PEARSON
Addison Wesley

COMMEMORATIVE ISSUE

The Feynman LECTURES ON PHYSICS

FEYNMAN · LEIGHTON · SANDS

VOLUME II

费恩曼物理学讲义 第2卷

世界图书出版公司

COMMEMORATIVE ISSUE

The Feynman
LECTURES ON PHYSICS

FEYNMAN • LEIGHTON • SANDS

VOLUME II

书　　名：The Feynman Lectures on Physics Vol.2
作　　者：Feynman et al.
中 译 名：费恩曼物理学讲义 第2卷
出 版 者：世界图书出版公司北京公司
印 刷 者：北京世图印刷厂
发　　行：世界图书出版公司北京公司（北京朝内大街137号　100010）
联系电话：010-64015659, 64038347
电子信箱：kjsk@vip.sina.com
开　　本：大16　　印　张：36.75
出版年代：2004年11月
书　　号：7-5062-7248-2/O・517
版权登记：图字：01-2004-5397
定　　价：98.00元

Original English language title from Proprietor's edition of the Work.
Original English language title: The Feynman lectures on physics vol.2, Richard Feynman, Robert B. Leighton, Matthew L. Sands, copyright ©1965,1989

Published by arrangement with the original publisher, Pearson Education, Inc., publishing as Prentice Hall, Inc.

本书影印版由 Prentice Hall, Inc.和世界图书出版公司北京公司合作出版发行。

ISBN 0-201-51004-9

About Richard Feynman

Born in 1918 in Brooklyn, Richard P. Feynman received his Ph.D. from Princeton in 1942. Despite his youth, he played an important part in the Manhattan Project at Los Alamos during World War II. Subsequently, he taught at Cornell and at the California Institute of Technology. In 1965 he received the Nobel Prize in Physics, along with Sin-Itero Tomanaga and Julian Schwinger, for his work in quantum electrodynamics.

Dr. Feynman won his Nobel Prize for successfully resolving problems with the theory of quantum electrodynamics. He also created a mathematical theory that accounts for the phenomenon of superfluidity in liquid helium. Thereafter, with Murray Gell-Mann, he did fundamental work in the area of weak interactions such as beta decay. In later years Feynman played a key role in the development of quark theory by putting forward his parton model of high energy proton collision processes.

Beyond these achievements, Dr. Feynman introduced basic new computational techniques and notations into physics, above all, the ubiquitous Feynman diagrams that, perhaps more than any other formalism in recent scientific history, have changed the way in which basic physical processes are conceptualized and calculated.

Feynman was a remarkably effective educator. Of all his numerous awards, he was especially proud of the Oersted Medal for Teaching which he won in 1972. *The Feynman Lectures on Physics,* originally published in 1963, were described by a reviewer in Scientific American as "tough, but nourishing and full of flavor. After 25 years it is *the* guide for teachers and for the best of beginning students." In order to increase the understanding of physics among the lay public, Dr. Feynman wrote *The Character of Physical Law* and *Q. E. D.: The Strange Theory of Light and Matter.* He also authored a number of advanced publications that have become classic references and textbooks for researchers and students.

Richard Feynman was a constructive public man. His work on the Challenger commission is well known, especially his famous demonstration of the susceptibility of the O-rings to cold, an elegant experiment which required nothing more than a glass of ice water. Less well known were Dr. Feynman's efforts on the California State Curriculum Committee in the 1960's where he protested the mediocrity of textbooks.

A recital of Richard Feynman's myriad scientific and educational accomplishments cannot adequately capture the essence of the man. As any reader of even his most technical publications knows, Feynman's lively and multi-sided personality shines through all his work. Besides being a physicist, he was at various times a repairer of radios, a picker of locks, an artist, a dancer, a bongo player, and even a decipherer of Mayan Hieroglyphics. Pérpetually curious about his world, he was an exemplary empiricist.

Richard Feynman died on February 15, 1988, in Los Angeles.

The Feynman Lectures on Physics, Special Preface

Toward the end of his life, Richard Feynman's fame had transcended the confines of the scientific community. His exploits as a member of the commission investigating the space shuttle Challenger disaster gave him widespread exposure; similarly, a best-selling book about his picaresque adventures made him a folk hero almost of the proportions of Albert Einstein. But back in 1961, even before his Nobel Prize increased his visibility to the general public, Feynman was more than merely famous among members of the scientific community—he was legendary. Undoubtedly, the extraordinary power of his teaching helped spread and enrich the legend of Richard Feynman.

He was a truly great teacher, perhaps the greatest of his era and ours. For Feynman, the lecture hall was a theater, and the lecturer a performer, responsible for providing drama and fireworks as well as facts and figures. He would prowl about the front of a classroom, arms waving, "the impossible combination of theoretical physicist and circus barker, all body motion and sound effects," wrote the New York Times. Whether he addressed an audience of students, colleagues, or the general public, for those lucky enough to see Feynman lecture in person, the experience was usually unconventional and always unforgettable, like the man himself.

He was the master of high drama, adept at riveting the attention of every lecture hall audience. Many years ago, he taught a course in Advanced Quantum Mechanics, a large class comprised of a few registered graduate students and most of the Caltech physics faculty. During one lecture, Feynman started explaining how to represent certain complicated integrals diagrammatically: time on this axis, space on that axis, wiggly line for this straight line, etc. Having described what is known to the world of physics as a Feynman diagram, he turned around to face the class, grinning wickedly. "And this is called *THE* diagram!" Feynman had reached the denouement, and the lecture hall erupted with spontaneous applause.

For many years after the lectures that make up this book were given, Feynman was an occasional guest lecturer for Caltech's freshman physics course. Naturally, his appearances had to be kept secret so there would be room left in the hall for the registered students. At one such lecture the subject was curved-space time, and Feynman was characteristically brilliant. But the unforgettable moment came at the beginning of the lecture. The supernova of 1987 has just been discovered, and Feynman was very excited about it. He said, "Tycho Brahe had his supernova, and Kepler had his. Then there weren't any for 400 years. But now I have mine." The class fell silent, and Feynman continued on. "There are 10^{11} stars in the galaxy. That used to be a *huge* number. But it's only a hundred billion. It's less than the national deficit! We used to call them astronomical numbers. Now we should call them economical numbers." The class dissolved in laughter, and Feynman, having captured his audience, went on with his lecture.

