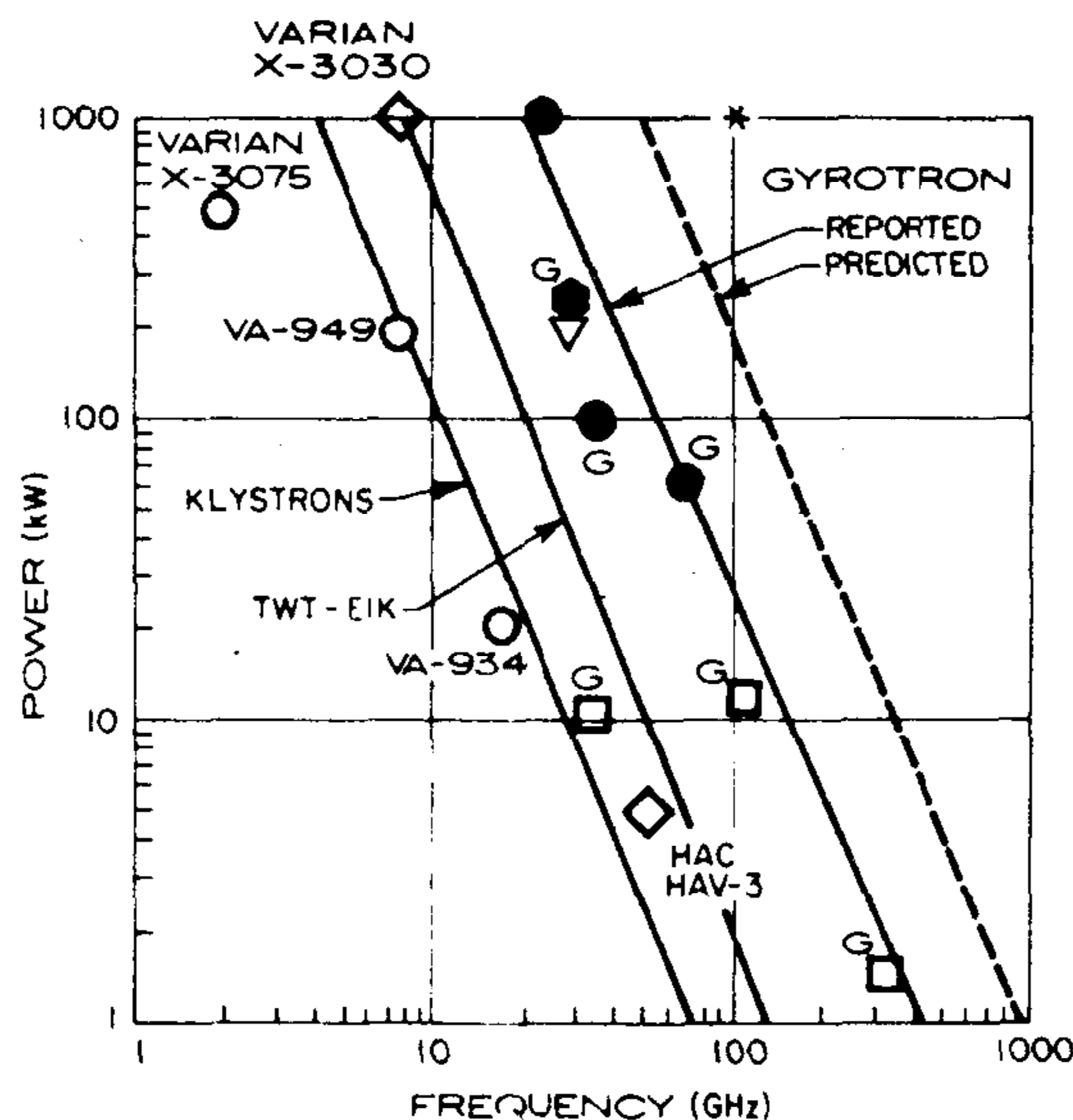


FIG. 1. Microwave tube power: (\square) published Russian CW gyrotron data; (\bullet) Russian millisecond pulsed gyrotrons; (\circ) klystrons; (\circ) TWTs and extended interaction klystrons; (\bullet) Varian millisecond pulsed gyrotron; (∇) Varian CW gyrotron; (*) Russian 100- μ sec pulsed gyrotron.

