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Contributions to Group Theory

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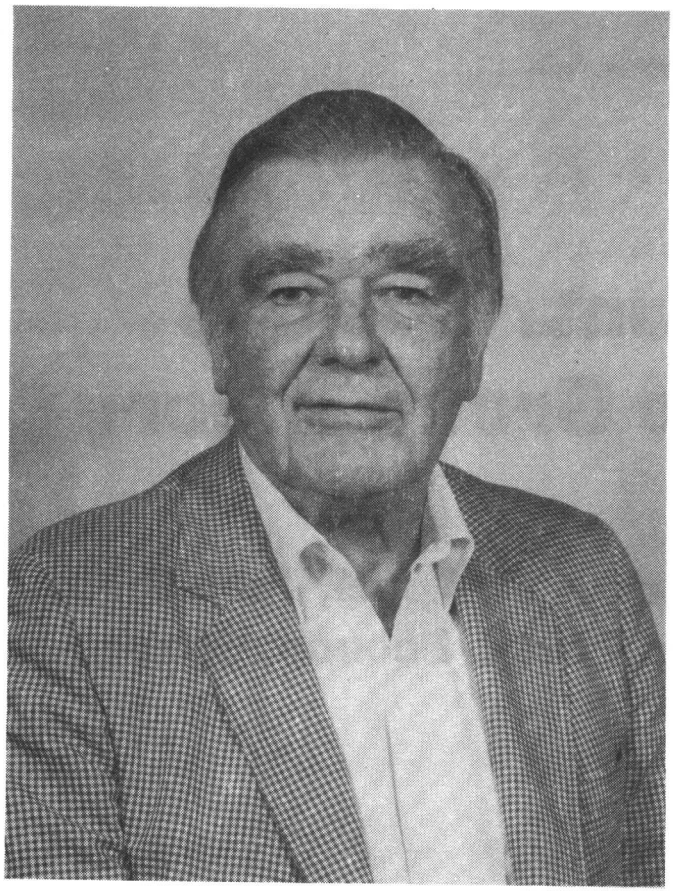

Papers dedicated to

ROGER C. LYNDON

on the occasion
of his sixty-fifth birthday.

Preface

This volume consists of five short articles on Roger Lyndon and his contributions to mathematics, and twenty-seven invited research papers. The research articles involve topics in combinatorial group theory and closely related areas. Several of the articles fall into subfields of combinatorial group theory in which much of the initial work was done by Lyndon. Most of the rest are in areas in which he has made major contributions. It is a tribute to Lyndon's mathematical breadth that papers covering such a wide array of topics are all closely related to the work he has done.

The articles on Lyndon and his work include a biographical essay by Kenneth Appel and expository articles by Saunders Mac Lane, John Ratcliffe, Jerome Keisler, and Paul Schupp. Mac Lane describes the results in Lyndon's doctoral thesis and explains how they fit into the early history of spectral sequences. Ratcliffe describes Lyndon's fundamental work in cohomology of groups in the early part of his career. Keisler discusses Lyndon's work in logic, especially his fundamental results in model theory in the mid 1950s. Schupp then describes Lyndon's work in group theory over the past twenty years.

The volume grew out of a desire by the editors to honor our teacher, Roger Lyndon, on the occasion of his sixty-fifth birthday. It has been a labor of love for us to gather together papers of solid mathematical interest dedicated to Roger Lyndon by his students, colleagues and friends, all of whom have shown great enthusiasm for this project.

We would like to thank all of the contributors and all the people who have made this collection possible. All the papers in this volume have been referred.

Kenneth I. Appel
John G. Ratcliffe
Paul E. Schupp

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ROGER C. LYNDON: A BIOGRAPHICAL AND PERSONAL NOTE

Kenneth I. Appel

We who are privileged to be Roger Lyndon's students know him as a genuine "Doctor-father," a man who patiently guided us in our first efforts at producing mathematics, and insisted that we learned to present our arguments in decent English. Although we have worked in many areas of mathematics since leaving Ann Arbor, he has continued to be interested in our mathematical work and has often made helpful suggestions. Because of his essential modesty and his enthusiasm for discussing our work rather than his own, some of us did not initially appreciate the extent and depth of his contributions to mathematics. Each of us knew of the profound work that he had done in our thesis specialties -- logic, homological algebra, many areas of group theory -- but we later discovered that people who were not familiar with his work in our own particular specialties considered him a mathematician of the first rank for his work in their areas of expertise.

