GEOMETRICAL OPTICS

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PREFACE

This book is an attempt to outline the essentials of geometrical optics for undergraduates studying physics, and is intended primarily for use in the first and second post-intermediate years.

In the author's opinion books of this standard already available are of two kinds: those which include and lay greater emphasis on physical optics, which tend to treat geometrical aspects in an unduly brief and partial way; and those in which the physical principles and general utility of the subject are somewhat obscured by an excessively mathematical treatment. Books of a more advanced type, mainly intended for specialists in applied optics, are much more satisfactory as regards accuracy and comprehensiveness, but cannot be recommended to non-specialist readers because of their length and detail.

The aim has therefore been to give a more balanced account of the subject, in which the underlying physical principles and the ultimate application to optical instruments and their design are always kept in view. As the subject forms only a part of a student's interests, an endeavour has been made to achieve this aim within as small a compass as possible. An impulse to develop the subject further in some sections has therefore been resisted, but guidance to more advanced reading is given in the bibliography. Some of the examples are also designed to amplify matters dealt with only briefly in the text. No reference is made to spectacle lenses, this subject being relatively unimportant for the student for whom the book is intended, and also adequately covered in other books.

The so-called new Cartesian sign convention for the treatment of optical systems is used. This convention is often favoured in university teaching, and is most in line with advanced optical practice.

I wish to express my gratitude to Professor E. C. Stoner, F.R.S., who was good enough to read the manuscript critically, and whose comments and advice have been most helpful. My thanks are also due to Mr. F. A. Long, who made useful suggestions on some sections, and to others of my colleagues who showed interest in various ways. My interest in the subject was greatly stimulated by valuable discussions with the late Dr. M. M. Nicolson, which have doubtless left their imprint on these pages. This book could not have been written without a close acquaintance with more advanced books on optics, notably Conrady's

" Applied Optics and Optical Design " and Martin's " Technical Optics ", and I acknowledge my indebtedness to these works. Finally, I am grateful to the Leeds Philosophical and Literary Society, Ltd., for permission to make the quotation from the Proceedings which appears in Appendix II, and to Mrs. Allen who agreed that her late husband's work should be quoted in this way.

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INTRODUCTION AND PRELIMINARIES

Scope of geometrical optics.—Geometrical optics is concerned with the deductive consequences and applications of one of the most obvious characteristics of light, the fact that it travels in straight lines; and no assumptions are made regarding the nature of light. The concept of rays of light is introduced, these rays being straight lines in a uniform medium and representing the directions of advance of the light. From the known behaviour of light rays at interfaces between optical media, their course through optical systems may be traced and the formation of images of objects studied, and alternatively, instruments for producing various effects may be designed.

It is important, however, to be aware of the limitations no less than the scope of this geometrical treatment, and to make these clear, allusion must be made to some conclusions from study of the branch of optics known as physical optics. Physical optics covers matters connected in a more detailed way with the nature and mode of propagation of light, such as interference, diffraction, polarization, and dispersion. The very existence of such phenomena establishes the wave nature of light, and experiments on these effects may be devised to give actual determinations of the wavelengths of light. The wavelengths of visible light radiations are found to lie within the approximate limits of 4×10^{-5} cm. to $7{\cdot}5 \times 10^{-5}$ cm. There are other radiations of similar nature having wavelengths outside these limits, but they do not give the sensation of light when received by the eye. Within the wavelength range of visual sensitivity the sensation of colour is associated with the magnitude of the wavelengths received by the eye. The distribution of the light energy over the wavelength range determines the colour seen. The longest wavelengths correspond to red, and the shortest to violet sensations, intermediate wavelengths to other colours.

The extremely small order of magnitude of the wavelength of light is the basic reason for the rectilinear propagation of light in all normal circumstances. The effect known as diffraction may be considered as a departure from rectilinear propagation due to the wave nature of light, and becomes pronounced and measurable only when light impinges on small obstacles or apertures; so small in fact that the wavelength of the light may no longer be considered insignificant as compared with their size. Rectilinear propagation of light is very closely adhered to, and hence the assumptions of the geometrical treatment of optical systems are closely true, provided the apertures and lenses, etc., used in the systems are of macroscopic dimensions relative to the wavelength of light. It must always be borne in mind, however, that the assumptions, though closely true, are never absolutely so, and in a few cases the effect of diffraction is important, even though produced by a lens of normal or large size, and will need to be briefly considered as we proceed.

