X-RAYS IN PRACTICE

BY

WAYNE T. SPROULL, PH. D.

Physicist, Research Laboratories Division
General Motors Corporation

New York *London*
McGRAW-HILL BOOK COMPANY, INC.
1946

X-RAYS IN PRACTICE

THE MAPLE PRESS COMPANY, YORK, PA.

PREFACE

The publication of this book very nearly coincides with the fiftieth anniversary of the discovery of x-rays and with the hundredth anniversary of the birth of their discoverer, W. C. Röntgen. The knowledge that has been accumulated about these rays, and the ways in which they have been applied for the benefit of mankind during the past fifty years, comprise the broad subject which we have tried to present to the reader within the confines of a volume of ordinary size.

Special effort has been made to index the book so completely that it will serve the needs of the person desiring a ready reference giving quick answers to specific questions about x-rays. Extensive bibliographical footnotes have been included so that this type of reader will usually find key references serving as a starting point for more extensive reading in the scientific literature. The tables in the Appendix should prove helpful to all types of reader.

On the other hand, a serious endeavor has been made to arrange the material in a logical sequence and to make it sufficiently readable so that the student interested in x-rays and their applications will find the book convenient as a text. The student with little or no previous training in physics or mathematics will find the treatment of some of the quantitative aspects difficult in places, but an effort has been made to interpret such mathematical passages in words so that their chief significance is readily grasped. If one skips over a portion that seems too involved or uninteresting, it should not be difficult to resume the thread of the story farther on. Before reading beyond Chap. 13, it would be definitely advisable for one unacquainted with trigonometry to consult a trigonometry book to become acquainted with the concept of the sine of an angle. The questions at the end of each chapter should help the student to check the thoroughness of his reading as he proceeds.

The inclusion of only a single chapter on medical applications, as contrasted to the numerous chapters on diffraction, is not to be construed as an indication of the relative importance of the two fields. This circumstance arises from the fact that x-ray diffraction theory has been extensively developed in a quantitative manner. It is inseparably interwoven through the modern theories of solids and crystallography. On the other hand, x-ray techniques have infiltrated into medical science in a somewhat supplementary way not involving such extensive mathematical treatment.

It is a pleasant obligation to acknowledge that this book could never have been written at all without the aid of the vast literature on x-rays already existing. Among the books from which figures, tables, and information have been borrowed, special mention is due to the following: W. H. Bragg and W. L. Bragg, "The Crystalline State," George Bell & Sons, Ltd., and the Macmillan Company; A. H. Compton and S. K. Allison, "X-rays in Theory and Practice," D. Van Nostrand Company, Inc.; M. J. Buerger, "X-ray Crystallography," John Wiley & Sons, Inc.; G. W. Files, "Medical Radiographic Technic," Charles C. Thomas, Publisher; C. D. Hodgman, "Handbook of Chemistry and Physics," Chemical Rubber Publishing Co.; L. G. H. Sarsfield, "Electrical Engineering in Radiology," Chapman & Hall, Ltd.; M. Siegbahn, "X-ray Spectroscopy," Oxford University Press; G. P. Thomson and W. Cochrane, "Theory and Practice of Electron Diffraction," the Macmillan Company; R. W. G. Wyckoff, "Structure of Crystals," Reinhold Publishing Corporation; also, such McGraw-Hill books as G. L. Clark, "Applied X-rays"; W. P. Davey, "A Study of Crystal Structure and Its Applications"; and F. K. Richtmyer and E. H. Kennard, "Introduction to Modern Physics."

Equally important are the numerous figures, tables, and information secured from a large number of periodicals. Of these, most are acknowledged in legends and footnotes.

Special mention is also due to General Motors Corporation for permission to publish numerous radiographs, diffraction patterns, and other material, and for excellent library facilities and other kind cooperation. The photographs and diagrams of x-ray equipment have been generously furnished by the General Electric X-ray Corporation, Machlett Laboratories, Inc., North American Philips Co., Inc., Picker X-ray Corp., Victoreen Instrument Co., and Westinghouse Electric & Mfg. Co., X-ray Division.

