Functional Integrals in Quantum Field Theory and Statistical Physics

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FUNCTIONAL INTEGRALS IN QUANTUM FIELD THEORY AND STATISTICAL PHYSICS

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FOREWORD

Functional integration is one of the most powerful methods of contemporary theoretical physics, enabling us to simplify, accelerate, and make clearer the process of the theoretician's analytical work. Interest in this method and the endeavour to master it creatively grows incessantly. This book presents a study of the application of functional integration methods to a wide range of contemporary theoretical physics problems.

The concept of a functional integral is introduced as a method of quantizing finite-dimensional mechanical systems, as an alternative to ordinary quantum mechanics. The problems of systems quantization with constraints and the manifolds quantization are presented here for the first time in a monograph.

The application of the functional integration methods to systems with an infinite number of degrees of freedom allows one to uniquely introduce and formulate the diagram perturbation theory in quantum field theory and statistical physics. This approach is significantly simpler than the widely accepted method using an operator approach.

A large part of this book is devoted to the development of nonstandard methods of perturbation theory using specific examples, the first of which is the theory of gauge fields. The method of functional integration with necessary modifications is used for the quantization of the electromagnetic field, the Yang–Mills field, and the gravitational field. Attempts to construct a unified gauge invariant theory of electromagnetic and weak interactions are explored here, too. The next example of functional integration is the derivation of the infrared asymptotic behaviour of the Green's function of quantum electrodynamics. We shall examine the application of functional integration to problems of scattering of high-energy particles and the formulae for a doubly-logarithmic asymptotic and eikonal approximation will be obtained.

The applications of functional integrals to problems of statistical physics begin with examples of superfluidity, superconductivity, and plasma theory. A modified perturbation theory for superfluid Bose and Fermi systems is used in the microscopic approach to the construction of the hydrodynamic Hamiltonian of the system and the equations of superfluid hydrodynamics. For the first time in a monograph, the question of superfluidity of two-dimensional and one-dimensional Bose systems is elucidated. The method describing quantum vortices in Bose and Fermi systems is developed and applied, specifically, to the theory of superconductivity of the second type. The method for using the hydrodynamical Hamiltonian for systems with Coulomb interaction is illustrated in application to the theory of plasma oscillations. The example of a problem which allows such a solution in functional integration formalism is the Ising model. In the last chapters dealing with statistical physics, the Wilson method is studied which utilizes functional integration for the theory of phase transitions. The closing chapter of the book extends the notion of excitations of the quantum vortex type which is customary in statistical physics onto the quantum field theory. This trend, bent on diminishing the number of fundamental fields, is a fairly recent development.

The book need not be read successively. After one becomes acquainted with the definition of the functional integral and the methods of construction of diagram perturbation theory (see (Chapters 1, 2), it is possible to concentrate on those applications of functional integrals to physical systems, which are most interesting. The choice of specific examples is, to a great extent, determined by the scientific interests of the author.

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CONTENTS

CHAPTER 1

FUNCTIONAL INTEGRALS AND QUANTUM MECHANICS

1. Introduction

Functional integrals were introduced into mathematics during the twenties by Wiener as a method for solving the problems of diffusion and Brown's motion [1]. In physics, functional integrals were rediscovered in the forties by Feynman and used by him for the reformulation of quantum mechanics. In the late forties, Feynman constructed a new formulation of quantum electrodynamics, based on the method of functional integration, and developed the now-famous diagram technique of perturbation theory [2–4]. This new theory has substantially simplified calculations and has helped to construct the theory of renormalization. The latter has turned out to be an important leap in solving the problem of divergencies which have arisen in quantum electrodynamics since its formulation in the 1929 paper of Heisenberg and Pauli [5]. At the present time, the theory of electromagnetic interactions agrees with experiments up to the seventh decimal digit, and this is one of the merits of the new perturbation theory.

Since the fifties, functional integrals, arising when solving functional equations in quantum field theory (Schwinger equation [6]), have been intensively studied. The functional formulation of quantum field theory has been investigated in the works of Bogoliubov [7], Gel'fand and Minlos [8], Matthews and Salam [9], Khalatnikov [10] and Fradkin [11].