Standard wavelengths employed in optics.—Several of the optical properties of transparent media depend on the wavelength of the light being transmitted, and among these the variation of refractive index with wavelength is of special importance within the field of geometrical optics. It is usual in glassmakers' catalogues and physical tables to give refractive indices of materials for a number of discrete wavelengths, and for this purpose some of the so-called Fraunhofer lines are customarily employed. Certain sources of light, for example gas discharge tubes containing hydrogen or other gases at a suitable pressure, emit light which when analysed by spectroscopic means is found to consist of a number of separate discrete wavelengths. The spectrum appears as a number of sharp coloured *lines* at various positions within the visual range of wavelengths, and is called a line spectrum. It is possible in many cases to determine the wavelengths of these lines with high accuracy, and the results are most usually expressed in Ångström units (Å.), one such unit being equal to 10^{-8} cm. The following are the wavelengths in these units of the main lines used in this branch of optics:

C	red	6563 Å.	H
D	yellow	5893 Å.	Na
F	blue	4862 Å.	H
G′	violet	4340 Å.	H

Each line is known by the letter shown on the left in the table, and the atom in the source giving rise to emission of the line is shown on the right. With the exception of the G′ line, the letters by which the lines are known were first used by Fraunhofer, who discovered them as absorption lines in the sun's spectrum. The G′ line is very close to the position of the Fraunhofer G line, but is more suitable for accurate measurements.

A further important quantity to which reference will occasionally be made is the *wavelength of maximum visual sensitivity*. Within the range of wavelengths detectable by the eye, sensitivity in normal conditions is highest near the middle of the range and falls off to zero at the red and violet extremes. The curve of sensitivity against wavelength has a flat maximum in the green region, the highest sensitivity being usually considered to be at the wavelength 5550 Å.

Sign conventions.—Conventions as to signs for both distances and angles must necessarily be introduced in the treatment of optical systems. In a report on the teaching of geometrical optics by a committee of the Physical Society, London (1934), the question of the merits of various sign conventions is discussed. Of the two alternatives recommended we adopt the former, denoted in the report by I(i). The convention may be stated as follows:

(1) *Convention for distances*

Two types of distances must first be distinguished—

(a) *Longitudinal distances.*—These are distances along the axis of a system measured **from** some chosen point or points within the system to objects, images, centres of curvature, etc. They are considered positive when they are in the direction of the incident light, and negative when against it.

(*b*) *Transverse distances.*—These are distances at right angles to the axis of a system, such as heights of objects and images. These are considered positive when above the axis and negative when below it.

Wherever possible in this book figures are drawn showing the light advancing from left to right, so that positive distances are to the right, and negative distances to the left. The sign convention thus agrees with the normal system of rectangular axes used in co-ordinate geometry and usually termed the Cartesian system of co-ordinates.

(2) *Convention for angles*

Acute angles between rays and normals to surfaces, or between rays and the axis of a system, are measured, and are considered positive when an anti-clockwise rotation is necessary to move from the ray to the normal or axis through the angles concerned.

Terms and symbols.—The term **object space** may be used to denote the region in which objects may be situated relative to an optical system if it is to produce images by reflection or refraction, while the term **image space** refers in a similar way to the region in which images may be formed. In a general case both objects and images may be real or virtual and on either side of, or even within, the system. The two spaces may thus be co-extensive along the axis of the system. It is convenient to preserve the distinction between them, however, and to use parallel symbols for similar quantities in the two spaces. Plain symbols are used in the object space and dashed symbols in the image space. For example, y refers to the transverse distance of an object point from the axis, and y' to the corresponding image-point distance. Any symbol referring to the object space is connected with the entrant rays, while image-space symbols are related to the emergent rays.