Much of the material for the final chapter on electron diffraction was taken from the work of Dr. W. W. Beeman, carried out while he worked with the author at General Motors. Thanks are also due to Prof. J. T. Norton of Massachusetts Institute of Technology and to Dr. E. J. Martin and A. E. Roach of General Motors for reviewing the manuscript and for helpful suggestions. Finally, the assistance and encouragement of my wife, Ethel, have greatly lightened the task.

WAYNE T. SPROULL.

DETROIT, MICH.,
January, 1946.

CONTENTS

CHAPTER 1

HISTORY OF X-RAYS

About 1890 the basic outline of the science of physics appeared to be practically complete. That is, it appeared that the general extent of the final structure was fairly well delineated by the portion already erected. Newton's laws of motion and gravitation, the concept of conservation of energy, the inverse square laws of magnetism and electrostatics, the kinetic theory of gases, Maxwell's equations, and Faraday's laws fully accounted for everything from the motion of the planets and the operation of an engine to the Brownian movement. The design of a telephone system, a bridge, or an electric power station was based on the same known fundamentals.

This physics of 1890 is now known as "classical physics." In 1890 it accounted for practically all known natural phenomena except a few obscure academic curiosities like the photoelectric effect and the spectral distribution of black-body radiation.

This happy picture of a science so complete that only minor details remained to be filled in did not long endure, however. The photoelectric effect was discovered in 1887. In rapid succession there followed the discovery of x-rays in 1895, radioactivity in 1896, and the electron in 1897. To cap the climax, in 1900 Planck succeeded in accounting for the spectral distribution of black-body radiation by assuming that the radiation consisted of integral indivisible "quanta" rather than continuous electromagnetic waves.

All these new developments were disconcerting. They not only failed to fit into the classical picture but often forced one to conclusions contradictory to it. Thus the discovery of x-rays is to be regarded as not merely another scientific discovery but one of a small group that initiated a new scientific era.

Late in 1895 some experiments on the properties of cathode rays were being conducted at the University of Würzburg in Germany by Wilhelm Conrad Röntgen, professor of physics. Many other scientists such as Hittorf, Crookes, and Lenard had been studying the properties of cathode rays for several years, but Röntgen noticed an effect that had escaped all the others. Since he was studying the fluorescence caused by the cathode rays, he was working in a dark room, and he was led to cover the cathode-ray tube with a black cardboard box, which of course prevented any visible light or ultraviolet radiation from escaping. In

spite of this, however, Röntgen observed that a screen coated with barium platinocyanide glowed brilliantly when brought near the covered tube. He made this epochal discovery on Nov. 8, 1895.[1]

Investigating this phenomenon, he soon concluded that it was caused by the generation of some hitherto unknown type of radiation from the cathode-ray tube, the new rays being invisible but able to

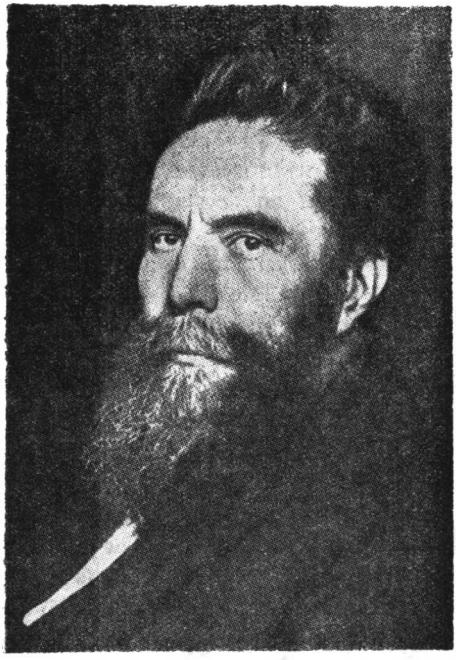


FIG. 1-1.—W. C. Röntgen. (Mar. 27, 1845–Feb. 10, 1923.)

