

# Unsolved Problems in Astrophysics

**JOHN N.  
BAHCALL  
and  
JEREMIAH P.  
OSTRIKER,  
Editors**



John N. Bahcall  
and  
Jeremiah P. Ostriker, Editors

# **UNSOLVED PROBLEMS IN ASTROPHYSICS**

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## PREFACE

The articles in this volume were written in response to the following hypothetical situation. A second year graduate student walks into the author's office and says: "I am thinking of doing a thesis in your area. Are there any good problems for me to work on?"

Leading astrophysical researchers answer the student's question in this collection by providing their views as to what are the most important problems on which major progress may be expected in the next decade. The authors summarize the current state of knowledge, observational and theoretical, in their areas. They also suggest the style of work that is likely to be necessary in order to make progress. The bibliographical notes at the end of each paper are answers to the parting question by the hypothetical graduate student: "Is there anything I should read to help me make up mind about a thesis?"

As everyone knows who reads a newspaper or listens to the daily news, astrophysics is in the midst of a technologically driven renaissance; fundamental discoveries are being made with astonishing frequency. Measured by the number of professional researchers, astrophysics is a small field. But, astronomical scientists have the entire universe outside planet earth as their exclusive laboratory. In the last decade, new detectors in space, on earth, and deep underground have, when coupled with the computational power of modern computers, revolutionized our knowledge and understanding of the astronomical world. This is a great time for a student of any age to become acquainted with the remarkable universe in which we live.

In order to make the texts more useful to students, each of the papers was "refereed" by cooperative graduate students and colleagues. On average, each paper was refereed four times. We would like to express our gratitude to the referees; their work made the papers clearer, more accessible, and in some cases, more correct. We are grateful to each of the authors for wonderful manuscripts and for their cooperation in what must have at times seemed like an endless series of iterations. The excellent quality of the final texts justifies their hard work.

In looking over the material as it now appears, we believe that these papers may have a wider readership than we originally anticipated. Most of the articles are accessible to junior or senior undergraduate students with a good science background. The book can therefore be useful as an undergraduate introduction to some of the important topics in modern astrophysics. We hope that readers who are

graduate students now or in the future will solve many of the problems listed here as unsolved. Anyone, from an undergraduate science major to a senior science faculty member, who would like to know more about some of the active areas of contemporary astrophysics can profit by reading about what these researchers think are the most important solvable problems.

The articles collected here were originally presented as invited talks at a conference entitled ‘Some Unsolved Problems in Astrophysics’ that was held at the Institute for Advanced Study on April 27-29, 1995. This conference was sponsored in part by the Sloan Foundation, to whom we express our gratitude. The dates for the conference were related to the 60th birthday (and 25th year at the Institute) of one of us (JNB), but nearly every effort was made to focus the meeting on science, not anniversaries. However, a large fraction of the attendees and speakers were alumni of the Institute’s postdoctoral program in astronomy and astrophysics.

The manuscript for this book was expertly prepared by Margaret Best. All of us are grateful to Maggie for her exceptional editorial and TeX skills and for her constant good nature.

John N. Bahcall, Jeremiah P. Ostriker  
Princeton, June 1996

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## CHAPTER 1

### THE COSMOLOGICAL PARAMETERS

P. J. E. PEEBLES

Joseph Henry Laboratories, Princeton University, Princeton, NJ

#### ABSTRACT

The tests of the relativistic cosmological model are well understood; what is new is the development of means of applying them in a broad variety of ways capable of producing a network of consistency checks. When the network is tight enough we will have learned either that general relativity theory passes a highly nontrivial test or that something is wrong with classical physics. I see no reason to look for the latter but we should keep it in mind. In the former case we will have gained the boundary conditions for a deeper cosmology, and a new set of puzzles to study.

#### 1.1 INTRODUCTION

The first thing to know about the measurement of the parameters of the standard relativistic cosmological model is that the problem has been with us for a long time. By the 1930s people understood the physics of the evolving relativistic cosmology and how astronomical observations might be used to test and constrain the values of its parameters. The first large-scale application of the astronomical tests, the count of galaxies as a function of apparent magnitude (Hubble 1936), had already reached redshift  $z \sim 0.4$  (inferred from the values of Hubble's counts and more recent measurements of the mean redshift-count relation). The application of the cosmological tests was a "key project" for the 200 inch telescope when it was under construction in the 1930s; now it is a key project for the Hubble Space Telescope and the Keck Telescope.

There has been ample time for the development of strong opinions on what the results of the measurements are likely to be, and for a tendency to lose sight of

the reasons for measuring the parameters. To my mind there are two main goals: extend the tests of the physics of the standard model, and seek clues to a cosmology based on a deeper level of physics. Despite the sobering record there is reason to believe we may actually be witnessing the end game in finding useful measurements of the parameters.