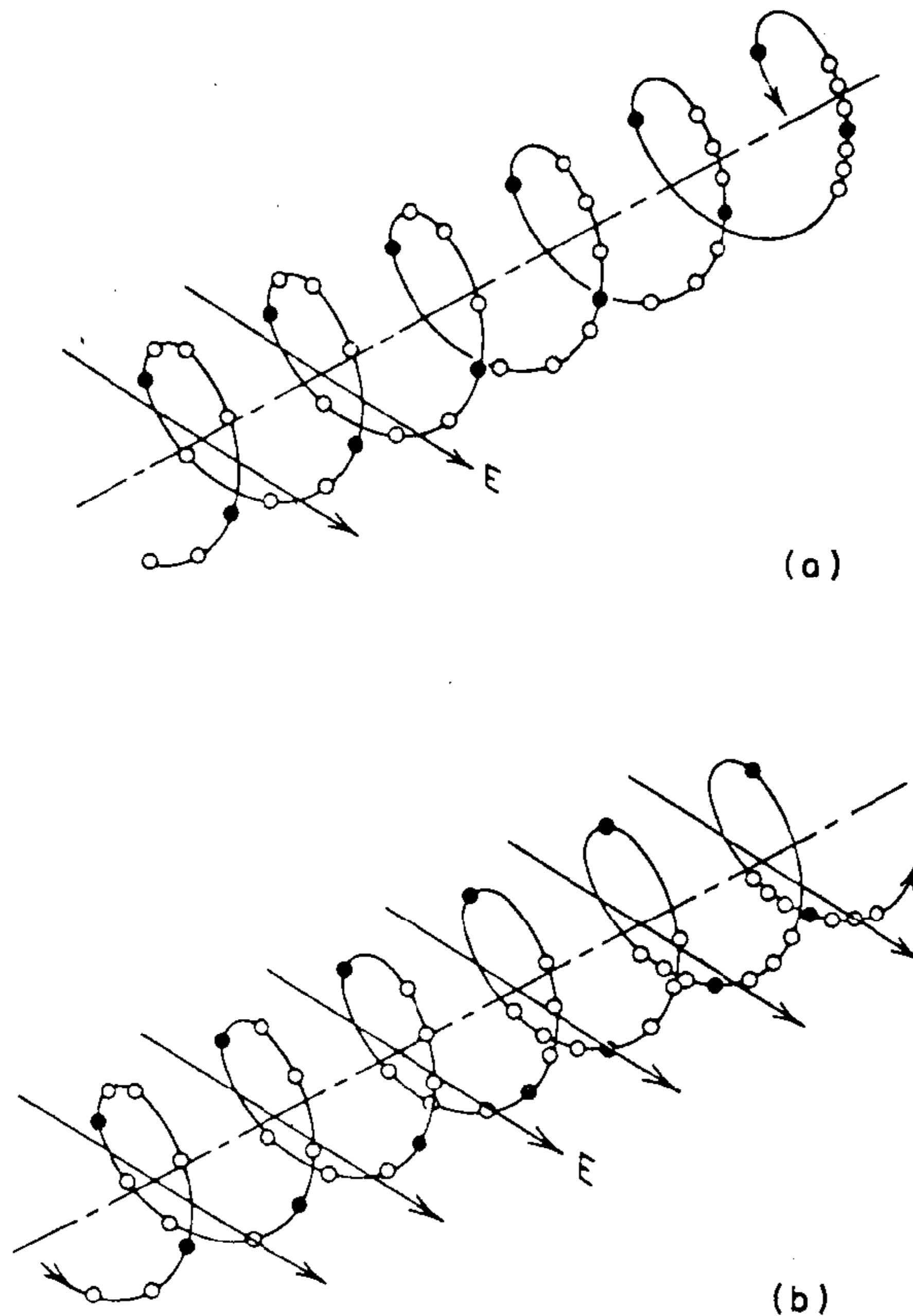


FIG. 2. Schematic representation of cyclotron bunching for $\Omega = \omega$ (a) and $\Omega < \omega$ (b).

