MECHANICAL VIBRATIONS

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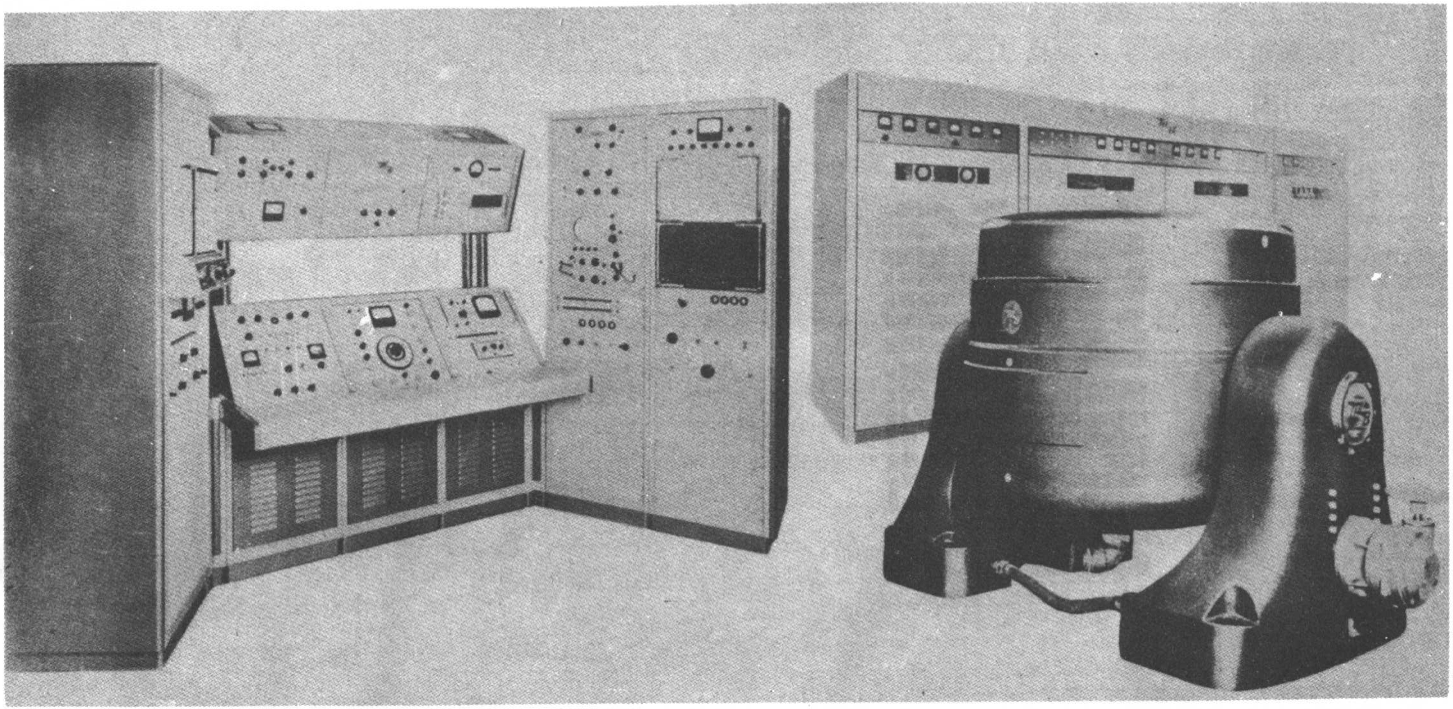
Allyn and Bacon, Inc. Boston, 1963

Library of Congress Catalog Card Number: 63-9257

Printed in the United States of America



Forced vibration test system. A 28,000 pound-force exciter is shown in the right foreground, control console on the left, and power supply unit in the background. (courtesy MB Electronics, A Division of Textron Electronics, Inc.)

PREFACE

The subject of vibration deals with the oscillatory motion of physical systems. The object of a vibration study is to determine the effect of vibrations on the performance and safety of the systems under consideration. The study of oscillatory motion is an important step toward this goal.

The purpose of this book is to present the fundamentals of vibration theory and to provide a background for advanced study in the field. No attempt has been made to cover all phases of vibrations, as the subject is very extensive.

This book is written primarily for mechanical engineering students of senior-year-college and beginning-graduate levels. The reader is assumed to have an elementary knowledge of dynamics, strength of materials, and differential equations. To provide adequate background, differential equations and other mathematical techniques used in the book are reviewed in the appendices.