In the sixties, a new field of applications of functional integrals appeared – the quantization of gauge fields. The electromagnetic field, the Einstein gravitational field, the Yang–Mills field and the chiral field can serve as examples of gauge fields. The action functionals of those fields are invariant under gauge transformations which depend on one or several arbitrary functions. From a mathematical point of view, gauge fields are fields of geometrical origin which are connected with fibrations over four-dimensional space-time. The specificity of geometrical fields has to be taken into account when quantizing them; otherwise incorrect results may be obtained.

This was noticed for the first time by Feynman in the Yang–Mills and gravitational fields. He has shown that quantization according to a method analogous to the Fermi method in quantum electrodynamics, violates the unitarity condition. Feynman also proposed a method for the removal of the difficulties shown.

Later, as a result of the work of several authors, the problem of gauge-field quantization was studied and the functional integration method [13–18] has turned out to be the most convenient method for solving that problem.

A special place among gauge fields is occupied by the gravitational field. The question of its quantization is connected with the hope that it is a natural physical regularizator cutting the interaction on small distances. The first results supporting this point of view were obtained by De Witt [19] and Khriplovich [20].

At the present time, the method of functional integration is most frequently applied to problems which are some how connected with gauge fields. It is necessary to underline its utilization in attempts to construct a unified theory of electromagnetic and weak interactions [21–23].

There exist many other applications of functional integration to quantum field theory. For example, by using this method it becomes simple to derive various asymptotic formulae for infrared and ultraviolet asymptotics of the Green functions and scattering amplitudes [60, 67–71]. The utilization of functional integrals also turns out to be interesting in dual models.

In the late sixties and early seventies, the theory of automodel (scaling) behaviour of quantum field theory amplitudes at high energies, which has much in common with the theory of phase transitions of the second kind, was developed. Here, the method of functional integration helps to qualitatively describe the picture of high-energy particle scattering and of critical phenomena, and to approximately evaluate the power indices (critical indices).

Lately, a new field of applications of functional integrals has appeared which is connected with the search for excitations in quantum field theory and is analogous to the quantum vortices in statistical physics. The idea that some of the elementary particles can be looked upon as collective excitations of the interacting fundamental fields system allows one to reduce the number of fundamental fields. The method of functional integration is perhaps the only acceptable approach to the solution of the problems appearing here. Some of the results obtained this way are expounded in Chapter 11 of this book.

The application of functional integrals in statistical physics allows one to derive many interesting results which are only obtained with difficulty by other methods. Feynman applied this approach to polaron theory and to the liquid helium theory and he succeeded in accurately evaluating the self-energy of a polaron and in investigating the qualitative features of the λ-transition in liquic helium.

The theory of phase transitions of the second kind, superfluidity, superconductivity, lasers, plasma, Kondo effect, Ising model – this is an incomplete list of problems, for which the application of the functional integration method appears to be very useful. In some of the problems, it allows us to prove results obtained by other methods, clarify the possibilities of their applicability and outline the evaluation of corrections. If there is a possibility of an exact solution, the method of functional integration gives a simple way of obtaining it. In problems far from being exactly solvable (general theory of phase transitions), the application of functional integrals helps to build up the qualitative picture of the phenomenon and to develop the approximative methods of calculations.

Functional integrals are especially useful for the description of collective excitations, such as plasma oscillations in the theory of the system of particles with Coulomb interaction, quantum vortices and long-wave photons in the theory of superfluidity and superconductivity. That is the case when standard perturbation theory should be modified. Functional integrals represent a sufficiently flexible mathematical apparatus, adjusted for such a revision and suggesting the method for its concrete realization.

Functional integration is an 'integral evaluation' adjusted to the needs of contemporary physics. At present, however, the exact mathematical theory and correct definition of functional integrals used in quantum field theory and statistical physics is lacking.

The exact definition and correct mathematical theory can be constructed for functional integrals which give solutions of partial differential equations, including the equations of quantum mechanics and diffusion theory. Mathematical questions of the theory of functional integrals are expounded in the surveys of Gel'fand and Yaglom [26], Kovalchik [27], in the books of Kac [28] and Berezin [29]. Let us also mention the works of Berezin [30], Daletzki [31], Evgraphov [32], Alimov and Buslaiev [33], devoted to the exact definition of some functional integrals.