The main symbols employed are listed below:

n, n'	refractive indices in object and image spaces
U, U'	axial object and image conjugate points
B, B'	off-axial object and image conjugate points
F, F'	first and second focal points
P, P'	first and second principal points
N, N'	first and second nodal points
W, W'	axial points in entrance and exit pupils
f, f'	first and second focal distances
l, l'	longitudinal object and image distances from respective principal points
x, x'	longitudinal object and image distances from respective focal points
y, y'	transverse distances from axis of conjugate object and image points
μ	relative refractive index
$\mathbf{P}$	power of a system
m	transverse linear magnification
M	magnifying power
ω	dispersive power
ν	constringence
S	spherical aberration
C	coma
A	astigmatism
P	Petzval curvature
D	distortion
LC	longitudinal chromatic aberration
TC	transverse chromatic aberration

CHAPTER I

REFLECTION AND REFRACTION AT PLANE SURFACES, PRISMS, DISPERSION

Laws of reflection and refraction

When a beam of light falls on a surface separating two optical media the rays of light may be reflected back into the medium in which the light was first travelling, or refracted into the second medium. Usually both processes occur together, part of the light being reflected and part refracted, the relative proportion being dependent on many factors, such as the reflecting power of the surface and the angle of incidence. The two processes take place in accordance with the relations found experimentally and known as the laws of reflection and refraction. In Figs. 1A and

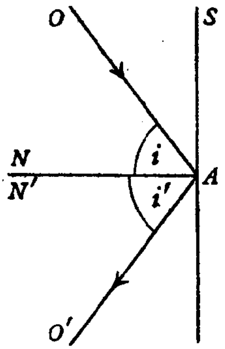

Fig. 1A.

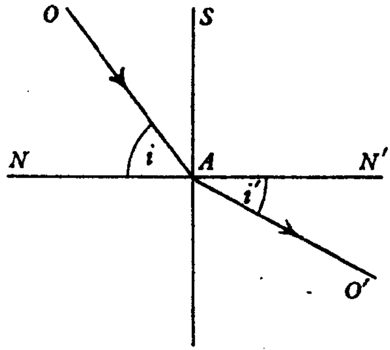

Fig. 1B.

1B the two effects are separately illustrated as they occur at a plane surface AS. In both figures A is the point of incidence of the *incident ray* OA making the *angle of incidence* i with the normal AN to the surface.

In Fig. 1A AO' is the *reflected ray* making an angle i', the *angle of reflection*, with the normal. In Fig. 1B AO' is the *refracted ray* making the angle i', the *angle of refraction*, with the normal.

The laws of reflection may be stated as follows:

First law.—The reflected ray lies in the plane of incidence, which is the plane containing the normal and the incident ray (i.e., the plane of the paper of Fig. 1A).

Second law.—The angles of incidence and reflection are equal in magnitude, the reflected ray being on the other side of the normal from the incident ray.

This may be written

$$i = -i' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 1.1$$

since the angles are of opposite sign according to our convention.

The laws of refraction are as follows:

FIRST LAW.—The refracted ray lies in the plane of incidence.

SECOND LAW (often termed Snell's law).—The angle of refraction depends on the angle of incidence in such a way that the ratio $\frac{\sin i}{\sin i'}$ is constant. The value of this ratio depends on the two media involved, and also on the wavelength of the light.

Experiment shows that this constant ratio of the sines of the angles in the two media is equal to the ratio of the velocities of light in the media, a fact which is predictable from the wave theory of light, i.e.,

$$\frac{\sin i}{\sin i'} = \frac{v}{v'} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 1.2$$

where v = velocity of light in the medium in which the light is incident, and v' = velocity of light in the medium into which the light is refracted.

The refractive index of a medium (for which we shall use the symbol n) is defined as the value of $\frac{\sin i}{\sin i'}$ when the light is incident *in vacuo* and refracted into the medium. Refractive indices are most commonly stated for sodium **D** light of wavelength 5893 Å.

Thus
$$n = \frac{\sin(\text{incident angle } \textit{in vacuo})}{\sin(\text{refraction angle in medium})}$$
$$= \frac{\text{velocity of light } \textit{in vacuo}}{\text{velocity of light in medium}}$$

For a second medium having refractive index n'

$$n' = \frac{\text{velocity of light } \textit{in vacuo}}{\text{velocity of light in second medium}}$$

Hence, dividing,

$$\frac{n'}{n} = \frac{\text{velocity of light in first medium}}{\text{velocity of light in second medium}}$$

and using equation 1.2

$$\frac{n'}{n} = \frac{\sin i}{\sin i'}, \quad \text{or} \quad n' \sin i' = n \sin i \quad . \quad . \quad . \quad 1.3$$

This expression gives the ratio of sines of angles of incidence and refraction in terms of the refractive indices (referred to a vacuum) of the two media bounded by the surface, and is the most general expression for the second law of refraction.