Showmanship aside, Feynman's pedagogical technique was simple. A summation of his teaching philosophy was found among his papers in the Caltech archives, in a note he had scribbled to himself while in Brazil in 1952:

> "First figure out why you want the students to learn the subject and what you want them to know, and the method will result more or less by common sense."

What came to Feynman by "common sense" were often brilliant twists that perfectly captured the essence of his point. Once, during a public lecture, he was trying to explain why one must not verify an idea using the same data that suggested the idea in the first place. Seeming to wander off the subject, Feynman began talking about license plates. "You know, the most amazing thing happened to me tonight. I was coming here, on the way to the lecture, and I came in through the parking lot. And you won't believe what happened. I saw a car with the license plate ARW 357. Can you imagine? Of all the millions of license plates in the state, what was the chance that I would see that particular one tonight? Amazing!" A point that even many scientists fail to grasp was made clear through Feynman's remarkable "common sense."

In 35 years at Caltech (from 1952 to 1987), Feynman was listed as teacher of record for 34 courses. Twenty-five of them were advanced graduate courses, strictly limited to graduate students, unless undergraduates asked permission to take them (they often did, and permission was nearly always granted). The rest were mainly introductory graduate courses. Only once did Feynman teach courses purely for undergraduates, and that was the celebrated occasion in the academic years 1961 - 1962 and 1962 - 1963, with a brief reprise in 1964, when he gave the lectures that were to become *The Feynman Lectures on Physics.*

At the time there was a consensus at Caltech that freshman and sophomore students were getting turned off rather than spurred on by their two years of compulsory physics. To remedy the situation, Feynman was asked to design a series of lectures to be given to the students over the course of two years, first to freshmen, and then to the same class as sophomores. When he agreed, it was immediately decided that the lectures should be transcribed for publication. That job turned out to be far more difficult than anyone had imagined. Turning out publishable books required a tremendous amount of work on the part of his colleagues, as well as Feynman himself, who did the final editing of every chapter.

And the nuts and bolts of running a course had to be addressed. This task was greatly complicated by the fact that Feynman had only a vague outline of what he wanted to cover. This meant that no one knew what Feynman would say until he stood in front of a lecture hall filled with students and said it. The Caltech professors who assisted him would then scramble as best they could to handle mundane details, such as making up homework problems.

Why did Feynman devote more than two years to revolutionize the way beginning physics was taught? One can only speculate, but there were probably three basic reasons. One is that he loved to have an audience, and this gave him a bigger theater than he usually had in graduate courses. The second was that he genuinely cared about students, and he simply thought that teaching freshmen was an important thing to do. The third and perhaps most important reason was the sheer challenge of reformulating physics, as he understood it, so that it could be presented to young students. This was his specialty, and was the standard by which he measured whether something was really understood. Feynman was once asked by a Caltech faculty member to explain why spin 1/2 particles obey Fermi-Dirac statistics. He gauged his audience perfectly and said, "I'll prepare a freshman lecture on it." But a few days later he returned and said, "You know, I couldn't do it. I couldn't reduce it to the freshman level. That means we really don't understand it."

This specialty of reducing deep ideas to simple, understandable terms is evident throughout *The Feynman Lectures on Physics,* but nowhere more so than in his treatment of quantum mechanics. To aficionados, what he has done is clear. He has presented, to beginning students, the path integral method, the technique of his own devising that allowed him to solve some of the most profound problems in physics. His own work using path integrals, among other achievements, led to the 1965 Nobel Prize that he shared with Julian Schwinger and Sin-Itero Tomanaga.

Through the distant veil of memory, many of the students and faculty attending the lectures have said that having two years of physics with Feynman was the experience of a lifetime. But that's not how it seemed at the time. Many of the students dreaded the class, and as the course wore on, attendance by the registered students started dropping alarmingly. But at the same time, more and more faculty and graduate students started attending. The room stayed full, and Feynman may never have known he was losing some of his intended audience. But even in Feynman's view, his pedagogical endeavor did not succeed. He wrote in the 1963 preface to the *Lectures:* "I don't think I did very well by the students." Rereading the books, one sometimes seems to catch Feynman looking over his shoulder, not at his young audience, but directly at his colleagues, saying, "Look at that! Look how I finessed that point! Wasn't that clever?" But even when he thought he was explaining things lucidly to freshmen or sophomores, it was not really they who were able to benefit most from what he was doing. It was his peers—scientists, physicists and professors—who would be the main beneficiaries of his magnificent achievement, which was nothing less than to see physics through the fresh and dynamic perspective of Richard Feynman.

Feynman was more than a great teacher. His gift was that he was an extraordinary teacher of teachers. If the purpose in giving *The Feynman Lectures on Physics* was to prepare a roomful of undergraduate students to solve examination problems in physics, he cannot be said to have succeeded particularly well. Moreover, if the intent was for the books to serve as introductory college textbooks, he cannot be said to have achieved his goal. Nevertheless, the books have been translated into 10 foreign languages and are available in four bilingual editions. Feynman himself believed that his most important contribution to physics would not be QED, or the theory of superfluid helium, or polarons, or partons. His foremost contribution would be the three red books of *The Feynman Lectures on Physics*. That belief fully justifies this commemorative issue of these celebrated books.