cause visible fluorescence in certain materials such as barium platinocyanide. In his first published paper[2] describing the new rays, which he called x-rays, he mentioned that they would penetrate a thousand-page book, a sheet of aluminum 3½ millimeters thick, and lead foil. He also mentioned that when he held his hand in front of the screen coated with barium platinocyanide the outlines of the bones in his hand could be seen on the screen. He stated that the rays acted upon a photographic plate and suggested that this opened the possibility of

[1] See Otto Glasser, "Wilhelm Conrad Röntgen and the Early History of the Roentgen Rays," Charles C. Thomas, Publisher, Springfield, Ill., 1934.

[2] W. C. Röntgen, *Ann. Physik*, **64**, 1 (1898), reprinted from an earlier publication in 1895.

taking pictures in a light room by enclosing the photographic plate in a lighttight holder, or "cassette," as he called it.

Many others immediately began experiments with these strange, new, penetrating rays, but it was not until 1912 that any really significant advance was made in the scientific understanding of their fundamental nature. Between 1895 and 1912 medical men learned to use x-rays as a powerful new tool in diagnosis. Almost immediately they were used as a means of photographing the bones of living people and animals, for detecting and setting fractures, and for detecting and locating foreign objects such as bullets and swallowed pins. This use of x-rays is known as *radiography*. The terms "radiography" and "radiograph" are analogous to the terms "photography" and "photograph," except that x-rays or gamma rays (to be discussed later) play the role played by light in photography. As the technique was improved, medical men eventually learned to recognize such phenomena as tubercular lesions, etc., in a radiographic picture.

During this period many of the pioneer experimenters suffered severe x-ray burns, which were found to be incurable in many cases; often fingers, hands, and arms were lost. Eventually, physicians learned to make use of this ability of the x-rays to kill living tissues by directing the rays at tumors and other diseased regions that they desired to destroy. It was found that such diseased tissues could be killed by a smaller dose of x-rays than that required to kill normal healthy tissue. Thus the technique of *x-ray therapy* was gradually developed, in which the basic idea is to apply a lethal dose of x-rays to diseased regions without exceeding the tolerance limit of the adjacent healthy tissue.

From the scientific point of view, however, the first milestone in the history of x-rays was their discovery by Röntgen in 1895. The next was the discovery of x-ray diffraction by crystals by M. Laue, W. Friedrich, and P. Knipping[1] in 1912.

Earlier experimenters had often tried to produce interference and diffraction effects with x-rays but had met with only indifferent success. However, they were able to conclude that x-rays consisted of waves having a length of the order of 10^{-9} centimeter. On the other hand, crystallographers as early as 1850 had concluded that the atoms in a crystal must be arranged in a geometric space lattice in which the spacing between atoms was of the order of 10^{-8} cm. Apparently Laue was the first to whom these two considerations suggested the idea that a crystal should serve as a diffraction grating for x-rays. Laue, being a theoretical physicist, asked Friedrich and Knipping, two experimental physicists, to try to observe the diffraction of x-rays by a single crystal. They

[1] M. Laue, W. Friedrich, and P. Knipping, *Ann. Physik*, **41**, 971 (1913), reprinted from an earlier publication in 1912.

set up suitable apparatus and soon discovered that diffraction patterns of the type predicted by Laue were indeed produced by crystals of copper sulfate (blue vitriol), zinc blende, rock salt, and galena.

This discovery immediately provided a powerful new method for investigating the nature of x-rays by diffracting them with crystals and, perhaps even more important, for investigating the nature of solids (most of which are composed of crystals) by using them to diffract x-rays. New possibilities and applications of this method are still being discovered and invented, and the work already done in this field is so extensive as to constitute a whole new branch of physics entirely unknown 30 years ago. Although hundreds of workers have made their names well known in the field of x-ray diffraction, there are two whose work is so extensive and important as to deserve special mention—W. H. Bragg and W. L. Bragg.