The quantities n and n' are the absolute refractive indices of the media concerned, being measured relative to a vacuum. On occasions it will be sufficient to speak of the constant ratio $\frac{\sin i}{\sin i'}$, as the **relative refractive index** between the media on the two sides of the boundary, and the symbol μ is commonly used for this quantity. If we denote by $\underset{1\to 2}{\mu}$ the relative refractive index when the light passes from medium 1 into medium 2, it becomes clear that

$$\underset{1\to 2}{\mu} = \underset{1\to 3}{\mu} \times \underset{3\to 2}{\mu} \quad . \quad . \quad . \quad . \quad . \quad . \quad 1.4$$

This follows immediately from the substitution of n_2/n_1 for $\underset{1\to 2}{\mu}$ according to equation 1.3, and of similar expressions for $\underset{1\to 3}{\mu}$ and $\underset{3\to 2}{\mu}$; where n_1, n_2, etc., are the absolute refractive indices of the media 1, 2, etc.

The relative refractive index between air and a medium is closely equal to the absolute refractive index of the medium. This is because the absolute refractive index of air under normal conditions is closely equal to unity.

We have $\qquad \underset{1\to 2}{\mu} = n_2/n_1$

or $\qquad \underset{1\to 2}{\mu} = n_2$ very closely

if medium 1 is air or any gas having refractive index very close to unity. The refractive index of air at N.T.P. is about 1·0003, so that the approximation made in taking the relative index between air and the medium as the absolute index is correct to about 0·03%. Also since the refractive index of air changes with the pressure and temperature, the refractive index of a material relative to air must vary slightly with the air conditions, and hence is not an absolute constant. Glass-makers' tables usually give refractive indices relative to air at N.T.P. rather than the absolute indices of glasses as being, in general, more immediately useful.

Reflection as a special case of refraction.—Returning now to equation 1.3, the general expression for the second law of refraction, it is seen that it also includes the second law of reflection (equation 1.1), if reflection is considered as the special case for which $n' = -n$. This is not only true in the simple case we have illustrated. It will later be seen that reflection formulæ at curved surfaces are simply special cases (for which $n' = -n$) of the corresponding refraction formulæ.

Plane surfaces considered as simple optical systems

Image formation by plane surfaces.—We shall first consider point objects, and later extend the treatment to extended objects.

Point objects.—*Reflection.*—Fig. 1c shows a pencil of light from a point object B reflected from the plane surface. After reflection the pencil appears to diverge from the point image B'. B' is termed a virtual image as the rays do not

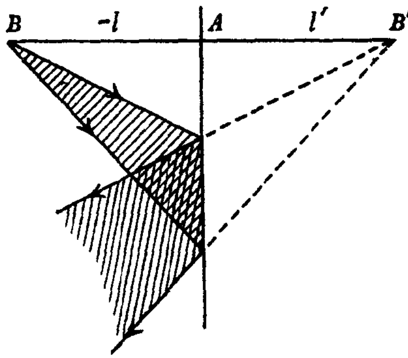

FIG. 1c.

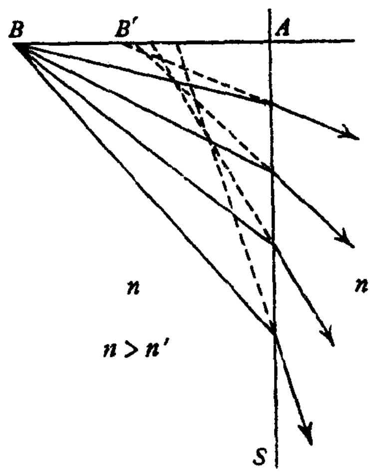

FIG. 1D.

actually pass through it; after reflection they proceed as though they had originated at B'. The laws of reflection easily lead to the conclusion that this image lies on the continuation of the perpendicular BA from the object to the mirror. Also it is as far behind the mirror as the object is in front. If the object distance from the surface is denoted by l, and the image distance by l', we have,

$$AB = AB'$$

or

$$-l = l'$$

l being negative according to the convention of signs chosen, i.e.,

$$\frac{l'}{l} = -1 \qquad \qquad 1.5$$

Refraction.—Fig. 1D shows four rays from a point object B before and after refraction at the plane surface AS between two media. In a case such as this the refracted pencil does not generally emerge as from a point image. This is the first and simplest example of *aberration* produced by an optical system, that is to say its failure to produce a point image corresponding to a point object.