The first three chapters constitute the core of an elementary terminal course. Chapter 1 describes the general concepts of vibration and simple harmonic motion. Chapter 2 treats systems having one degree of freedom through the study of a single second-order linear ordinary differential equation. The significance of each of the terms in this equation is explained. Then it is shown that this equation is applicable to the study of a large number of physical systems. The concept of vibration modes is introduced in Chapter 3. Although the discussion is primarily centered on the two-degree-of-freedom system, it gives the physical concepts and prepares the groundwork for studying multi-degree-of-freedom systems by other mathematical techniques.

The remainder of the book deals with more advanced topics. The Lagrange equations, introduced in Chapter 4, provide a potent tool for solving problems in vibrations. Chapters 5 and 6 illustrate additional techniques and practical applications. Chapter 7 uses matrix algebra to solve multi-degree-of-freedom systems. It may serve as a foundation for studying vibration problems on the digital computer. Mechanical transients and electromechanical systems are discussed in Chapter 8 by the use of the Laplace transform method. It is believed that transient study is sufficiently important to justify a separate chapter in a course in vibrations.

Chapter 9 describes the application of the electronic analogue computer in solving vibration problems, and it is regarded as an extension of Chapter 8 on analogue study. It is found that the operation of the computer can easily be grasped by mechanical engineering students, and it aids in their understanding of vibration phenomena. The use of the analogue computer as a tool is presented in some detail. Since laboratory work is essential for the understanding of the capabilities of the computer, a list of suggested experiments is included in Appendix A.

The material in this book has been used for a number of years by different instructors in Vibrations I and II taught at the Michigan State University. There is a laboratory section associated with each of these courses. Chapters 1 to 3 and part of Chapter 9 are being used for the first course, the remaining chapters for the second course. Chapter 9 is used principally for the laboratory sections. The analogue computer is used to supplement the vibration laboratory instead of the digital because students can fully participate in the experiments, only an elementary knowledge of electrical circuits is required, and small-size analogue computers are relatively inexpensive. Except for Chapters 1 to 3, the chapters are organized as independently as possible, but without repetition. Thus, after the first three chapters, the selection of topics can be very flexible.

To limit the scope of the book to systems described by linear ordinary differential equations, certain topics, such as vibration of a continuous medium, acoustics, and nonlinear vibrations, are purposely omitted. Nonlinear equations, however, are briefly discussed in Chapter 9.

No attempt has been made to compile a complete bibliography of the literature, which is very extensive. The authors, however, wish to acknowledge their indebtedness to the writers who have contributed to this field of study and to the authors of the texts listed as references. The authors are especially thankful to Dr. C. U. Ip for his valuable suggestions.

CONTENTS

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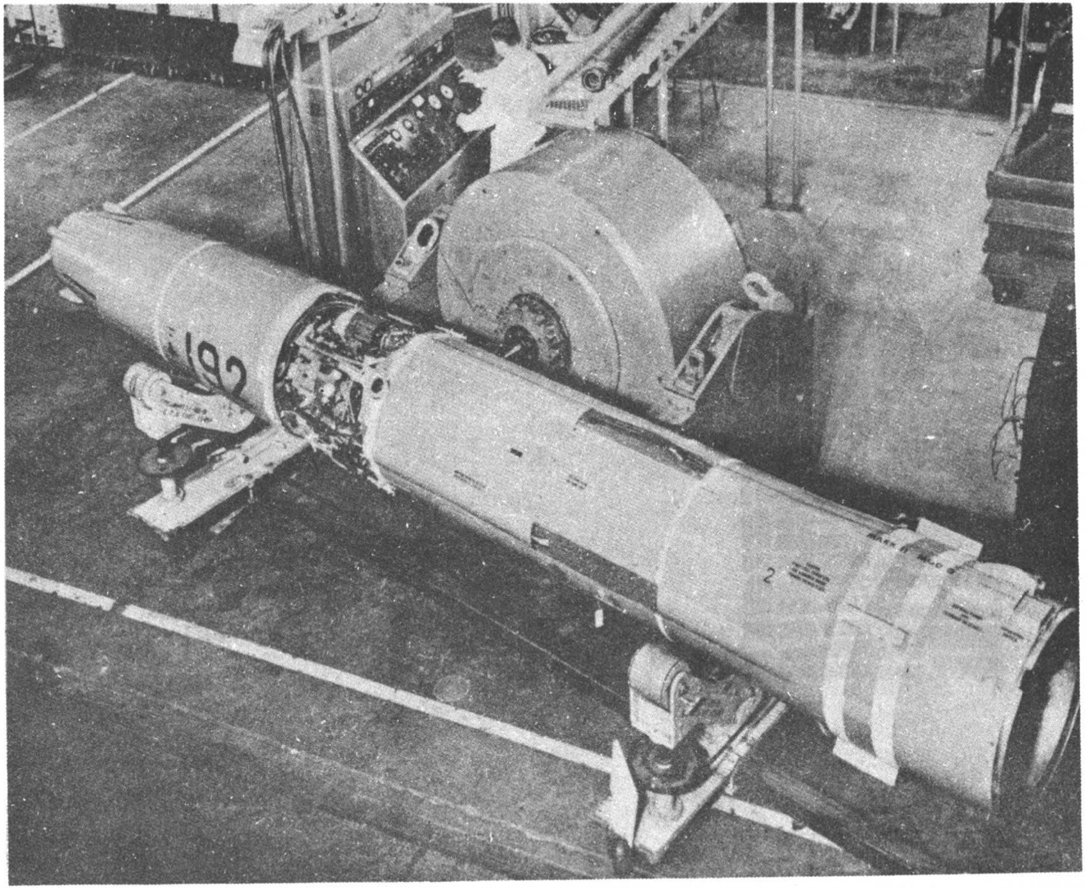