David L. Goodstein
Gerry Neugebauer
California Institute of Technology

April 1989

Contents

Chapter 1. Electromagnetism

1–1 Electrical forces 1–1
1–2 Electric and magnetic fields 1–3
1–3 Characteristics of vector fields 1–4
1–4 The laws of electromagnetism 1–5
1–5 What are the fields? 1–9
1–6 Electromagnetism in science and technology 1–10

Chapter 2. Differential Calculus of Vector Fields

2–1 Understanding physics 2–1
2–2 Scalar and vector fields—T and h 2–2
2–3 Derivatives of fields—the gradient 2–4
2–4 The operator ∇ 2–6
2–5 Operations with ∇ 2–7
2–6 The differential equation of heat flow 2–8
2–7 Second derivatives of vector fields 2–9
2–8 Pitfalls 2–11

Chapter 3. Vector Integral Calculus

3–1 Vector integrals; the line integral of $\nabla\psi$ 3–1
3–2 The flux of a vector field 3–2
3–3 The flux from a cube; Gauss' theorem 3–4
3–4 Heat conduction; the diffusion equation 3–6
3–5 The circulation of a vector field 3–8
3–6 The circulation around a square; Stokes' theorem 3–9
3–7 Curl-free and divergence-free fields 3–10
3–8 Summary 3–11

Chapter 4. Electrostatics

4–1 Statics 4–1
4–2 Coulomb's law; superposition 4–2
4–3 Electric potential 4–4
4–4 $E = -\nabla\phi$ 4–6
4–5 The flux of E 4–7
4–6 Gauss' law; divergence of E 4–9
4–7 Field of a sphere of charge 4–10
4–8 Field lines; equipotential surfaces 4–11

Chapter 5. Application of Gauss' Law

5–1 Electrostatics is Gauss's law plus . . . 5–1
5–2 Equilibrium in an electrostatic field 5–1
5–3 Equilibrium with conductors 5–2
5–4 Stability of atoms 5–3
5–5 The field of a line charge 5–3
5–6 A sheet of charge; two sheets 5–4
5–7 A sphere of charge; a spherical shell 5–4
5–8 Is the field of a point charge exactly $1/r^2$? 5–5
5–9 The fields of a conductor 5–7
5–10 The field in a cavity of a conductor 5–8

Chapter 6. The Electric Field in Various Circumstances

6–1 Equations of the electrostatic potential 6–1
6–2 The electric dipole 6–2
6–3 Remarks on vector equations 6–4
6–4 The dipole potential as a gradient 6–4
6–5 The dipole approximation for an arbitrary distribution 6–6
6–6 The fields of charged conductors 6–8
6–7 The method of images 6–8
6–8 A point charge near a conducting plane 6–9
6–9 A point charge near a conducting sphere 6–10
6–10 Condensers; parallel plates 6–11
6–11 High-voltage breakdown 6–13
6–12 The field-emission microscope 6–14

Chapter 7. The Electric Field in Various Circumstances (Continued)

7–1 Methods for finding the electrostatic field 7–1
7–2 Two-dimensional fields; functions of the complex variable 7–2
7–3 Plasma oscillations 7–5
7–4 Colloidal particles in an electrolyte 7–8
7–5 The electrostatic field of a grid 7–10

Chapter 8. Electrostatic Energy

8–1 The electrostatic energy of charges. A uniform sphere 8–1
8–2 The energy of a condenser. Forces on charged conductors 8–2
8–3 The electrostatic energy of an ionic crystal 8–4
8–4 Electrostatic energy in nuclei 8–6
8–5 Energy in the electrostatic field 8–9
8–6 The energy of a point charge 8–12

Chapter 9. Electricity in the Atmosphere

9–1 The electric potential gradient of the atmosphere 9–1
9–2 Electric currents in the atmosphere 9–2
9–3 Origin of the atmospheric currents 9–4
9–4 Thunderstorms 9–5
9–5 The mechanism of charge separation 9–7
9–6 Lightning 9–10

Chapter 10. Dielectrics

10–1 The dielectric constant 10–1
10–2 The polarization vector P 10–2
10–3 Polarization charges 10–3
10–4 The electrostatic equations with dielectrics 10–6
10–5 Fields and forces with dielectrics 10–7

CHAPTER 11. INSIDE DIELECTRICS

11–1 Molecular dipoles 11–1
11–2 Electronic polarization 11–1
11–3 Polar molecules; orientation polarization 11–3
11–4 Electric fields in cavities of a dielectric 11–5
11–5 The dielectric constant of liquids; the Clausius-Mossotti equation 11–6
11–6 Solid dielectrics 11–8
11–7 Ferroelectricity; $BaTiO_3$ 11–8

CHAPTER 12. ELECTROSTATIC ANALOGS

12–1 The same equations have the same solutions 12–1
12–2 The flow of heat; a point source near an infinite plane boundary 12–2
12–3 The stretched membrane 12–5
12–4 The diffusion of neutrons; a uniform spherical source in a homogeneous medium 12–6
12–5 Irrotational fluid flow; the flow past a sphere 12–8
12–6 Illumination; the uniform lighting of a plane 12–10
12–7 The "underlying unity" of nature 12–12

CHAPTER 13. MAGNETOSTATICS

13–1 The magnetic field 13–1
13–2 Electric current; the conservation of charge 13–1
13–3 The magnetic force on a current 13–2
13–4 The magnetic field of steady currents; Ampere's law 13–3
13–5 The magnetic field of a straight wire and of a solenoid; atomic currents 13–5
13–6 The relativity of magnetic and electric fields 13–6
13–7 The transformation of currents and charges 13–11
13–8 Superposition; the right-hand rule 13–11

CHAPTER 14. THE MAGNETIC FIELD IN VARIOUS SITUATIONS

14–1 The vector potential 14–1
14–2 The vector potential of known currents 14–3
14–3 A straight wire 14–4
14–4 A long solenoid 14–5
14–5 The field of a small loop; the magnetic dipole 14–7
14–6 The vector potential of a circuit 14–8
14–7 The law of Biot and Savart 14–9

CHAPTER 15. THE VECTOR POTENTIAL

15–1 The forces on a current loop; energy of a dipole 15–1
15–2 Mechanical and electrical energies 15–3
15–3 The energy of steady currents 15–6
15–4 $\boldsymbol{B}$ versus $\boldsymbol{A}$ 15–7
15–5 The vector potential and quantum mechanics 15–8
15–6 What is true for statics is false for dynamics 15–14

CHAPTER 16. INDUCED CURRENTS

16–1 Motors and generators 16–1
16–2 Transformers and inductances 16–4
16–3 Forces on induced currents 16–5
16–4 Electrical technology 16–8