do not interact are shown as solid dots. Electrons which interact with the field are shown as small circles. Initially, the electrons are distributed uniformly along the path. As the electrons pass through the region of oscillating electric field, there is a cumulative interaction between the oscillating field and the orbiting electrons. This produces a sinusoidal energy modulation of electrons along the path and a consequent angular velocity modulation because of the change in relativistic mass with energy. It is important to recognize that electrons which gain energy slow down in angular velocity, and electrons which lose energy speed up. The change in the radius of gyration is a small, one-time effect, but the angular velocity modulation produces continuing drifts. As a result, the electrons tend to form a rotating bunch of charge as shown in Fig. 2a. The figure shows the situation when the relativistic cyclotron frequency equals the operating frequency.

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If the cavity is sufficiently long and the cavity frequency exceeds the cyclotron frequency, the bunch falls back in phase so the electrons are decelerated by the electric field over the latter part of the interaction region. This situation, shown in Fig. 2b, causes a negative electronic conductance or oscillation. In a practical gyrotron or gyrokystron there are many such helical paths with both parallel and colinear axes which fill the region of high electric field in the cavity resonators. For this case, when there are many helical paths around one axis, it can be seen that the rotating bunches on the helices form a "rod" of charge rotating around the axis.

The spatial relation of the rotating charge "rods" on parallel axes is determined by the transverse mode pattern in the cavity resonator or waveguide. Figures 3a-c show schematically how the bunches form for hollow beams in circular waveguides supporting the TE_{11}^o , TE_{21}^o and TE_{01}^o modes, respectively. The circles represent projections of the helical electron orbits on a cross section perpendicular to the beam and waveguide axis. The dark shaded portion of each circle represents a cut through the rod-like bunch which is rotating around the helix axis. For the TE_{01}^o mode, Fig. 3c shows that the motion of all the charge "rods" in aggregate makes the bunched hollow beam rotate first to the left and then to the right while it grows and shrinks in diameter. For the TE_{11}^o case shown in Fig. 3a, the beam does not change in diameter, but moves in an eccentric fashion. Figure 3b shows a rotating cycloidal pattern for the TE_{21}^o mode.