A 12,500 pound-force shaker performing lateral vibration test on a Talos missile (courtesy Bendix Products Aerospace Division, South Bend, Indiana)

1 INTRODUCTION

PRIMARY OBJECTIVES

The subject of vibration deals with the oscillatory motion of *dynamic systems*. A dynamic system is a combination of matter which possesses mass and whose parts are capable of relative motion. All bodies possessing mass and elasticity are capable of vibration. The mass is inherent in the body, and the elasticity is due to the relative motion of the parts of the body. The system considered may be in the form of a structure, a machine or its components, or a group of machines. The oscillatory motion of the system may be objectionable, trivial, or necessary for performing a task.

The objective of the designer is to control or minimize the vibration when it is objectionable and to utilize and enhance the vibration when it is desirable. Objectionable vibrations in a device may cause the loosening of parts or the malfunctioning or eventual failure of a machine. On the other hand, shakers in foundries and vibrators in testing machines require vibration. The proper functioning of many instruments depends on the proper control of the vibrational characteristics of the devices.

The primary objective of our study is to analyze the oscillatory motion of dynamic systems and the forces associated with the motion. It should be remembered that the ultimate goal in the study of vibration is to determine its effect on the performance and safety of the system under consideration. The analysis of the oscillatory motion is an important step toward this goal.

Our study begins with the description of the elements in a vibratory system, the introduction of some terminology, and a discussion of simple harmonic motion. These concepts will be used throughout the text. Other concepts and terminology will be introduced in the appropriate places as needed.

1-2. ELEMENTS OF A VIBRATORY SYSTEM

The elements that constitute a vibratory system are illustrated in Fig. 1-1. They are idealized and called (1) the *mass*, (2) the *spring*, (3) the *damper*, and (4) the *excitation* elements. The first three elements are the *parameters* descriptive of the physical system. For example, it can be said that the given system consists of a mass, a spring, and a damper arranged as shown in the figure. Energy may be stored in the mass and the spring and dissipated in the damper in the form of heat. These parameters are called the inactive or *passive elements*. To simplify the mathematics involved in the treatment of the subject, the passive elements are assumed to be *invariant with time*. Energy enters the physical system through the application of an excitation to the system. Hence the excitation is called an *active element*, and its magnitude varies according to a prescribed function of time. As shown in Fig. 1-1, an excitation force is applied externally to the system.

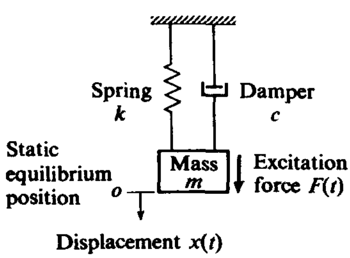

Fig. 1-1. *Elements of a vibratory system*

Furthermore, the parameters are assumed to be "lumped" together and are symbolized by the corresponding elements. Not all physical systems have *lumped parameters*. For example, a coil spring possesses both mass and elasticity. In order to consider it as a spring element, either its mass is assumed to be negligible or an appropriate portion of its mass is lumped together with the other masses of the system. A beam has its mass and elasticity inseparably distributed along its length. Hence the vibrational characteristics of a beam, or more generally of an elastic body, can be studied by this approach only if the elastic body is approximated by a finite number of lumped parameters. This method, however, is a practical approach to the study of some very complicated structures such as that of an aircraft.