CHAPTER 17. THE LAWS OF INDUCTION

17–1 The physics of induction 17–1
17–2 Exceptions to the "flux rule" 17–2
17–3 Particle acceleration by an induced electric field; the betatron 17–3
17–4 A paradox 17–5
17–5 Alternating-current generator 17–6
17–6 Mutual inductance 17–9
17–7 Self-inductance 17–11
17–8 Inductance and magnetic energy 17–12

CHAPTER 18. THE MAXWELL EQUATIONS

18–1 Maxwell's equations 18–1
18–2 How the new term works 18–3
18–3 All of classical physics 18–5
18–4 A travelling field 18–5
18–5 The speed of light 18–8
18–6 Solving Maxwell's equations; the potentials and the wave equation 18–9

CHAPTER 19. THE PRINCIPLE OF LEAST ACTION

A special lecture—almost verbatim 19–1
A note added after the lecture 19–14

CHAPTER 20. SOLUTIONS OF MAXWELL'S EQUATIONS IN FREE SPACE

20–1 Waves in free space; plane waves 20–1
20–2 Three-dimensional waves 20–8
20–3 Scientific imagination 20–9
20–4 Spherical waves 20–12

CHAPTER 21. SOLUTIONS OF MAXWELL'S EQUATIONS WITH CURRENTS AND CHARGES

21–1 Light and electromagnetic waves 21–1
21–2 Spherical waves from a point source 21–2
21–3 The general solution of Maxwell's equations 21–4
21–4 The fields of an oscillating dipole 21–5
21–5 The potentials of a moving charge; the general solution of Liénard and Wiechert 21–9
21–6 The potentials for a charge moving with constant velocity; the Lorentz formula 21–12

CHAPTER 22. AC CIRCUITS

22–1 Impedances 22–1
22–2 Generators 22–5
22–3 Networks of ideal elements; Kirchhoff's rules 22–7
22–4 Equivalent circuits 22–10
22–5 Energy 22–11
22–6 A ladder network 22–12
22–7 Filters 22–14
22–8 Other circuit elements 22–16

CHAPTER 23. CAVITY RESONATORS

23–1 Real circuit elements 23–1
23–2 A capacitor at high frequencies 23–2
23–3 A resonant cavity 23–6
23–4 Cavity modes 23–9
23–5 Cavities and resonant circuits 23–10

CHAPTER 24. WAVEGUIDES

24–1 The transmission line 24–1
24–2 The rectangular waveguide 24–4
24–3 The cutoff frequency 24–6
24–4 The speed of the guided waves 24–7
24–5 Observing guided waves 24–7
24–6 Waveguide plumbing 24–8
24–7 Waveguide modes 24–10
24–8 Another way of looking at the guided waves 24–10

CHAPTER 25. ELECTRODYNAMICS IN RELATIVISTIC NOTATION

25–1 Four-vectors 25–1
25–2 The scalar product 25–3
25–3 The four-dimensional gradient 25–6
25–4 Electrodynamics in four-dimensional notation 25–8
25–5 The four-potential of a moving charge 25–9
25–6 The invariance of the equations of electrodynamics 25–10

CHAPTER 26. LORENTZ TRANSFORMATIONS OF THE FIELDS

26–1 The four-potential of a moving charge 26–1
26–2 The fields of a point charge with a constant velocity 26–2
26–3 Relativistic transformation of the fields 26–5
26–4 The equations of motion in relativistic notation 26–11

CHAPTER 27. FIELD ENERGY AND FIELD MOMENTUM

27–1 Local conservation 27–1
27–2 Energy conservation and electromagnetism 27–2
27–3 Energy density and energy flow in the electromagnetic field 27–3
27–4 The ambiguity of the field energy 27–6
27–5 Examples of energy flow 27–6
27–6 Field momentum 27–9

CHAPTER 28. ELECTROMAGNETIC MASS

28–1 The field energy of a point charge 28–1
28–2 The field momentum of a moving charge 28–2
28–3 Electromagnetic mass 28–3
28–4 The force of an electron on itself 28–4
28–5 Attempts to modify the Maxwell theory 28–6
28–6 The nuclear force field 28–12

CHAPTER 29. THE MOTION OF CHARGES IN ELECTRIC AND MAGNETIC FIELDS

29–1 Motion in a uniform electric or magnetic field 29–1
29–2 Momentum analysis 29–1
29–3 An electrostatic lens 29–2
29–4 A magnetic lens 29–3
29–5 The electron microscope 29–3
29–6 Accelerator guide fields 29–4
29–7 Alternating-gradient focusing 29–6
29–8 Motion in crossed electric and magnetic fields 29–8

CHAPTER 30. THE INTERNAL GEOMETRY OF CRYSTALS

30–1 The internal geometry of crystals 30–1
30–2 Chemical bonds in crystals 30–2
30–3 The growth of crystals 30–3
30–4 Crystal lattices 30–3
30–5 Symmetries in two dimensions 30–4
30–6 Symmetries in three dimensions 30–7
30–7 The strength of metals 30–8
30–8 Dislocations and crystal growth 30–9
30–9 The Bragg-Nye crystal model 30–10

CHAPTER 31. TENSORS

31–1 The tensor of polarizability 31–1
31–2 Transforming the tensor components 31–3
31–3 The energy ellipsoid 31–3
31–4 Other tensors; the tensor of inertia 31–6
31–5 The cross product 31–8
31–6 The tensor of stress 31–9
31–7 Tensors of higher rank 31–11
31–8 The four-tensor of electromagnetic momentum 31–12

CHAPTER 32. REFRACTIVE INDEX OF DENSE MATERIALS

32–1 Polarization of matter 32–1
32–2 Maxwell's equations in a dielectric 32–3
32–3 Waves in a dielectric 32–5
32–4 The complex index of refraction 32–8
32–5 The index of a mixture 32–8
32–6 Waves in metals 32–10
32–7 Low-frequency and high-frequency approximations; the skin depth and the plasma frequency 32–11

CHAPTER 33. REFLECTION FROM SURFACES

33–1 Reflection and refraction of light 33–1
33–2 Waves in dense materials 33–2
33–3 The boundary conditions 33–4
33–4 The reflected and transmitted waves 33–7
33–5 Reflection from metals 33–11
33–6 Total internal reflection 33–12