In a gyro traveling wave tube, the electrons travel along helix systems

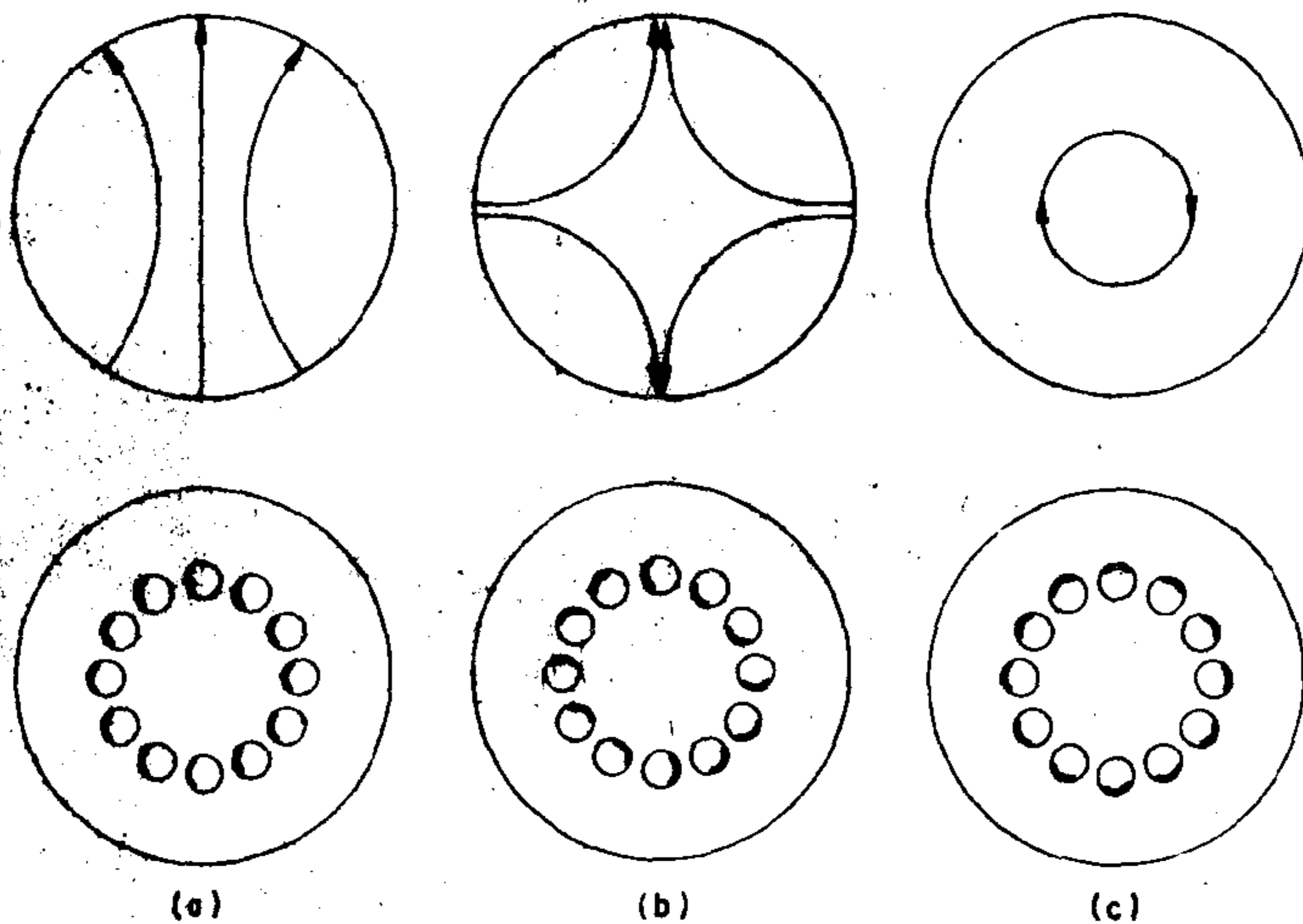


Fig. 3. Bunching for various waveguide modes: (a) TE_{11} ; (b) TE_{21} ; (c) TE_{01} .