In the works performed on the physical level of exactness, the functional integral is used as a heuristic means for the construction of perturbation theory and for the transition from one perturbation theory to another. From this point of view the functional integrals are studied in this book.

2. Functional integrals in quantum mechanics

We present here the definition of the functional integral in quantum mechanics. Feynman in his 1948 article [2] introduced and studied the functional integral in configuration space. For many applications, however, the most suitable form seems to be the expression given by Feynman in 1951 [4], where integration is taken along trajectories in the phase space.

Let us investigate the one-dimensional mechanical system determined by its Hamilton function $H(q,p)$, where q is the coordinate and p is the canonically conjugated momentum. The principle of canonical quantization of such a system consists of replacing the coordinate q and momentum p by operators $\hat{q}$ and $\hat{p}$ according to the rule

$$q \to \hat{q} \equiv q, \qquad p \to \hat{p} = -\mathrm{i}\,\hbar \frac{\partial}{\partial q}, \tag{2.1}$$

where $\hbar$ is the Planck constant. In the following we shall use the system of units with $\hbar = 1$. The operators act on a Hilbert space of complex functions $\Psi(q)$. According to (2.1), the effect of the coordinate operator on the function $\Psi(q)$ is a multiplication of that function by the variable q and the operator of momentum is proportional to the differentiation operator $\partial/\partial q$.

The time evolution of a state is determined by the Heisenberg equation

$$\mathrm{i}\frac{\partial \Psi}{\partial t} = \hat{H}\Psi, \tag{2.2}$$

where $\hat{H}$ is the energy operator obtained from the classical Hamiltonian function $H(q,p)$ by replacing q and p, according to (2.1), with operators $\hat{q}$ and $\hat{p}$ ordered in a certain way. We can write down the formal solution of equation (2.2) as

$$\Psi(t) = \hat{U}(t,t_0)\Psi(t_0), \tag{2.3}$$

where the evolution operator

$$\hat{U}(t,t_0) = \exp\left(\mathrm{i}(t_0 - t)\hat{H}\right) \tag{2.4}$$

is the exponential of the energy operator $\hat{H}$.

The method of functional integration allows one to express the matrix element of the evolution operator as a mean value of the expression

$$\exp(\mathrm{i}S[t_0, t]), \tag{2.5}$$

over trajectories in the phase space, where

$$S[t_0, t] = \int_{t_0}^{t} (p(\tau)\dot{q}(\tau) - H(q(\tau), p(\tau)))\,\mathrm{d}\tau \tag{2.6}$$

is a classical action corresponding to the trajectory $(q(\tau), p(\tau)), (t_0 \le \tau \le t)$, in the phase space, $\dot{q}(\tau) \equiv \mathrm{d}q(\tau)/\mathrm{d}\tau$.

The mean value over trajectories is called the *Feynman functional integral.* Usually this is defined as a limit of finite-dimensional integrals. We shall present here one of the possible definitions.

We divide the interval $[t_0, t]$ with $\tau_1, \ldots, \tau_{N-1}$ points into N equal parts. Let us consider the functions $p(\tau)$, defined on the interval $[t_0, t]$, which are constant on the intervals

$$[t_0, \tau), (\tau_1, \tau_2), \ldots, (\tau_{N-1}, t], \tag{2.7}$$

and the continuous functions $q(\tau)$ linear on the intervals (2.7). We fix the values of the function $q(\tau)$ at the end points of the interval $[t_0, t]$, putting

$$q(t_0) = q_0\,; \qquad q(t) = q. \tag{2.8}$$

The trajectory $(q(\tau), p(\tau))$ is determined by values of the piecewise linear function $q(\tau)$ in points $\tau_1, \ldots, \tau_{N-1}$ (we denote them by $q_1, \ldots, q_{N-1}$) and by values of the piecewise constant function $p(\tau)$ on intervals (τ_k, τ_{k+1}). We denote those values by $p_1, \ldots, p_N$.