In spite of the limitations, the lumped-parameter approach to the study of vibration problems is well justified for the following reasons: (1) Many physical systems are essentially lumped-parameter systems. (2) The concepts can be extended to analyze the vibration of elastic bodies. (3) Many physical systems are too complex to be investigated analytically as elastic bodies, and they are often studied through the use of their equivalent lumped-parameter systems. (4) The assumption of lumped parameters greatly simplifies the analytical effort required to obtain a solution.

The *mass* element is assumed to be a rigid body. It executes the vibrations and can gain or lose kinetic energy in accordance with the velocity change of the body. *Newton's law of motion* may be stated as follows: The product of the mass and its acceleration is equal to the force applied to the mass, and the acceleration takes place in the direction in which the force acts. *Work* is force times displacement in the direction of the force. The work done in changing the *kinetic energy* of a mass is *conserved*. The kinetic energy increases if work is positive and decreases if work is negative.

The *spring* element possesses elasticity and is assumed to be of negligible mass. A spring force exists only if the spring is deformed, such as the extension or the compression of a coil spring. Therefore the spring force exists if there is a *relative displacement* between the two ends of the spring. The work done in deforming a spring is *conserved* and is equal to the *strain energy* stored in the spring. This strain energy is often called the *potential energy*. A *linear spring* is one that obeys *Hooke's law*, that is, the spring force is proportional to the spring deformation. The constant of proportionality, measured in force per unit deformation, is called the spring constant k.

The *damping* element has neither mass nor elasticity. Damping force exists only if there is *relative motion* between the two ends of the damper. The work or energy input to a damper is converted into heat, and therefore the damping element is *nonconservative*. Many types of damping are encountered in engineering, and most of them are nonlinear. For example, the frictional drag of a body moving in a fluid is approximately proportional to the velocity squared, but the exact value of the exponent is dependent on many variables. *Coulomb* or dry friction damping is a function of the normal force between the bodies as well as the materials involved. The Coulomb damping force is generally assumed independent of the relative velocity between the sliding bodies. *Viscous damping*, in which the damping force is proportional to the velocity, is called linear damping. The mathematics for dealing with linear damping is relatively simple. Thus, viscous

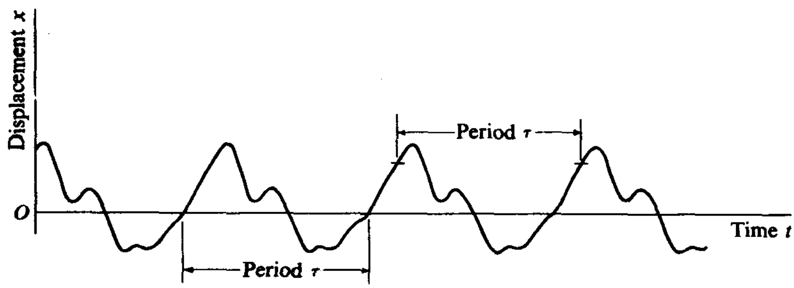

Fig. 1-2. *Periodic motion*

damping or its equivalent is generally assumed in engineering. The viscous-damping coefficient c is measured in force per unit velocity.

Energy enters a system through the application of an *excitation* to the system. Figure 1-1 shows an excitation force applied to the mass. The magnitude of the excitation varies in accordance with a prescribed function of time. Alternatively, if the system is suspended from a support, excitation may be applied to the system through imparting a prescribed motion to the support. In machinery, excitation often arises from the unbalance of the moving components. The vibrations of a dynamic system under the influence of an excitation are called *forced vibrations*. Forced vibrations, however, are often defined as the vibrations that are caused and maintained by a periodic excitation.

If the vibratory motion is *periodic*, the system repeats its motion at equal intervals of time, as shown in Fig. 1-2. The time required for the system to repeat its motion is called a *period* τ, which is the time required to complete one *cycle* of motion. Frequency f is the number