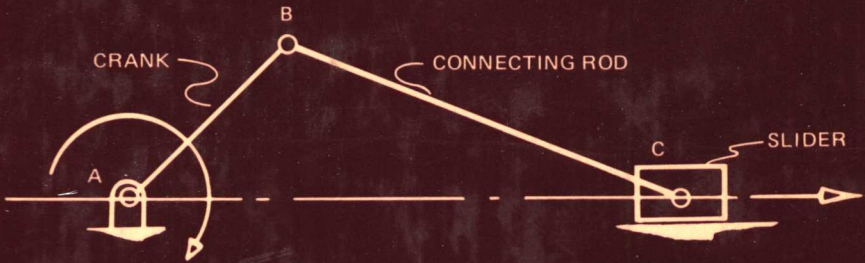


MECHANISM ANALYSIS

**SIMPLIFIED GRAPHICAL
AND ANALYTICAL TECHNIQUES**



LYNDON O. BARTON

MECHANISM ANALYSIS

Simplified Graphical and Analytical Techniques

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OTHER VOLUMES IN PREPARATION

To my wife Olive, my children Rhonda, Loren, Carol, and
Leon, and my mother Clarice.

Preface

This book is written primarily for mechanical design engineers and mechanical design engineering students who are concerned with the design of machines, in general, and in particular with mechanism analysis, a subject which forms a principal part of the study of kinematics of mechanisms. The principal aim of this volume is to place at the disposal of the reader a practical book that will serve (1) as a handy reference for simplified approaches to problems typically encountered in the analysis and synthesis of a mechanism, and (2) as a supplementary textbook for independent study or quick review of both principles and applications in mechanism analysis.

The book presents a wide assortment of graphical and analytical techniques, as well as complete listings in FORTRAN of computer programs and programmable calculator programs for the Hewlett Packard HP-41C for analysis of basic classes of mechanisms. Special emphasis has been given to relatively simple kinematic chains such as slider-crank, four-bar, quick-return, and sliding coupler mechanisms. These mechanisms have been selected because they form the basic elements of most machines and because they are easily adaptable to the teaching of fundamental kinematic principles. Once these principles are fully understood, it is comparatively easy to apply this knowledge to the analysis of more complex mechanisms.

Several novel approaches for simplification of the analytic process are presented. These include the rectilinear and angular motion diagrams presented in Chapter 2, the link extension concept for velocity analysis by instant centers in Chapter 5, the generalized procedure for constructing the acceleration polygon in Chapter 9, the Parallelogram Method for slider-crank analysis in Chapter 12, and the Simplified Vector Method and modified version of same in Chapters 14-18.

One important feature is that the Simplified Vector Method, unlike conventional methods which rely on calculus and other forms of sophisticated mathematics, relies mainly on basic algebra and trigonometry to obtain an analytical solution. This simplified mathematical approach has made it

possible to include several analytical problem solutions rarely found in kinematic textbooks. Hopefully, this approach will not only make this material accessible to a wide body of readers, but will also help to provide the quick insight often needed by designers in the analysis of a linkage.

The book is written for easy readability and comprehension, without reliance on any other source. Needed background material on topics such as Uniformly Accelerated Motion (Chapter 2), Properties of Vectors (Chapter 3), Complex Algebra (Chapter 13), and Trigonometry (Appendix B) is provided for review. Concepts are presented as concisely as possible, employing numerous illustrative examples and graphical aids, as well as step-by-step procedures for most graphical constructions. In addition, the topics are arranged in a logical sequence corresponding to that ordinarily followed in teaching a course in kinematics.

Some of the material in this book is based on several technical papers which the author has previously published (see References). Much of the material has been drawn from class notes which have been developed and used over several years of teaching kinematics of mechanisms as part of an Engineering Technology college curriculum.

The author gratefully acknowledges his indebtedness to the E. I. DuPont de Nemours and Company Engineering Department and the Delaware Technical Community College Mechanical Engineering Department for providing the engineering and teaching opportunities, respectively, that have enabled him to pursue and accumulate the knowledge and experience that form the basis of this book.

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- Teachers, relatives, and friends who have been a source of inspiration and encouragement in the author's career; and
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Lyndon O. Barton

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I

INTRODUCTORY CONCEPTS

Mechanism analysis (or kinematics of machines) is inherently a vital part in the design of a new machine or in studying the design of an existing machine. For this reason, the subject has always been of considerable importance to the mechanical engineer. Moreover, considering the tremendous advances that have been made within recent years in the design of high-speed machines, computers, complex instruments, automatic controls, and mechanical robots. It is not surprising that the study of mechanisms has continued to attract greater attention and emphasis than ever before.

Mechanism analysis may be defined as a systematic analysis of a mechanism based on principles of kinematics, or the study of motion of machine components without regard to the forces that cause the motion. To better appreciate the role of mechanism analysis in the overall design process, consider the following. Typically, the design of a new machine begins when there is a need for a mechanical device to perform a specific function. To fulfill this need, a conceptual or inventive phase of the design process is required to establish the general form of the device. Having arrived at a concept, the designer usually prepares a preliminary geometric layout of the machine or mechanism for a complete kinematic analysis. Here the designer is concerned not only that all components of the machine are properly proportioned so that the desired motions can be achieved (synthesis phase), but also with the analysis of the components themselves to determine such characteristics as displacements, velocities, and accelerations (analysis phase). At the completion of this analysis, the designer is ready to proceed to the next logical step in the design process: kinetic analysis, where individual machine members are analyzed further to determine the forces resulting from the motion.

Mechanism analysis therefore serves as a necessary prerequisite for the proper sizing of machine members, so that they can withstand the