The next milestone in the history of x-rays was the invention of the Coolidge tube by W. D. Coolidge.[1] From 1895 to 1913, all x-ray tubes were of the so-called "gas-filled" type. These were suitable for certain types of work at less than 50,000 volts and are still widely used today in diffraction work, but they were very awkward for radiographic work and impractical for the higher voltages such as 100 or 200 kilovolts now used for many common types of radiographic work. Except for a few special types of tubes for diffraction work, all modern x-ray tubes are of the Coolidge type. These tubes are exhausted to the best vacuum obtainable and the cathode consists of an incandescent tungsten filament.

Another milestone in the history of x-rays was also established in 1913 by H. G. J. Moseley,[2] who discovered that a portion of the x-rays radiated from the target of an x-ray tube has a wave length characteristic of and dependent upon the atomic number of the target material. Thus the elements chromium, manganese, iron, cobalt, nickel, copper, and zinc, having atomic numbers 24, 25, 26, 27, 28, 29, and 30, respectively, strongly emit x-rays having the following wave lengths, respectively, when they are bombarded with cathode rays: 2.3, 2.1, 1.9, 1.8, 1.7, 1.5, and 1.4 angstrom. This discovery led to the establishment of a satisfactory theory of the origin of characteristic x-ray spectra and also helped theoretical physicists such as Bohr and Rutherford to arrive at a fundamentally correct theory of the structure of the atom. The important and extensive research of A. H. Compton also deserves special mention.

During the 30 years that have elapsed since Moseley's discovery, there have been remarkable developments in the application of x-rays to many important new types of work. At the same time, techniques

[1] W. D. Coolidge, *Phys. Rev.*, **2**, 409 (1913).

[2] H. G. J. Moseley, *Phil. Mag.* **26**, 1024 (1913), **27**, 703 (1914).

have been vastly improved in many of the older fields. For example, today steel parts 6 inches thick and weighing many tons are radiographed for defects in a routine manner with million-volt equipment as quickly and as easily as Röntgen radiographed his own hand 50 years ago.

Quite recently, however, a new invention has appeared that appears likely to rank with the Coolidge tube as another important milestone in the history of x-rays. This is the electron induction accelerator invented and developed by D. W. Kerst,[1] which has already generated x-rays of great intensity with an energy of 100 million volts. This device has also been called a "betatron" and a "rheotron," and the theory of its operation has been worked out by Kerst and Serber.[2]

QUESTIONS

1. What is meant by classical physics?

2. Can x-rays be seen directly? Are they dangerous or harmful without proper protection?

3. Distinguish between radiography, x-ray diffraction, and x-ray therapy.

4. What are the two features that distinguish a Coolidge tube from gas-filled x-ray tubes?

5. What is a cassette?

[1] D. W. Kerst, *Phys. Rev.*, **60**, 47 (1941); *Rev. Sci. Instruments*, **13**, 387 (1942).

[2] D. W. Kerst and R. Serber, *Phys. Rev.*, **60**, 53 (1941).

CHAPTER 2

FUNDAMENTALS OF X-RAY TUBES

1. Generation of X-rays. Let us visualize a machine gun firing bullets in rapid succession against the face of a large, heavy block of iron. A person standing near the spot where the bullets strike the block would be able to hear a plinking noise, indicating that sound-wave pulses are generated by the bullets striking the block.

The generation of x-rays is very similar to this except that electrons are substituted for the bullets, the x-rays being analogous to the sound.

2. Cathode Rays. Electrons moving rapidly in a stream like machine-gun bullets are called "cathode rays." Of course, the electrons are incomparably lighter than bullets, and their motion is very much swifter.

Obviously, it would be impossible to produce a satisfactory stream of machine-gun bullets by firing the gun down a long hall filled solidly to the ceiling with baseballs. In a similar way, if one wishes to produce cathode rays, he would naturally choose a vacuum in which to work, rather than the open air, because the molecules in ordinary air obstruct the passage of electrons much as a hall full of baseballs obstructs the passage of bullets.