CHAPTER 34. THE MAGNETISM OF MATTER

34–1 Diamagnetism and paramagnetism 34–1
34–2 Magnetic moments and angular momentum 34–3
34–3 The precession of atomic magnets 34–4
34–4 Diamagnetism 34–5
34–5 Larmor's theorem 34–6
34–6 Classical physics gives neither diamagnetism nor paramagnetism 34–8
34–7 Angular momentum in quantum mechanics 34–8
34–8 The magnetic energy of atoms 34–11

CHAPTER 35. PARAMAGNETISM AND MAGNETIC RESONANCE

35–1 Quantized magnetic states 35–1
35–2 The Stern-Gerlach experiment 35–3
35–3 The Rabi molecular-beam method 35–4
35–4 The paramagnetism of bulk materials 35–6
35–5 Cooling by adiabatic demagnetization 35–9
35–6 Nuclear magnetic resonance 35–10

CHAPTER 36. FERROMAGNETISM

36–1 Magnetization currents 36–1
36–2 The field H 36–5
36–3 The magnetization curve 36–6
36–4 Iron-core inductances 36–8
36–5 Electromagnets 36–9
36–6 Spontaneous magnetization 36–11

CHAPTER 37. MAGNETIC MATERIALS

37–1 Understanding ferromagnetism 37–1
37–2 Thermodynamic properties 37–4
37–3 The hysteresis curve 37–5
37–4 Ferromagnetic materials 37–10
37–5 Extraordinary magnetic materials 37–11

CHAPTER 38. ELASTICITY

38–1 Hooke's law 38–1
38–2 Uniform strains 38–2
38–3 The torsion bar; shear waves 38–5
38–4 The bent beam 38–9
38–5 Buckling 38–11

CHAPTER 39. ELASTIC MATERIALS

39–1 The tensor of strain 39–1
39–2 The tensor of elasticity 39–4
39–3 The motions in an elastic body 39–6
39–4 Nonelastic behavior 39–8
39–5 Calculating the elastic constants 39–10

CHAPTER 40. THE FLOW OF DRY WATER

40–1 Hydrostatics 40–1
40–2 The equations of motion 40–2
40–3 Steady flow—Bernoulli's theorem 40–6
40–4 Circulation 40–9
40–5 Vortex lines 40–10

CHAPTER 41. THE FLOW OF WET WATER

41–1 Viscosity 41–1
41–2 Viscous flow 41–4
41–3 The Reynolds number 41–5
41–4 Flow past a circular cylinder 41–7
41–5 The limit of zero viscosity 41–9
41–6 Couette flow 41–10

CHAPTER 42. CURVED SPACE

42–1 Curved spaces with two dimensions 42–1
42–2 Curvature in three-dimensional space 42–5
42–3 Our space is curved 42–6
42–4 Geometry in space-time 42–7
42–5 Gravity and the principle of equivalence 42–8
42–6 The speed of clocks in a gravitational field 42–9
42–7 The curvature of space-time 42–11
42–8 Motion in curved space-time 42–12
42–9 Einstein's theory of gravitation 42–13

1

Electromagnetism

1-1 Electrical forces
1-2 Electric and magnetic fields
1-3 Characteristics of vector fields
1-4 The laws of electromagnetism
1-5 What are the fields?
1-6 Electromagnetism in science and technology

Review: Chapter 12, Vol. I, *Characteristics of Force*

1-1 Electrical forces

Consider a force like gravitation which varies predominantly inversely as the square of the distance, but which is about a *billion-billion-billion-billion* times stronger. And with another difference. There are two kinds of "matter," which we can call positive and negative. Like kinds repel and unlike kinds attract—unlike gravity where there is only attraction. What would happen?

A bunch of positives would repel with an enormous force and spread out in all directions. A bunch of negatives would do the same. But an evenly mixed bunch of positives and negatives would do something completely different. The opposite pieces would be pulled together by the enormous attractions. The net result would be that the terrific forces would balance themselves out almost perfectly, by forming tight, fine mixtures of the positive and the negative, and between two separate bunches of such mixtures there would be practically no attraction or repulsion at all.

There is such a force: the electrical force. And all matter is a mixture of positive protons and negative electrons which are attracting and repelling with this great force. So perfect is the balance, however, that when you stand near someone else you don't feel any force at all. If there were even a little bit of unbalance you would know it. If you were standing at arm's length from someone and each of you had *one percent* more electrons than protons, the repelling force would be incredible. How great? Enough to lift the Empire State Building? No! To lift Mount Everest? No! The repulsion would be enough to lift a "weight" equal to that of the entire earth!

With such enormous forces so perfectly balanced in this intimate mixture, it is not hard to understand that matter, trying to keep its positive and negative charges in the finest balance, can have a great stiffness and strength. The Empire State Building, for example, swings only eight feet in the wind because the electrical forces hold every electron and proton more or less in its proper place. On the other hand, if we look at matter on a scale small enough that we see only a few atoms, any small piece will not, usually, have an equal number of positive and negative charges, and so there will be strong residual electrical forces. Even when there are equal numbers of both charges in two neighboring small pieces, there may still be large net electrical forces because the forces between individual charges vary inversely as the square of the distance. A net force can arise if a negative charge of one piece is closer to the positive than to the negative charges of the other piece. The attractive forces can then be larger than the repulsive ones and there can be a net attraction between two small pieces with no excess charges. The force that holds the atoms together, and the chemical forces that hold molecules together, are really electrical forces acting in regions where the balance of charge is not perfect, or where the distances are very small.

You know, of course, that atoms are made with positive protons in the nucleus and with electrons outside. You may ask: "If this electrical force is so terrific, why don't the protons and electrons just get on top of each other? If they want to be in an intimate mixture, why isn't it still more intimate?" The answer has to do with the quantum effects. If we try to confine our electrons in a region that is very close to the protons, then according to the uncertainty principle they must have some mean square momentum which is larger the more we try to confine them. It is this motion, required by the laws of quantum mechanics, that keeps the electrical attraction from bringing the charges any closer together.