Confining attention to rays very nearly at perpendicular incidence, however, this aberrational effect disappears. For small angles $\sin i \simeq i \simeq \tan i$, and if such an approximation may be made a point image B' of the object point B is obtained. In considering rays at small enough angles to the normal for this approximation to be valid we confine our attention to the *paraxial region*, and such rays are termed *paraxial rays*. This is a sufficient definition of paraxial rays for the present purpose, though a fuller definition will be necessary in the consideration of curved surfaces. Fig. 1E shows a pencil of paraxial rays from B diverging

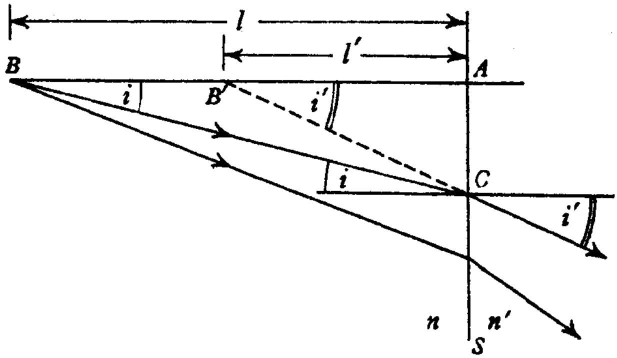

FIG. 1E.

after refraction from B', the virtual image of B. The angles i and i' are exaggerated in size for clarity. The relationship between the distances of object (l) and image (l') from the refracting surface may be deduced as follows. Consider the ray BC for which i and i' are the angles of incidence and refraction respectively.

Then $$\widehat{ABC} = i \quad \text{and} \quad \widehat{AB'C} = i'$$

Now $$\frac{n'}{n} = \frac{\sin i}{\sin i'} \simeq \frac{AC/AB}{AC/AB'} = \frac{l'}{l}$$

or $$\frac{l'}{l} = \frac{n'}{n} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 1.6$$

The result is independent of the position of C provided the assumption $\sin i = AC/AB$ remains valid, that is to say for all paraxial rays. B' is thus a true point image for paraxial rays only.

The use of this relationship in determination of refractive indices by the so-called "real and apparent depth" method will be familiar to many readers, but will not be enlarged on here.

It will be noted that in the case in which $n' = -n$ this equation reverts to the reflection relationship $l'/l = -1$. It should, however, be noted that the reflection relationship is true for all rays since when $n' = -n$ the relation

$$\frac{\sin i}{\sin i'} = -1 = \frac{i}{i'}$$

is not confined to small angles for its validity.

Extended objects.—*Reflection.*—In Fig. 1F any perpendicular AB to the reflecting surface AS is chosen as the "axis" of the

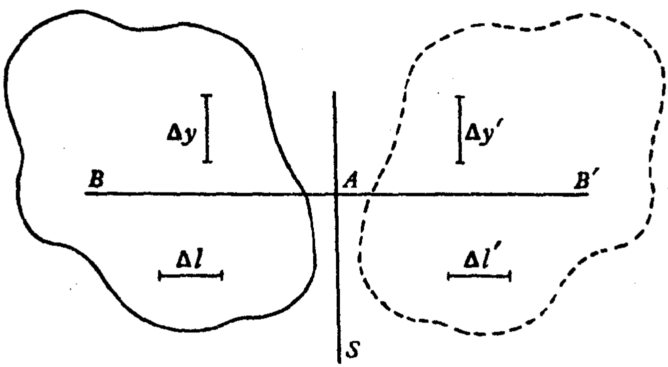

FIG. 1F.

system. Dimensions within an extended body perpendicular to the axis are termed *transverse* dimensions, while those parallel to the axis are termed *longitudinal* dimensions. The short lengths Δy and Δl are examples of the two types of distances within the extended body shown. It is clear that $\Delta y'$, the image of Δy, is equal to Δy and of the same sign. Thus $\frac{\Delta y'}{\Delta y} = +1$. This quantity is known as the *transverse linear magnification* of the image.

$\Delta l'$ is, however, equal to $-\Delta l$, as is clear in the figure or may be deduced from $l' = -l$. The *longitudinal magnification* is therefore -1. This implies inversion of the image in the longitudinal direction.

Refraction.—Referring back to Fig. 1E, transverse movement of B is accompanied by equal transverse movement of B'; hence transverse dimensions of extended objects are unaffected by