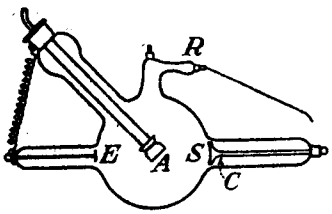


FIG. 2-1.—Diagram of early gas-filled x-ray tube.

3. Gas-filled X-ray Tubes. Let us consider how one obtains the electrons. Before 1913, all x-ray tubes were of the so-called "gas-filled" type. Gas-filled tubes are still used today for some types of diffraction work. The cathode rays are produced in a gas-filled tube somewhat as follows:

Figure 2-1 is a diagram of one of the early gas-filled x-ray tubes. A block of aluminum C, having a concave surface S, is located near one end of the partly evacuated glass tube, and a block of metal A is located near the center of the tube. These two pieces of metal are connected to a high-voltage electrical supply, and hence they are called "electrodes."

The concave aluminum electrode is negatively charged and hence is called the "cathode." The block of metal in the center is called the anticathode, or target from our machine-gun analogy. These early tubes were constructed with a third electrode E off to one side, which was also positively charged (being directly connected to the anticathode) and was called the "anode."

The tube is evacuated to a pressure of about 10^{-4} millimeter of mercury. Ordinary atmospheric pressure is about 760 mm. of mercury, so 10^{-4} mm. of mercury is about 10^{-7} atmosphere. Under these conditions, each cubic centimeter of space inside the tube still contains about $3\frac{1}{2}$ trillion molecules, but even so the mean free path is about 50 centimeters, which means that a molecule of nitrogen or oxygen in the tube can travel more than a foot in a straight line, on the average, before it will collide with another molecule.

Owing to such causes as the presence of minute radioactive impurities and cosmic rays, there are always a few stray electrons present here and there in the tube. When a high voltage is applied, so as to make the cathode negative and the anode and anticathode positive, these electrons will be attracted to the latter and repelled by the former and hence will rush at high speed through the gas present, toward the anticathode. In so doing, a few of them will collide with some of the atoms of oxygen or nitrogen of which the air molecules are composed, and in many cases the collisions will be sufficiently violent to wrench an electron loose from the atom. (Atoms consist of a swarm of electrons electrically attached to a central positively charged nucleus.) The atoms thus bereft of one of their usual complement of electrons are called *ions*, and they carry a net positive charge. For example, a normal neutral nitrogen atom consists of a swarm of 7 electrons (each negatively charged) surrounding a central nucleus having a positive charge of 7 units just sufficient to neutralize the 7 negative electronic charges. When one of the 7 electrons is wrenched loose from the atom, the ion that is left consists of the nucleus with a charge of $+7$ and the 6 remaining electrons with a charge of -6, leaving a net charge of $+1$ for the ion.

Thus electrons rushing through the air at high speed produce more electrons by ejecting them from the atoms composing the air, and at the same time positive ions are produced. The electrons ejected from the atoms add to those initially present, and these, in turn, rush through the gas and produce by collision still more electrons and more ions. Thus the process is cumulative, and soon an enormous number of electrons is rushing toward the anticathode and a huge swarm of ions is rushing at a slower, yet very great speed toward the cathode.

In this manner, an electric discharge (similar to the one seen in the familiar neon signs) starts, and the large number of electrons streaming through the gas away from the cathode toward the anticathode constitutes the cathode rays needed to generate the x-rays when they strike the target, which is the general name given to the anticathode or anode or other metallic object (depending upon the construction of the tube) that most of the cathode rays strike. The positive ions likewise strike the cathode with great energy, where they pick up an electron and so

become neutral atoms again. This positive-ion bombardment of the cathode causes it to emit additional electrons, which also contribute to the electron stream constituting the cathode rays. The concave shape of the cathode distributes the high-voltage electrostatic field so as to focus the cathode rays upon a limited "focal spot" on the target.