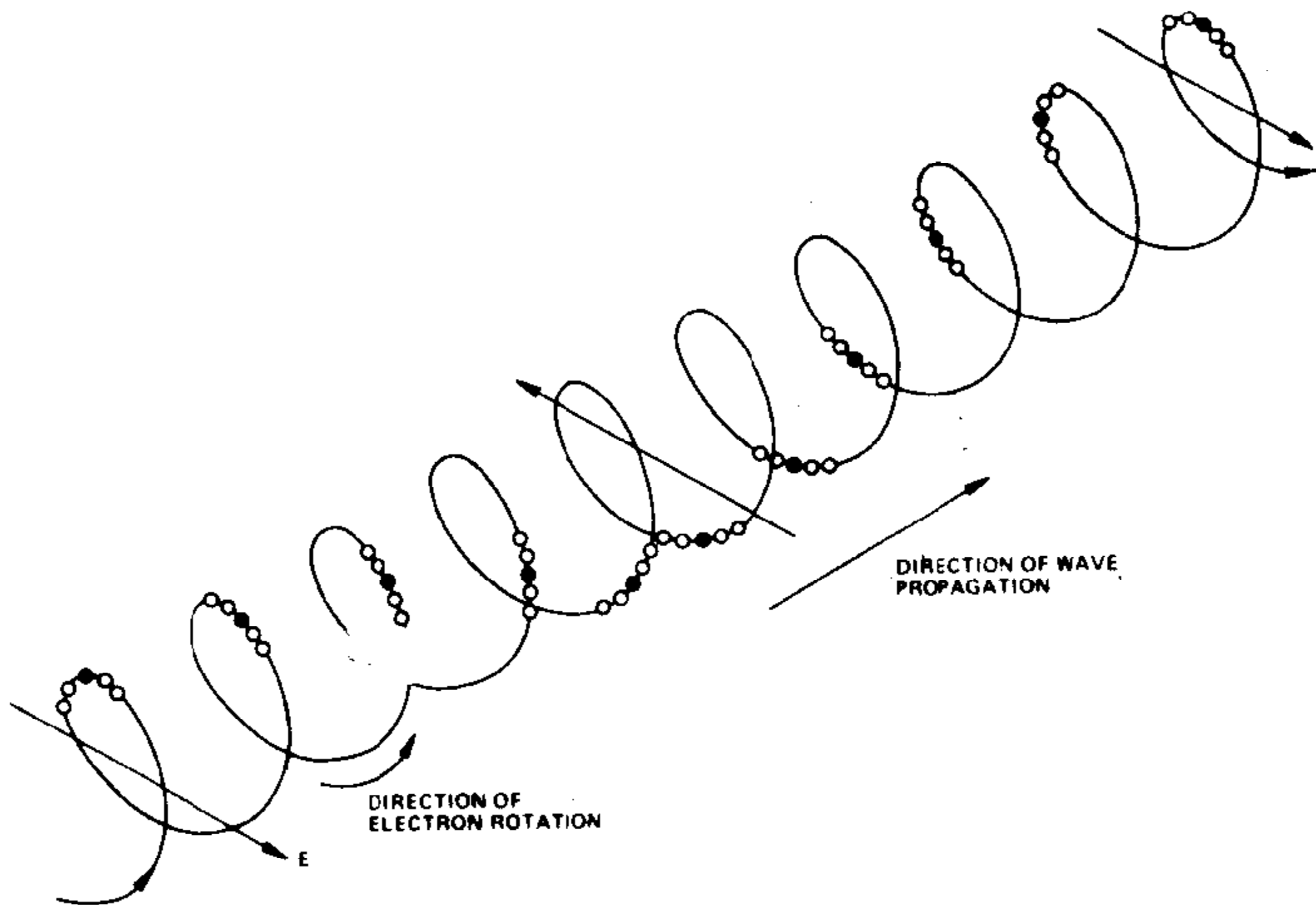


FIG. 4. Interaction of a helical beam and a traveling wave.

through the high electric field region of a waveguide with their axes parallel to the waveguide axis. In this case, again because the operating frequency is above the cyclotron frequency, the rod of charge formed by the bunching process twists around the helix system with a pitch substantially longer than the electron pitch, as shown in Fig. 4. As the electrons rotate and the fields propagate in the direction of the helix axis, the position where the electrons in the bunch move parallel to the electric field advances with a phase velocity which is in approximate synchronism with the phase velocity of the wave in the waveguide.

Gyro devices which operate at a harmonic of the electron cyclotron frequency utilize electromagnetic fields that are not parallel and uniform over the electron orbit. Rather, the electric and magnetic fields are sufficiently nonuniform for the instantaneous tangential force on the electron to have a finite average value over an electron cyclotron period. For example, Fig. 5a shows an electron orbiting in a field with a spatial reversal at the orbit center. If the electron advances to the position shown in Fig. 5b by the time the field has reversed, i.e., the electromagnetic field frequency ω is twice the electron cyclotron frequency Ω , the finite average condition is satisfied. This will result in a strong interaction at frequencies close to the second harmonic of the electron cyclotron frequency. In general, however, as the harmonic number becomes higher, the interaction will become weaker for the same strength of the electromagnetic fields in the cavity or the waveguide. Hence, the fundamental electron cyclotron resonance is preferable for high-average-power devices in which cavity or waveguide losses are a problem.