## 1.2 WHY MEASURE THE PARAMETERS?

### 1.2.1 *Testing the Physics*

In the standard cosmological model the universe is close to homogeneous and isotropic in the large-scale average, and it is homogeneously expanding and cooling. The evidence, which most cosmologists agree is quite strong, is summarized for example in Peebles, Schramm, Turner, and Kron (1991) and Peebles (1993). (I refer the reader to these references for details of the following comments.) The evolution of the standard model is described by general relativity theory. This part is not as closely probed, and one goal of the cosmological tests is to broaden the constraints on the underlying gravity physics. Here is the situation.

To begin, I assume the geometry of our spacetime is described by a single metric tensor, that is, a single line element which determines the relations among measured distance or time intervals between events in spacetime. The evidence is that our universe is close to homogeneous and isotropic in the large-scale average, and we know the line element of a homogeneous and isotropic spacetime is unique up to coordinate transformations; the Robertson-Walker form is<sup>1</sup>

$$ds^2 = dt^2 - a(t)^2 dl^2, \quad dl^2 = \frac{dr^2}{1 \pm r^2/R^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2. \quad (1.1)$$

In this equation a comoving observer, who moves so the universe is seen to be isotropic in the large-scale mean, has fixed position  $r, \theta, \phi$ . The proper time kept by such comoving observers is  $t$  (if the observers' clocks are synchronized so all see the same mean mass density  $\rho_i$  at a given time  $t_i$ ). The parameter  $R^{-2}$  in the expression  $dl^2$  for the spatial part of the line element measures the radius of curvature of the three-dimensional space sections of fixed world time  $t$ . If the prefactor of  $R^{-2}$  is negative space is closed, as in the surface of a balloon. The expansion factor  $a(t)$  in the expression for  $ds^2$  means the balloon in general may be expanding or contracting; the evidence is that our universe is expanding. If the prefactor of  $R^{-2}$  is negative space sections are open, the circumference of a circle of radius  $r$  being larger than  $2\pi r$ . In this case the nearly homogeneous

---

<sup>1</sup>Corrections for the departures from homogeneity are important for some of the cosmological tests, and will have to be reconsidered as the tests improve. The issues are discussed in Peebles (1993).

space we observe may extend to indefinitely large distances, or space might be periodic, or conditions beyond the distance we can observe might be very different. If  $R^{-2} = 0$  then  $dl^2$  is the familiar Cartesian form for flat space, and spacetime is said to be cosmologically flat.

The proper physical distance between comoving observers, measured at given world time, is  $D = la(t)$ , where the coordinate distance  $l$  is the result of integrating the second part of equation (1.1) along the geodesic connecting the observers at fixed  $t$ . The rate of change of the proper distance is

$$v = \frac{dD}{dt} = HD, \quad H = \frac{1}{a} \frac{da}{dt}. \quad (1.2)$$

If  $v$  is much less than the velocity of light this gives a good picture for the predicted linear relation between distance and relative velocity in a homogeneous expanding universe.<sup>2</sup> Thus a galaxy at distance  $D$  is moving away at recession velocity  $v = HD$ , and the resulting Doppler effect stretches the wavelength of the radiation received from the galaxy by the amount

$$z \equiv \frac{\delta\lambda}{\lambda} = HD/c. \quad (1.3)$$

The constant of proportionality  $H$  is Hubble's constant; its present value usually is written as  $H_0$ . The linear relation between redshift and distance, which is Hubble's law, has been tested to redshifts on the order of unity; for an example see Figure 7 in McCarthy (1993).

Hubble's law was one of the first pieces of evidence leading to the discovery of the relativistic cosmology, but we see that the functional form follows more generally from the observed large-scale homogeneity of the universe.

When the redshift is comparable to or larger than unity equation (1.2) does not directly apply (because  $D$  is measured along a surface of fixed cosmic time  $t$ , which is not how light from a distant galaxy travels to us). The easy way to analyze the redshift in this case is to imagine that the electromagnetic field is decomposed into normal modes of oscillation with fixed boundary conditions in the space coordinates of equation (1.1). The boundary conditions mean the physical wavelength  $\lambda$  of a mode stretches as the universe expands:

$$\lambda \propto a(t). \quad (1.4)$$

If the interaction of the radiation with other matter and fields is weak then adiabaticity tells us the number of photons in each mode is conserved, which is to

<sup>2</sup>As in equation (1.1), this assumes perfect homogeneity. In the real world the galaxies are moving at peculiar velocities  $\sim 500 \text{ km s}^{-1}$  relative to the ideal Hubble flow in equation (1.2).

say that equation (1.4) gives the time evolution of the wavelength of freely propagating radiation as measured by comoving observers placed along the path of the radiation. The cosmological redshift factor  $z$  is defined by the equation

$$1 + z \equiv \frac{\lambda_{\text{obs}}}{\lambda_{\text{em}}} = \frac{a(t_{\text{obs}})}{a(t_{\text{em}})}. \quad (1.5)$$

The wavelength at emission from a source at epoch  $t_{\text{em}}$  is  $\lambda_{\text{em}}$ , as measured by a comoving observer at the source, and the radiation is detected at time  $t_{\text{obs}}$  at wavelength  $\lambda_{\text{obs}}$ . If the time between emission and detection is small we can expand  $a(t)$  in a Taylor series to get equation (1.3). It has become customary to label epochs in the early universe by the redshift factor  $z$  considered as an expansion factor even when there is no chance of detection of radiation freely propagating to us from this epoch.