Roger Lyndon, who will always be associated with the Midwest because of his distinguished career at the University of Michigan, was born on December 18, 1917, at Calais, Maine, almost the easternmost point of the continental United States. His family lived in the small nearby town of Eastport, where his father, Percy Lyndon, was a Unitarian minister. His paternal grandfather had come to America from England at the age of two. Lyndon's maternal grandfather came to Maine from Louisiana and owned a busy wharf on the Bay of Fundy. When Roger was two years old, his mother, Ann Aymar Milliken Lyndon, died. In the next fifteen years Percy, Roger and his sister lived in various towns in Massachusetts and New York. After graduation

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from the Derby School, Lyndon enrolled at Harvard in 1935, intent on studying literature and becoming a writer.

Lyndon had always enjoyed mathematics. He soon discovered that he could get good grades in mathematics with no effort, while having to memorize the complete works of Shakespeare and a number of obscure poets seemed forbidding. Thus he completed his undergraduate degree in mathematics in 1939. This was the Harvard of G. D. Birkhoff, Mac Lane, Quine, Huntington, M. H. Stone, H. Whitney, J. L. Coolidge, G. Birkhoff, and L. Loomis -- as always, a major center of mathematics in America. Upon graduation, after having spent most of his life in New England, Lyndon decided to see what the rest of the country was like and spent almost a year working in a bank in Albuquerque. Then he returned to Harvard to earn a Master's degree in 1941.

Lyndon taught at Georgia Tech during the academic year 1941-42. After the war began he participated in a program for teaching navigation to Navy pilots. In 1942 he returned to Harvard to teach in the Navy V-12 program and remained to receive a Ph.D. in 1946. Lyndon's first mathematical interest was logic. He studied Quine's system and discovered that it contained a major error but never published this result. After becoming interested in relational algebras he finally decided to write a thesis in homological algebra. The thesis was a brilliant piece of mathematics; it anticipated much of the development of spectral sequences. Saunders Mac Lane, who was Lyndon's thesis adviser, describes this work in detail in another article in this volume. Alfred Tarski arrived in Cambridge during Lyndon's final year at Harvard and Lyndon attended Tarski's course. This was the beginning of a long personal and professional friendship that led to Lyndon's work in model theory a decade later.

After two years in London with the Office of Naval Research, Lyndon felt that to work in cohomology he should study topology further, and he went to work at Princeton University, the center of American topology at that time. The Princeton faculty then included Lefshetz, Steenrod, Artin, Church, Bochner,

Tucker, Spencer, and Fox. While at Princeton he was greatly influenced by Ralph Fox, whose course in knot theory he attended. The first semester of Fox's course covered the group theoretic background for knot theory and Lyndon became much more interested in the group theory than the knot theory. During that time he had close contact with K. Reidemeister who visited for a year, and S. Eilenberg whom he had known at Harvard and who came frequently to Princeton to collaborate with Steenrod. After five years as an instructor and assistant professor at Princeton, Lyndon joined the faculty of the University of Michigan as an assistant professor in 1953. He has remained at Michigan ever since except for visiting positions at the University of California, Berkeley (1956-57), the Institute for Defense Analyses, Princeton (1959-60), Queen Mary College, London (1960-61 and 1964-65), Morehouse College, Atlanta (1969), Université de Montpellier, France (1974-75), and Université de Picardie, France (1980-81). He was promoted to associate professor in 1956 and professor in 1959.

During his years on the Michigan faculty Lyndon has been the thesis adviser of twelve Ph.D. students, of whom I was privileged to have been the second. The diversity of topics of the theses of his students serves as well as his own bibliography to illustrate the breadth of his mathematical interests. These topics include decision problems, model theory, algorithmic problems, cohomology, combinatorial group theory, and group rings (the names of Lyndon's students and their thesis titles are listed separately).