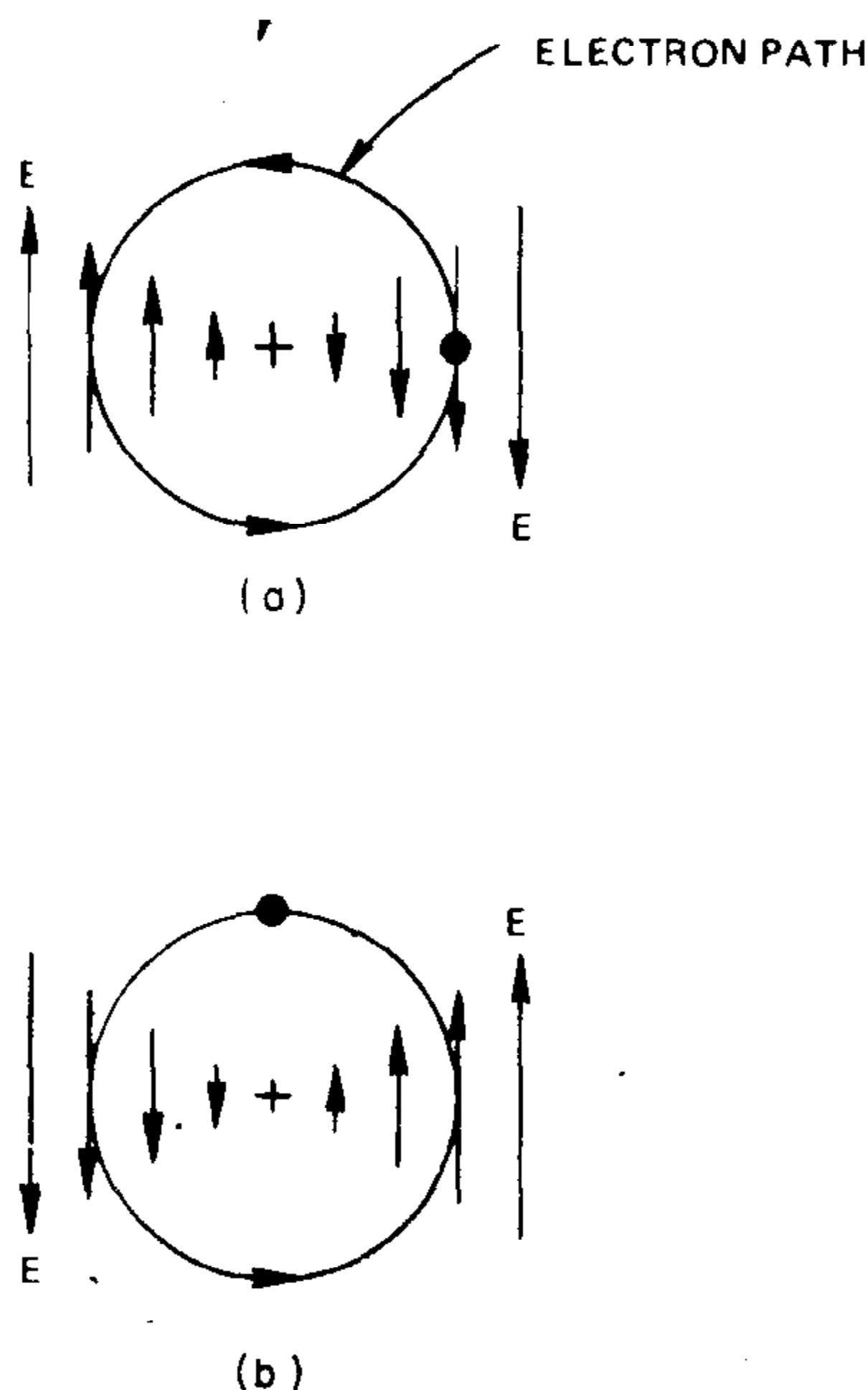


FIG. 5. Harmonic interaction of an electron and a field varying at twice the cyclotron frequency.

A large number of papers have been published on gyro devices. In any article on this subject it is appropriate to mention the seminal work of Twiss (1), Gaponov (2), and Schneider (3). They clearly understood the importance of relativistic mass change in the bunching process. This bunching mechanism is now sometimes called the cyclotron resonance maser (CRM) instability.

During the period between 1959 and 1964 other workers, including Pantell (4), Chow and Pantell (5), Bott (6, 7), and Feinstein (8), performed important experiments which produced very interesting results. Pantell, probably incorrectly, attributed the observed instability of the beam-wave system to longitudinal bunching caused by $\mathbf{v} \times \mathbf{B}$ forces. While such an instability exists and is now frequently referred to as the Weibel (9) instability, the CRM instability was most likely responsible for the results. Bott suggested both instabilities. Hirshfield and Wachtel (10) both observed the CRM instability and calculated its characteristics.