The thermal cosmic background radiation (the CBR) detected at wavelengths of millimeters to centimeters is very close to homogeneous — the surface brightness departs from isotropy by only about one part in  $10^5$  — and the spectrum is very close to blackbody at temperature  $T = 2.73$  K. To analyze the behavior of this radiation in an expanding universe suppose the homogeneous space of equation (1.1) at time  $t$  contains a homogeneous sea of thermal radiation at temperature  $T$ . The photon occupation number of a mode with wavelength  $\lambda$  is given by Planck's equation,

$$N = \frac{1}{e^{hc/kT\lambda} - 1}. \quad (1.6)$$

If the radiation is freely propagating the occupation number  $N$  is conserved, so we see from equation (1.4) that the mode temperature scales with time as

$$T \propto 1/a(t). \quad (1.7)$$

Since this is independent of the mode wavelength an initially thermal sea of radiation remains thermal, even in the absence of the traditional thermalizing grain of dust.

We know the universe now is transparent to the CBR, at least along some lines of sight, because distant galaxies are observed at CBR wavelengths. At some earlier epoch the universe could have been dense and hot enough to have been opaque and therefore capable of relaxing the radiation to the observed thermal spectrum. When this was happening the interaction of matter and radiation cannot be neglected, of course, but since the heat capacity of the radiation is much larger than that of the matter<sup>3</sup> equation (1.7) still applies to the coupled matter and radiation when sources or sinks of the radiation may be neglected.

<sup>3</sup>The energy density in the radiation is  $aT^4$ , where  $a$  is Stefan's constant. In a plasma of  $n$  protons per unit volume and a like number of free electrons the energy density is  $3nkT$ . The ratio is  $aT^3/(3nk) \sim 10^9$ , nearly independent of redshift.



The conclusion is that the CBR is a fossil of a time when our expanding universe was hotter and denser than it is now. This argument does not require general relativity theory, only conventional local physics, a nearly homogeneous and isotropic expansion described by a single line element (as in eq. [1.1]), and an expansion factor large enough to lead back to a time when the universe was opaque enough to have been capable of relaxing to equilibrium. I think all who have given the matter serious thought would agree with this; the issue is the minimum expansion factor needed to account for the observations. Hoyle, Burbidge, and Narlikar (1993) propose a Quasi-Steady State scenario in which the present expansion phase traces back to a redshift only slightly greater than the largest observed for galaxies. Others doubt that the properties of the postulated thermalizing dust grains can be chosen to relax the radiation to blackbody at such low redshifts while still allowing the observed visibility of high redshift galaxies at CBR wavelengths, though the issue certainly could be analyzed in more detail than has been done by either side so far. Most cosmologists accept the evidence for the origin of the light elements as remnants of the rapid expansion and cooling of the universe through temperatures on the order of 1 MeV, at redshift  $z \sim 10^{10}$ . This model for element formation depends on the rate of expansion through  $z \sim 10^{10}$  and thus tests the gravity theory, as follows.

In general relativity the expansion factor  $a(t)$  in equation (1.1) satisfies

$$H^2 = \left( \frac{1}{a} \frac{da}{dt} \right)^2 = \frac{8}{3} \pi G \rho \pm \frac{1}{a^2 R^2} + \frac{1}{3} \Lambda. \quad (1.8)$$

The mean mass density is  $\rho$ ,  $\Lambda$  is Einstein's cosmological constant, and the constant  $R^2$  appears with the same algebraic sign as in equation (1.1). (I simplify the equations by choosing units so the velocity of light is unity.) The equation of local energy conservation is  $\dot{\rho}/\rho = -3(\dot{a}/a)(\rho + P)$ , where  $P$  is the pressure. If the pressure is not negative the mass density varies with the expansion parameter at least as rapidly as  $a^{-3}$ , meaning it is the most rapidly varying term in equation (1.8), and hence the dominant term at high redshift. Thus the predicted expansion rate through the epoch of light element production is very well approximated as

$$\left( \frac{1}{a} \frac{da}{dt} \right)^2 = \frac{8}{3} \pi G \rho. \quad (1.9)$$

For a reasonable value of the baryon number density, and assuming the baryon distribution at high redshift is close to homogeneous and the lepton numbers are small, the predicted values of the light element abundances left over from the rapid expansion and cooling of the early universe are close to the observed abundances (with modest and reasonable corrections for the effects of nuclear burning in stars).