Although the general operation of a gas-filled x-ray tube has been described here in only a brief qualitative way, enough has been said to indicate that the phenomena involved are of considerable complexity, although the fundamental idea is merely to hurl electrons at high speed against a block of metal.

4. Practical Operation of Gas-filled X-ray Tubes. The electrons rushing through the tube away from the cathode and the positive ions rushing toward the cathode constitute an electric current passing through the tube, commonly called the "tube current." The magnitude of this current and the voltage between the electrodes is largely determined by the gas pressure in the tube. If the pressure is as great as 10^{-3} mm. of mercury, so that the mean free path is only 5 cm., it will be found that a current of perhaps 50 or 100 milliamperes will flow through the tube when a potential difference as low as perhaps only two or three thousand volts is applied. It will be found impossible to build up a potential difference of, say, 20 kilovolts across the tube because the cumulative nature of the ionization process discussed in Sec. 3 causes a very great increase in the current when only a slight increase is made in the applied voltage.

If the pressure in the tube is as low as 10^{-5} mm. of mercury, on the other hand, it will be found that a current of only a few microamperes can be forced through the tube, regardless of whether the applied potential is 5 kv. or 50 kv. This is due to the fact that the mean free path of the ions and electrons[1] in the gas is now about 5 meters so that very few of the electrons or ions strike any gas atoms in passing from one electrode to the other, and so there is no cumulative ionization process.

Between these two extremes, there is a limited critical range of pressure around 10^{-4} mm. of mercury, in which the potential drop and tube current can both be maintained at suitable values to produce x-rays of desirable intensity and penetration. Regulation of the high-voltage supply and the pressure of the gas in the tube will then permit one to maintain any tube current desired in the range of, say, 3 to 30 ma. and 5 to 50 kv.

If the tube is connected to a high-vacuum pumping system that is capable of exhausting it to a pressure of 10^{-5} mm. of mercury, it will be found that a properly designed leak valve connected so as to permit air

[1] The mean free path of electrons is probably five or six times that of molecules in the gas.

to leak in at a very slow but accurately adjustable rate will afford accurate and rapid control over the tube current and voltage.

In medical work, however, it is impractical to have the tube rigidly connected by a glass or metal tube to a vacuum system; prior to 1913, therefore, the gas-filled tubes used in such work were exhausted to the desired pressure and then sealed off so as to be portable. The pressure in these tubes decreased slowly as they were used, with a current of perhaps 5 ma. passing through the tube, probably because of adsorption of the gas by the glass and metal surfaces of the tube. Consequently the tube potential increased as the tube was used, causing a change in the character of the x-rays generated; if not corrected, the pressure would soon drop to a value making further operation impossible. Various devices were incorporated in these early tubes to permit the operator to restore the gas pressure to a suitable value. In some types, a discharge was passed through a "gasifier" (such as asbestos, mica, or glass wool) in a small side tube R (Fig. 2-1). Some of these early x-ray tubes operated with hydrogen in them instead of air because this reduced the sputtering of the metal parts, and in these the pressure was regulated by means of palladium tubes, which could be heated to let a little hydrogen diffuse in. These disadvantages in a sealed-off gas tube are overcome by the Coolidge type of tube, now used for most purposes.

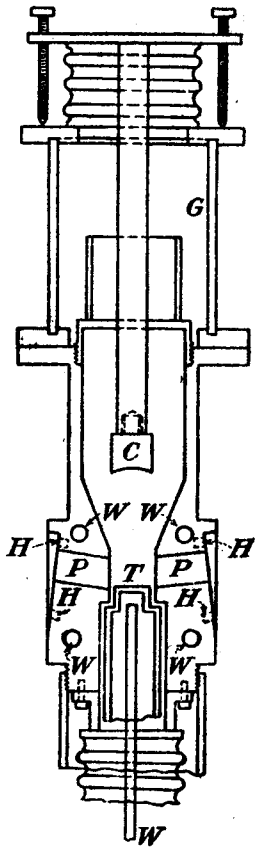
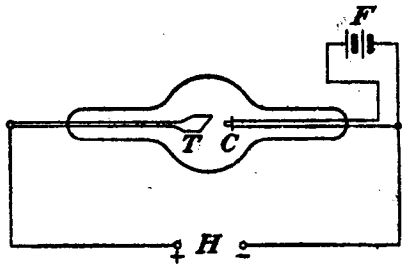