It is also important to recognize the contribution of the Russian experimentalists. Their work proved that electron guns of the magnetron injection type can produce electron beams with the requisite current and

transverse energy to make very high-power, millimeter-wave, gyro devices practical.

Recently there have been a number of excellent review papers, including ones by Andronov *et al.* (11), Hirshfield and Granatstein (12), and Flyagin *et al.* (13). These are well worth some study and provide additional reviews of the literature.

For every type of linear-beam microwave tube employing Langmuir wave or axial bunching, there is an analogous gyro device. In this article, however, the discussion is limited to those devices upon which there has been substantial work, i.e., gyrotron oscillators, gyroklystron amplifiers, and gyro TWT amplifiers. Following a section devoted to theory common to all gyro devices, there is a section on each kind of gyro device, which reviews specific theoretical and experimental work. After these, there is a section on mode competition, space charge, and velocity spread, which are problems in all gyro devices.

No extensive review of methods of forming electron beams for gyrotrons is included because the final design of these beams is usually determined by empirical procedures which make extensive use of computer modeling. However, the work of Tsimring (14) is useful in initial synthesis of magnetron injection guns as are the scaling laws which have been arrived at analytically and verified experimentally by Antakov *et al.* (15), Avdoshin *et al.* (16, 17) and others referenced in their works.

Early investigations of the CRM instability conducted by Bott (7) and Hirshfield and Wachtel (10) used beams in which the transverse energy was developed by passing a conventional laminar electron beam through a region in which the magnetic field lines were perturbed so as to be helical. This idea is due to Wingerson (18).

II. GYRO DEVICE THEORY

A. General

The analysis of gyro devices can be performed in three ways. One is to use the relativistic Vlasov equation or collisionless Boltzmann equation to find a perturbed electron distribution function resulting from the electromagnetic forces on the electrons, and thus, the rf beam current. (For example, Ott and Manheimer (19) have used this method in deriving dispersion relations for gyro TWTs.) A second is to use a Lagrangian formulation to find the rf beam current from the distortion of a charge element, as produced by the forces (see the analytic gyroklystron transconductance in Section IV). The third is to write an expression for the rate of change of energy of an electron as a result of the interaction of the electron with the fields [see, e.g.,

Roberts and Buchsbaum (20), Gaponov *et al.* (21), or Lindsay (22)]. For small field amplitudes, the implementation of these various techniques always involves the integration of a linearized equation for the property of interest along the unperturbed electron trajectory. For large fields, either the Lagrangian or particle energy formulation is used. The trajectories and/or energy changes are found by exact numerical integration of the equations of motion.

In any event, the starting point is always the Lorentz force equation

$$\frac{d}{dt}(\mathbf{p}) = \frac{d}{dt}(\gamma m_0 \mathbf{v}) = -|e|(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \quad (2.1)$$

in which e and m_0 are the electronic charge and rest mass, respectively, \mathbf{v} is the velocity of the electron, $\mathbf{p} = \gamma m_0 \mathbf{v}$ is the momentum, $\gamma = [1 - (v/c)^2]^{-1/2}$, \mathbf{E} and \mathbf{B} are the electric and magnetic fields, respectively, and $v = |\mathbf{v}|$.

B. The Vlasov Equation

When Eq. (2.1) is applied to a statistical distribution of electrons in which binary collisions can be neglected, the relativistic Vlasov equation,

$$W[\partial/\partial t + \mathbf{v} \cdot \nabla_r - |e|(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \nabla_p]f(\mathbf{r}, \mathbf{p}, t) = 0 \quad (2.2)$$

results. In this ∇_r and ∇_p are the divergence operators in position and momentum space, respectively, \mathbf{r} is a generalized position coordinate, and f is the distribution function for the electrons. If \mathbf{E} , \mathbf{B} , and f are expressed as the sum of an initial part plus a small, first-order perturbation, e.g., $f = f_0 + f^{(1)}$, then Eq. (2.2) when linearized becomes

$$[\partial/\partial t + \mathbf{v} \cdot \nabla_r - |e|(\mathbf{v} \times \mathbf{B}_0) \cdot \nabla_p]f^{(1)} = |e|[\mathbf{E}^{(1)} + \mathbf{v} \times \mathbf{B}^{(1)}] \cdot \nabla_p f_0 \quad (2.3)$$