Let us consider the finite-dimensional integral

$$(2\pi)^{-N} \int \mathrm{d}p_1\, \mathrm{d}q_1 \ldots \mathrm{d}q_{N-1}\, \mathrm{d}p_N \exp(\mathrm{i}S[t_0, t]) \equiv J_N(q_0, q; t_0, t), \tag{2.9}$$

where $S[t_0, t]$ is the action (2.6) for the described trajectory $(q(\tau), p(\tau))$, defined by the parameters $q_1, \ldots, q_N, p_1, \ldots, p_N$. The basic assertion says that the limit of the integral (2.9) for $N \to \infty$ is equal to the matrix element of the evolution operator

$$\lim_{N \to \infty} J_N(q_0, q; t_0, t) = \langle q | \exp(\mathrm{i}(t_0 - t)\hat{H}) | q_0 \rangle. \tag{2.10}$$

It is not hard to check this statement in those cases when the Hamiltonian H is a function of the coordinate or momentum only.

If $H = H(q)$ (H depends on the coordinate only), then the classical action for the above-mentioned trajectory $(q(\tau), p(\tau))$, takes the form

$$\int_{t_0}^{t} (p\dot{q} - H(q))\,\mathrm{d}\tau = p_1(q_1 - q_0) + p_2(q_2 - q_1) + \cdots$$

$$\cdots + p_N(q - q_{N-1}) - \int_{t_0}^{t} H(q(\tau))\,\mathrm{d}\tau. \tag{2.11}$$

Integrating in (2.9) over the momenta, we obtain the product of δ-functions:

$$\delta(q_1 - q_0)\delta(q_2 - q_1)\cdots\delta(q - q_{N-1}). \tag{2.12}$$

This product allows us to put the expression $\exp(-\mathrm{i}\int_{t_0}^{t} H(q(\tau))\,\mathrm{d}\tau)$ to be equal to $\exp(\mathrm{i}(t_0 - t)H(q_0))$ and place it in front of the integration symbol. Further integration over $q_1, \ldots, q_{N-1}$ coordinates eliminates all δ-functions except one and leads to the result

$$\delta(q_0 - q)\exp(\mathrm{i}(t_0 - t)H(q_0)), \tag{2.13}$$

identical to the matrix element of the evolution operator.

If $H = H(p)$ (H depends on momentum only) then the action takes the form

$$\int_{t_0}^{t} (p(\tau)\dot{q}(\tau) - H(p(\tau))\,\mathrm{d}\tau = p_1(q_1 - q_0) + p_2(q_2 - q_1) + \cdots$$

$$\cdots + p_N(q - q_{N-1}) - \int_{t_0}^{t} H(p(\tau))\,\mathrm{d}\tau. \tag{2.14}$$

Integrating in (2.9) first over coordinates $q_1, \ldots, q_{N-1}$ and then over all momenta $p_1, \ldots, p_N$ we obtain the expression

$$\frac{1}{2\pi}\int \mathrm{d}p \exp\{\mathrm{i}p(q - q_0) + \mathrm{i}(t_0 - t)H(p)\}, \tag{2.15}$$

equal to the matrix element of the evolution operator for the Hamiltonian $H = H(\hat{p})$.

The proof of formula (2.10) is more complicated if nontrivial dependence of the Hamiltonian on the coordinate and momentum occurs. In such a case, the prelimit expression (2.9) is not identical to its limit – the evolution operator matrix element. Proof of a formula analogous to (2.10) for the evolution operator of the system described by a parabolic-type equation is given, e.g., in Reference [32]. For the Schrodinger equation, the proof is known only if operator $\hat{H}$ is a sum of a function of coordinates and a function of momenta:

$$H = H_1(q) + H_2(p). \tag{2.16}$$

Namely, the Hamiltonians of the (2.16) type are used in nonrelativistic quantum mechanics.

We denote the functional integral, defined as the $N \to \infty$ limit of expression (2.9), by the symbol

$$\int_{q(t_0)}^{q(t)} \exp(\mathrm{i}S[t_0,t]) \prod_\tau \frac{\mathrm{d}p(\tau)\,\mathrm{d}q(\tau)}{2\pi}. \tag{2.17}$$

This form is convenient but it does not reflect the fact that in the prelimit expression (2.9) the number of integrations over momenta is higher by one order than that over the coordinates.