There is another question: "What holds the nucleus together"? In a nucleus there are several protons, all of which are positive. Why don't they push themselves apart? It turns out that in nuclei there are, in addition to electrical forces, nonelectrical forces, called nuclear forces, which are greater than the electrical forces and which are able to hold the protons together in spite of the electrical repulsion. The nuclear forces, however, have a short range—their force falls off much more rapidly than $1/r^2$. And this has an important consequence. If a nucleus has too many protons in it, it gets too big, and it will not stay together. An example is uranium, with 92 protons. The nuclear forces act mainly between each proton (or neutron) and its nearest neighbor, while the electrical forces act over larger distances, giving a repulsion between each proton and all of the others in the nucleus. The more protons in a nucleus, the stronger is the electrical repulsion, until, as in the case of uranium, the balance is so delicate that the nucleus is almost ready to fly apart from the repulsive electrical force. If such a nucleus is just "tapped" lightly (as can be done by sending in a slow neutron), it breaks into two pieces, each with positive charge, and these pieces fly apart by electrical repulsion. The energy which is liberated is the energy of the atomic bomb. This energy is usually called "nuclear" energy, but it is really "electrical" energy released when electrical forces have overcome the attractive nuclear forces.

Lower case Greek letters and commonly used capitals

α		alpha
β		beta
γ	Γ	gamma
δ	Δ	delta
ϵ		epsilon
ζ		zeta
η		eta
θ	Θ	theta
ι		iota
κ		kappa
λ	Λ	lambda
μ		mu
ν		nu
ξ	Ξ	xi (ksi)
o		omicron
π	Π	pi
ρ		rho
σ	Σ	sigma
τ		tau
υ	Υ	upsilon
ϕ	Φ	phi
χ		chi (khi)
ψ	Ψ	psi
ω	Ω	omega

We may ask, finally, what holds a negatively charged electron together (since it has no nuclear forces). If an electron is all made of one kind of substance, each part should repel the other parts. Why, then, doesn't it fly apart? But does the electron have "parts"? Perhaps we should say that the electron is just a point and that electrical forces only act between *different* point charges, so that the electron does not act upon itself. Perhaps. All we can say is that the question of what holds the electron together has produced many difficulties in the attempts to form a complete theory of electromagnetism. The question has never been answered. We will entertain overselves by discussing this subject some more in later chapters.

As we have seen, we should expect that it is a combination of electrical forces and quantum-mechanical effects that will determine the detailed structure of materials in bulk, and, therefore, their properties. Some materials are hard, some are soft. Some are electrical "conductors"—because their electrons are free to move about; others are "insulators"—because their electrons are held tightly to individual atoms. We shall consider later how some of these properties come about, but that is a very complicated subject, so we will begin by looking at the electrical forces only in simple situations. We begin by treating only the laws of electricity—including magnetism, which is really a part of the same subject.

We have said that the electrical force, like a gravitational force, decreases inversely as the square of the distance between charges. This relationship is called Coulomb's law. But it is not precisely true when charges are moving—the electrical forces depend also on the motions of the charges in a complicated way. One part of the force between moving charges we call the *magnetic* force. It is really one aspect of an electrical effect. That is why we call the subject "electromagnetism."

There is an important general principle that makes it possible to treat electromagnetic forces in a relatively simple way. We find, from experiment, that the force that acts on a particular charge—no matter how many other charges there are or how they are moving—depends only on the position of that particular charge, on the velocity of the charge, and on the amount of charge. We can write the force $\boldsymbol{F}$ on a charge q moving with a velocity $\boldsymbol{v}$ as

$$\boldsymbol{F} = q(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}). \tag{1.1}$$

We call $\boldsymbol{E}$ the *electric field* and $\boldsymbol{B}$ the *magnetic field* at the location of the charge. The important thing is that the electrical forces from all the other charges in the universe can be summarized by giving just these two vectors. Their values will depend on *where* the charge is, and may change with *time*. Furthermore, if we replace that charge with another charge, the force on the new charge will be just in proportion to the amount of charge so long as all the rest of the charges in the world do not change their positions or motions. (In real situations, of course, each charge produces forces on all other charges in the neighborhood and may cause these other charges to move, and so in some cases the fields *can* change if we replace our particular charge by another.)

We know from Vol. I how to find the motion of a particle if we know the force on it. Equation (1.1) can be combined with the equation of motion to give

$$\frac{d}{dt}\left[\frac{m\boldsymbol{v}}{(1-v^2/c^2)^{1/2}}\right] = \boldsymbol{F} = q(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}). \tag{1.2}$$

So if $\boldsymbol{E}$ and $\boldsymbol{B}$ are given, we can find the motions. Now we need to know how the $\boldsymbol{E}$'s and $\boldsymbol{B}$'s are produced.

One of the most important simplifying principles about the way the fields are produced is this: Suppose a number of charges moving in some manner would produce a field $\boldsymbol{E}_1$, and another set of charges would produce $\boldsymbol{E}_2$. If both sets of charges are in place at the same time (keeping the same locations and motions they had when considered separately), then the field produced is just the sum

$$\boldsymbol{E} = \boldsymbol{E}_1 + \boldsymbol{E}_2. \tag{1.3}$$

This fact is called *the principle of superposition* of fields. It holds also for magnetic fields.

This principle means that if we know the law for the electric and magnetic fields produced by a *single* charge moving in an arbitrary way, then all the laws of electrodynamics are complete. If we want to know the force on charge *A* we need only calculate the $\boldsymbol{E}$ and $\boldsymbol{B}$ produced by each of the charges *B*, *C*, *D*, etc., and then add the $\boldsymbol{E}$'s and $\boldsymbol{B}$'s from all the charges to find the fields, and from them the forces acting on charge *A*. If it had only turned out that the field produced by a single charge was simple, this would be the neatest way to describe the laws of electrodynamics. We have already given a description of this law (Chapter 28, Vol. I) and it is, unfortunately, rather complicated.