This may be integrated along the unperturbed trajectories, giving the solution

$$f^{(1)}(\mathbf{r}, \mathbf{p}, t) = \int_{t_1}^t dt' |e| [\mathbf{E}^{(1)}(\mathbf{r}'t') + \mathbf{v}' \times \mathbf{B}^{(1)}(\mathbf{r}'t')] \cdot \nabla_p f_0 \quad (2.4)$$

in which t_1 is a time before the electrons entered the electromagnetic fields. The rf beam current density is then

$$\mathbf{i}^{(1)} = -|e| \int f^{(1)} \mathbf{v} d^3 \mathbf{p} \quad (2.5)$$

and the power transferred to the cavity or waveguide fields is

$$P = -\frac{1}{2\pi} \int_0^{2\pi} d(\omega t) \int \mathbf{i}^{(1)} \cdot \mathbf{E}^{(1)} d^3 \mathbf{r} \quad (2.6)$$

C. Guiding Center Equations of Motion

If it is desired to use a Lagrangian formulation or a particle energy formulation, while it is possible to resolve Eq. (2.1) into orthogonal components and proceed directly to equations for the electron trajectory or energy, it is frequently convenient and instructive (in terms of understanding the electron motions) to derive, from the Lorentz force equation, equations which describe the behavior of electrons in terms of their motions around a "guiding center."

One such guiding-center formulation, which is relativistically correct, has been used by Zhurakhovskiy (23) and Rapoport *et al.* (24). They have derived equations of motion for relativistic electrons in a strong, steady, uniform magnetic field and a perturbing rf electromagnetic field. These equations characterize the motion in terms of a velocity v_z in the direction of the magnetic field and a velocity v_t tangential to the projection of the electron path in a plane perpendicular to the magnetic field. Also used is an angular coordinate ($\Omega t + \Phi$).

These equations are particularly convenient for the analysis of gyrotrons and gyroklystrons because the energy of an electron can be found from the two velocities, v_t and v_z . Also, the angular coordinate Φ describes the angular velocity modulation fully.

Starting from the relativistic equations of motions in rectangular coordinates for electrons in time-varying electromagnetic fields, Zhurakhovskiy and Rapoport *et al.* find solutions which have the form

$$x + jy = X + jY + r \exp[j(\Omega t + \Phi)] \quad (2.7)$$

$$\dot{x} + j\dot{y} = v_x + jv_y = jv_t \exp[j(\Omega t + \Phi)] \quad (2.8)$$

$$r\Omega = v_t; \quad \dot{z} = v_z$$

by imposing the additional condition

$$\dot{X} + j\dot{Y} + (\dot{r} + jr\Phi) \exp[j(\Omega t + \Phi)] = 0$$

In Eqs. (2.7) and (2.8) x and y are the coordinates of the electron in the direction transverse to the steady magnetic field B_0 , and z is the coordinate in the direction of B_0 and the beam; X and Y are coordinates of the guiding center and r , v_t , v_z , and Φ are running values of electron helix radius, transverse velocity, parallel velocity, and phase with respect to Ωt , respectively; t is time, and Ω is the relativistic cyclotron frequency at $t = 0$ given by

$$\Omega = \frac{|e|B_0}{m_0} (1 - \beta_\perp^2 - \beta_\parallel^2)^{1/2} = \Omega_0 (1 - \beta_\perp^2 - \beta_\parallel^2)^{1/2} = \frac{\Omega_0}{\gamma_0} \quad (2.9)$$

where $\beta_\perp = v_\perp/c$ and $\beta_\parallel = v_\parallel/c$ are initial values; $\beta_t = v_t/c$ and $\beta_z = v_z/c$,