Let us remark that the functional integral, defined by formula (2.10) as a limit of the finite-dimensional one, depends on the method of approximation to the $(q(\tau), p(\tau))$ trajectory. This is connected with the fact that we have no natural prescription for the ordering when replacing the arguments of the function $H(p, q)$ by noncommuting operators $\hat{q}$ and $\hat{p}$. However, the operators with a physical meaning correspond, as a rule, to the functions for which the replacement of arguments by noncommuting operators leads to an unambiguous results. This is true for the energy operator in nonrelativistic quantum mechanics which is equal to the sum of a quadratic function of momenta and a function of coordinates. In such cases the functional integral leads to unambiguous results, too.

We generalize the functional integral formalism to a system with an arbitrary finite number of degrees of freedom.

The action of a mechanical system with n degrees of freedom has the form

$$S[t_0, t] = \int \left(\sum_{i=1}^{n} p_i \dot{q}^i - H(q, p) \right) \mathrm{d}\tau. \tag{2.18}$$

Here q^i is the ith canonical coordinate; p_i is the canonically conjugated momentum;

$$H(q, p) \equiv H(q^1, \dots, q^n; p_1, \dots, p_n) \text{ is the Hamiltonian.}$$

By definition, the functional integral for the evolution operator matrix element is a limit of the finite-dimensional integral obtained from (2.9) by the replacement

$$(2\pi)^{-N} \to (2\pi)^{-Nn}; \qquad dq_k \to \prod_{i=i}^{n} dq_k^i; \qquad dp_k = \prod_{i=i}^{n} dp_{i,k}, \tag{2.19}$$

where q_k^i are the values of the ith coordinate at the point $\tau_k(k = 1, \ldots, N-1)$ and p_{ik} are the values of the ith momentum on (τ_{k-1}, τ_k) interval. It is necessary to keep all the coordinates $q^1, \ldots, q^n$ simultaneously fixed at both ends of the time interval $[t_0, t]$.

We will denote the functional integral defined in such a way by the symbol

$$\int_{q(t')=q'}^{q(t'')=q''} \exp(\mathrm{i}S) \prod_t \prod_{i=1}^{n} \frac{dq^i(t)\, dp_i(t)}{2\pi}. \tag{2.20}$$

3. Quantization of systems with constraints

In the previous section the quantization of finite-dimensional mechanical systems with the action of Hamilton type (2.18) was studied using the technique of functional integration. Field theory can be looked upon as an infinite-dimensional analogy of a mechanical system with action (2.18). In such an approach the theory of gauge fields is an analogy of mechanical systems with constraints. The quantization of finite-dimensional systems with constraints requires the modification of the functional integral.

A classical action of the finite-dimensional mechanical system with constraints

$$S = \int \left(\sum_{i=1}^{n} p_i \dot{q}^i - H(q,p) - \sum_{a=1}^{m} \lambda_a \varphi^a(q,p) \right) \tag{3.1}$$

also contains, besides the coordinates q and momenta p, the variables λ_a, which come in linearly and play the role of Lagrange multiplicators. The coefficients $\varphi^a(q,p)$ have the meaning of constraints. The variables q, p generate the phase space of dimension $2n$. The number of constraints shall be denoted as m. We suppose that $m < n$ and that the constraints φ^a and Hamiltonian H are in involution, i.e., that they fulfil the conditions

$$\{H, \varphi^a\} = \sum_b c_b^a \varphi^b; \qquad \{\varphi^a, \varphi^b\} = \sum_d c_d^{ab} \varphi^d. \tag{3.2}$$

In these formulae c_b^a, c_d^{ab} are functions of q and p and $\{f, q\}$ is the Poisson bracket:

$$\{f,g\} = \sum_{i=1}^{n} \left(\frac{\partial f}{\partial p_i} \frac{\partial g}{\partial q^i} - \frac{\partial f}{\partial q^i} \frac{\partial g}{\partial p_i} \right). \tag{3.3}$$

The system of equations of motion for action (3.1) contains, besides canonical equations

$$\dot{q}^i = \frac{\partial H}{\partial p_i} + \sum_{a=1}^{m} \lambda_a \frac{\partial \varphi^a}{\partial p_i}; \qquad \dot{p}_i = -\frac{\partial H}{\partial q^i} - \sum_{a=1}^{m} \lambda_a \frac{\partial \varphi^a}{\partial q^i}. \tag{3.4}$$

also the constraint equations

$$\varphi^a(q,p) = 0, \quad a = 1, \ldots, m. \tag{3.5}$$

It is apparent from Equations (3.5) that some of the variables q, p are spurious, i.e., the solution of constraint equations often turns out to be rather difficult. It is therefore desirable to have a formalism where explicit solutions of constraint equations are not required.