I fondly remember my experiences working as a thesis student in the years 1957-59. My conversations with Lyndon often took place in the newly established commons room, which was supplied with a large coffee percolator. As a "morning person," I would want to discuss my work with Lyndon as soon as he arrived at Angell Hall. It soon became clear to me that no mathematical discussion was possible until he had consumed at least two cups of coffee, so during my last two years of graduate work I was the unofficial coffee preparer for the commons room.

Lyndon produces elegant mathematics and thinks in terms of broad and deep ideas. Paul Schupp and John Ratcliffe, who have styles similar to his, tell me that as students they had a very easy time communicating their ideas to him and understanding his. I tend to work from the combinatorial aspects of particular problems without thinking of abstract structural considerations. While I was a graduate student, many of my conversations with Lyndon proceeded as follows. I would announce proudly that I could prove a lemma and write it on the blackboard, and he would copy it down. I would then proceed through a detailed if rough hewn proof. When I had finished, he would admit that he had not followed much of what I had said. A day later, working from the statement of my lemma and a few recollections of my methods, he would show me an extremely elegant proof. It often took me quite some time to understand what he had done, and I learned a good bit of mathematics in the process.

I have become fascinated with several problems that he has attacked in his papers and the methods he has developed to work on them. This interest had led me to study these problems further and proceed a bit further along the path. Thus he has continued to inspire my work in the more than twenty years since I have left Ann Arbor. I am sure that this is the case with several of his other students. In reading Lyndon's papers, I am impressed by the mathematical power that he uses, even in the proof of specialized results. I once asked him whether there was a thread common to his diverse work in so many fields of mathematics; he replied that he felt that the problems on which he worked had all been combinatorial in nature. In agreeing with this method of classifying his work, one would certainly have to put him in the very first rank of those who have used combinatorial techniques in the last forty years.

The articles by Keisler, Mac Lane, Ratcliffe, and Schupp in this volume help us to appreciate Lyndon the mathematician. To appreciate Lyndon the man, one should read his article "Saunders Mac Lane as a shaper of mathematics and mathematicians" [52]. In that tribute to Mac Lane, Lyndon provides a

catalogue of his own personal values and evidence of his own modesty that would be hard to match here.

A tall man of distinguished appearance with a deep voice, Lyndon is the kind of person who quickly makes people feel comfortable working with him. Mathematical conversations with him have the same effect that locker room orations of a great football coach have on team members -- one leaves with the enthusiasm to work on a level far above one's ability and a feeling that it might be possible to do so, forgetting, perhaps because of Lyndon's self-effacing manner, that he brings to similar tasks an extraordinary talent and energy that few others possess.

Ph.D. Students of Roger C. Lyndon

		Thesis Title
Gerald O. Losey	1958	Group Rings and Dimension Subgroups
Kenneth I. Appel	1959	Two Investigations on the Borderline of Logic and Algebra
Calvin C. Elgot	1960	Decision Problems of Finite Automata Design and Related Arithmetic
Paul E. Schupp	1966	On Dehn's Algorithm and the Conjugacy Problem
Charles S. Holmes	1967	Projectivities of Free Products
Arthur G. Conn	1969	Continuous Model Theory and Set Theory
Nancy E. King	1969	Real Length Functions in Groups
Gerald E. Meike	1969	A Decision Procedure for the Prefix Class
Norman J. Frisch	1970	Automorphisms of the Fundamental Group of an Orientable 2-Manifold
John H. Remmers	1971	Some Algorithmic Problems for Semigroups: A Geometric Approach
Ian M. Chiswell	1973	On Groups Acting on Trees
John G. Ratcliffe	1977	The Theory of Crossed Modules with Applications to Cohomology of Groups and Combinatorial Homotopy Theory (jointly directed by James Kister)
Libo Lo	1984	On the Computational Complexity of the Theory of Abelian Groups

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