It turns out that the form in which the laws of electrodynamics are simplest are not what you might expect. It is *not* simplest to give a formula for the force that one charge produces on another. It is true that when charges are standing still the Coulomb force law is simple, but when charges are moving about the relations are complicated by delays in time and by the effects of acceleration, among others. As a result, we do not wish to present electrodynamics only through the force laws between charges; we find it more convenient to consider another point of view—a point of view in which the laws of electrodynamics appear to be the most easily manageable.

1-2 Electric and magnetic fields

First, we must extend, somewhat, our ideas of the electric and magnetic vectors, $\boldsymbol{E}$ and $\boldsymbol{B}$. We have defined them in terms of the forces that are felt by a charge. We wish now to speak of electric and magnetic fields *at a point* even when there is no charge present. We are saying, in effect, that since there are forces "acting on" the charge, there is still "something" there when the charge is removed. If a charge located at the point (x, y, z) at the time t feels the force $\boldsymbol{F}$ given by Eq. (1.1) we associate the vectors $\boldsymbol{E}$ and $\boldsymbol{B}$ with *the point* in space (x, y, z). We may think of $\boldsymbol{E}(x, y, z, t)$ and $\boldsymbol{B}(x, y, z, t)$ as giving the forces that *would be* experienced at the time t by a charge located at (x, y, z), *with the condition* that placing the charge there *did not disturb* the positions or motions of all the other charges responsible for the fields.

Following this idea, we associate with *every* point (x, y, z) in space two vectors $\boldsymbol{E}$ and $\boldsymbol{B}$, which may be changing with time. The electric and magnetic fields are, then, viewed as *vector functions* of x, y, z, and t. Since a vector is specified by its components, each of the fields $\boldsymbol{E}(x, y, z, t)$ and $\boldsymbol{B}(x, y, z, t)$ represent three mathematical functions of x, y, z, and t.

It is precisely because $\boldsymbol{E}$ (or $\boldsymbol{B}$) can be specified at every point in space that it is called a "field." A "field" is any physical quantity which takes on different values at different points in space. Temperature, for example, is a field—in this case a scalar field, which we write as $T(x, y, z)$. The temperature could also vary in time, and we would say the temperature field is time-dependent, and write $T(x, y, z, t)$. Another example is the "velocity field" of a flowing liquid. We write $\boldsymbol{v}(x, y, z, t)$ for the velocity of the liquid at each point in space at the time t. It is a vector field.

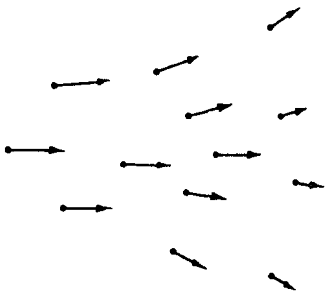

Fig. 1–1. A vector field may be represented by drawing a set of arrows whose magnitudes and directions indicate the values of the vector field at the points from which the arrows are drawn.

Returning to the electromagnetic fields—although they are produced by charges according to complicated formulas, they have the following important characteristic: the relationships between the values of the fields at *one point* and the values at a *nearby point* are very simple. With only a few such relationships in the form of differential equations we can describe the fields completely. It is in terms of such equations that the laws of electrodynamics are most simply written.

There have been various inventions to help the mind visualize the behavior of fields. The most correct is also the most abstract: we simply consider the fields as mathematical functions of position and time. We can also attempt to get a mental picture of the field by drawing vectors at many points in space, each of which gives the field strength and direction at that point. Such a representation is shown in Fig. 1–1. We can go further, however, and draw lines which are everywhere tangent to the vectors—which, so to speak, follow the arrows and keep track of the direction of the field. When we do this we lose track of the *lengths* of the vectors, but we can keep track of the strength of the field by drawing the lines far apart when the field is weak and close together when it is strong. We adopt the convention that the *number of lines per unit area* at right angles to the lines is proportional to the *field strength*. This is, of course, only an approximation, and it will require, in general, that new lines sometimes start up in order to keep the number up to the strength of the field. The field of Fig. 1–1 is represented by field lines in Fig. 1–2.

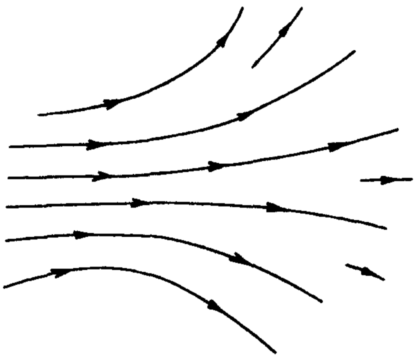

Fig. 1–2. A vector field can be represented by drawing lines which are tangent to the direction of the field vector at each point, and by drawing the density of lines proportional to the magnitude of the field vector.

1–3 Characteristics of vector fields

There are two mathematically important properties of a vector field which we will use in our description of the laws of electricity from the field point of view. Suppose we imagine a closed surface of some kind and ask whether we are losing "something" from the inside; that is, does the field have a quality of "outflow"? For instance, for a velocity field we might ask whether the velocity is always outward on the surface or, more generally, whether more fluid flows out (per unit time) than comes in. We call the net amount of fluid going out through the surface per unit time the "flux of velocity" through the surface. The flow through an element of a surface is just equal to the component of the velocity perpendicular to the surface times the area of the surface. For an arbitrary closed surface, the *net outward flow*—or *flux*—is the average outward normal component of the velocity, times the area of the surface:

$$\text{Flux} = (\text{average normal component})\cdot(\text{surface area}). \tag{1.4}$$

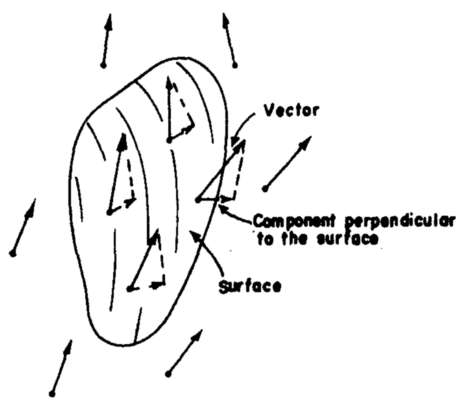

Fig. 1–3. The flux of a vector field through a surface is defined as the average value of the normal component of the vector times the area of the surface.