Constraint Equations (3.5) define the surface M of the $2n - m$ dimensions in phase space Γ. The involution conditions (3.2) guarantee, for arbitrary functions $\lambda_a(t)$, the fulfilment of constraint conditions (3.5), provided those equations are satisfied for initial conditions. In other words, a trajectory which starts on the manifold M does not leave it.

We shall regard as observables on the manifold M the variables which are not influenced by arbitrariness in the choice of $\lambda_a(t)$. This requirement is fulfilled by the functions $f(q,p)$, which obey the conditions

$$\{f, \varphi^a\} = \sum_b d_b^a \varphi^b. \tag{3.6}$$

Indeed, in the equations of motion for those functions

$$\dot{f} = \{H, f\} + \sum_a \lambda_a \{\varphi^a, f\} \tag{3.7}$$

the λ_a-depending terms vanish on M.

The function $f(q,p)$ defined on M and satisfying conditions (3.6) does not in fact depend on all variables. Conditions (3.6) can be looked upon as a system of m differential equations of the first order on M for which Equations (3.2) are conditions of integrability. The function f is therefore unambiguously defined by its values on a submanifold of the system's initial conditions which has the dimension $(2n - m) - m = 2(n - m)$. It is convenient to take as such a manifold a surface Γ^*, defined by constraint equations (3.5) and m additional conditions:

$$\chi_a(q,p) = 0, \; a = 1, \ldots, m. \tag{3.8}$$

The functions χ_a must satisfy the condition

$$\det \| \{\chi_a, \varphi^b\} \| \neq 0, \tag{3.9}$$

because only in that case can Γ^* play the role of an initial surface for Equation (3.6). It is convenient to suppose that χ_a mutually commute*:

$$\{\chi_a, \chi_b\} = 0. \tag{3.10}$$

In such a case it is possible to introduce canonical variables onto the manifold Γ^*. Indeed, if condition (3.9) is satisfied, then, using canonical transformation in Γ, we can introduce a new set of variables where χ_a take a simple form:

$$\chi_a(q, p) = p_a, \quad a = 1, \ldots, m, \tag{3.11}$$

where $p_a (a = 1, \ldots, m)$ is a subset of canonical momenta of the new system of variables. Condition (3.9) can be written, in terms of the new variables, as

$$\det \left\| \frac{\partial \varphi^a}{\partial q^b} \right\| \neq 0, \tag{3.12}$$

and the constraint Equation (3.6) can therefore be solved with respect to q^a. Finally, the surface Γ^* is given by the equations

$$p_a = 0, \qquad q^a = q^a(q^*, p^*), \tag{3.13}$$

on Γ, so that q^* and p^* are independent, canonically constructed, variables on Γ^*.

Let us study now what the functional integral for the finite-dimensional mechanical system with constraints looks like. We shall introduce additional conditions $\chi_a(q, p)$ so that relations (3.9) and (3.10) are satisfied. The basic assertion is that the evolution operator matrix element is given by the functional integral

$$\int \exp\left\{ i \int_{t_0}^{t} \left(\sum_{i=1}^{n} p_i \dot{q}^i - H(q, p) \right) d\tau \right\} \prod_{\tau} d\mu(q(\tau), p(\tau)), \tag{3.14}$$

where the integration measure is given by the formula

$$d\mu(\tau) = (2\pi)^{m-n} \det \| \{\chi_a, \varphi^b\} \| \prod_a \delta(\chi_a)\delta(\varphi^a) \prod_{i=1}^{n} dq^i(\tau) dp_i(\tau). \tag{3.15}$$

*As from now, we shall mean by a commutator of functions f and g on phase space the Poisson bracket $\{f, g\}$ (3.3). The functions commute if the Poisson bracket is equal to zero.