In the case of an electric field, we can mathematically define something analogous to an outflow, and we again call it the flux, but of course it is not the flow of any substance, because the electric field is not the velocity of anything. It turns out, however, that the mathematical quantity which is the average normal component of the field still has a useful significance. We speak, then, of the *electric flux*—also defined by Eq. (1.4). Finally, it is also useful to speak of the flux not only through a completely closed surface, but through any bounded surface. As before, the flux through such a surface is defined as the average normal component of a vector times the area of the surface. These ideas are illustrated in Fig. 1–3.

There is a second property of a vector field that has to do with a line, rather than a surface. Suppose again that we think of a velocity field that describes the flow of a liquid. We might ask this interesting question: Is the liquid circulating?

By that we mean: Is there a net rotational motion around some loop? Suppose that we instantaneously freeze the liquid everywhere except inside of a tube which is of uniform bore, and which goes in a loop that closes back on itself as in Fig. 1–4. Outside of the tube the liquid stops moving, but inside the tube it may keep on moving because of the momentum in the trapped liquid—that is, if there is more momentum heading one way around the tube than the other. We define a quantity called the *circulation* as the resulting speed of the liquid in the tube times its circumference. We can again extend our ideas and define the "circulation" for any vector field (even when there isn't anything moving). For any vector field the *circulation around any imagined closed curve* is defined as the average tangential component of the vector (in a consistent sense) multiplied by the circumference of the loop (Fig. 1–5).

$$\text{Circulation} = (\text{average tangential component})\cdot(\text{distance around}). \tag{1.5}$$

You will see that this definition does indeed give a number which is proportional to the circulation velocity in the quickly frozen tube described above.

With just these two ideas—flux and circulation—we can describe all the laws of electricity and magnetism at once. You may not understand the significance of the laws right away, but they will give you some idea of the way the physics of electromagnetism will be ultimately described.

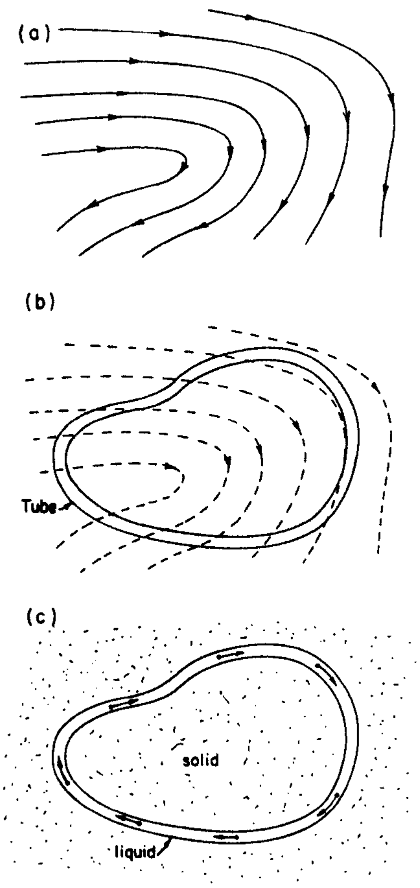

Fig. 1–4. (a) The velocity field in a liquid. Imagine a tube of uniform cross section that follows an arbitrary closed curve as in (b). If the liquid were suddenly frozen everywhere except inside the tube, the liquid in the tube would circulate as shown in (c).

1–4 The laws of electromagnetism

The first law of electromagnetism describes the flux of the electric field:

$$\text{The flux of } \boldsymbol{E} \text{ through any closed surface} = \frac{\text{the net charge inside}}{\epsilon_0}, \tag{1.6}$$

where ϵ_0 is a convenient constant. (The constant ϵ_0 is usually read as "epsilon-zero" or "epsilon-naught".) If there are no charges inside the surface, even though there are charges nearby outside the surface, the *average* normal component of $\boldsymbol{E}$ is zero, so there is no net flux through the surface. To show the power of this type of statement, we can show that Eq. (1.6) is the same as Coulomb's law, provided only that we also add the idea that the field from a single charge is spherically symmetric. For a point charge, we draw a sphere around the charge. Then the average normal component is just the value of the magnitude of $\boldsymbol{E}$ at any point, since the field must be directed radially and have the same strength for all points on the sphere. Our rule now says that the field at the surface of the sphere, times the area of the sphere—that is, the outgoing flux—is proportional to the charge inside. If we were to make the radius of the sphere bigger, the area would increase as the square of the radius. The average normal component of the electric field times that area must still be equal to the same charge inside, and so the field must decrease as the square of the distance—we get an "inverse square" field.

If we have an arbitrary stationary curve in space and measure the circulation of the electric field around the curve, we will find that it is not, in general, zero (although it is for the Coulomb field). Rather, for electricity there is a second law that states: for any surface S (not closed) whose edge is the curve C,

$$\text{Circulation of } \boldsymbol{E} \text{ around } C = \frac{d}{dt}(\text{flux of } \boldsymbol{B} \text{ through } S). \tag{1.7}$$

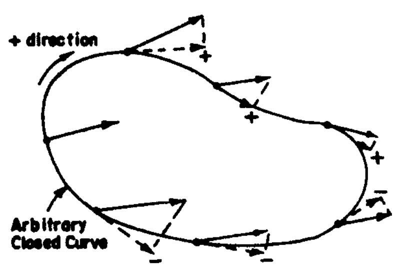

Fig. 1–5. The circulation of a vector field is the average tangential component of the vector (in a consistent sense) times the circumference of the loop.

We can complete the laws of the electromagnetic field by writing two corresponding equations for the magnetic field $\boldsymbol{B}$.

$$\text{Flux of } \boldsymbol{B} \text{ through any closed surface} = 0. \tag{1.8}$$

For a surface S bounded by the curve C,

$$c^2(\text{circulation of } \boldsymbol{B} \text{ around } C) = \frac{d}{dt}(\text{flux of } \boldsymbol{E} \text{ through } S) + \frac{\text{flux of electric current through } S}{\epsilon_0}. \tag{1.9}$$