FIG. 2-2.—Self-rectifying water-cooled gas-type x-ray tube for diffraction work. C, cathode; T, target; P, ports for x-rays; W, water passages; H, screw holes for attaching diffraction cameras. (*Wyckoff and Lagsdin; courtesy of American Institute of Physics.*)

Gas tubes are still used for diffraction work to a considerable extent, and therefore the practical details of their operation are of some interest. These modern gas tubes are used permanently connected to a vacuum system, however, and equipped with a leak valve.

It is clear that ordinary gas-filled tubes must be supplied with high voltage of the correct polarity. That is, the cathode must be negative and the anode positive. The use of an alternating electric voltage cannot ordinarily be tolerated because electrons would bombard the cathode instead of the anode when the polarity is opposite to that intended by the tube designer. However, in recent years, those using gas-filled

tubes in diffraction work have discovered that they can be so designed[1] that they may be directly connected to a high-voltage electrical supply of alternating polarity, such as a transformer. Gas-filled tubes of such modern design (Fig. 2-2) will rectify their own current, like a Coolidge tube. Ordinary gas tubes will not and therefore must be operated from an electrical supply having a rectifier or other provision for supplying direct current.



Coolidge tube
C=Cathode cup T=Target
F=Filament battery
H=High voltage source

FIG. 2-3.

5. The Coolidge X-ray Tube. Reference to the Coolidge tube has already been made in Chap. 1.[2] The great majority of the x-ray tubes used today are of this type. The tube is evacuated to the best vacuum attainable. Before it is sealed off from the pumps, it is operated under full power for several hours to drive the gas out of the portions which get hot, so that, when it is sealed off, subsequent operation will not cause any appreciable amount of gas to be released inside the tube.

The cathode of this tube is not a block of aluminum, as in the gas tube, but consists instead of a spiral tungsten wire filament, inside a small metal cup. The filament can be heated to incandescence by passing an electric current through it, just like the filament of an ordinary incandescent lamp. The surrounding metal cup causes the electrostatic field between the anode and cathode, set up by the high voltage applied to the tube, to distribute itself in such a way that electrons emitted from the hot filament inside the cup will be focused into a narrow beam of cathode rays, which will strike the anode within a small, definite spot on its surface (called the "focal spot"). With no focusing cup, the electrons would diverge from the filament. A Coolidge tube ordinarily has only two electrodes, cathode and anode, the anode serving as target.

In the Coolidge tube, the electrons needed to provide the cathode rays are emitted from the surface of the hot tungsten filament. This will be explained in greater detail in the next section.

6. Free Electrons and Thermionic Emission. A normal tungsten atom consists of a central massive nucleus carrying a positive charge exactly sufficient to neutralize the negative charges of the surrounding swarm of 74 electrons. In a tungsten wire, the tungsten atoms which

[1] R. W. G. Wyckoff and J. B. Lagsdin, *Rev. Sci. Instruments*, **7**, 35 (1936).
[2] W. D. Coolidge, *Phys. Rev.*, **2**, 409 (1913).

compose it are fixed in a rigid framework in such a way that each one may vibrate with a small amplitude to and fro in any direction about its fixed central equilibrium position in the framework to which it is figuratively "attached" by the electrical forces of interaction with its neighboring atoms. However, each atom keeps to itself only 73 of its electrons[1] instead of the 74 present in a neutral atom. The seventy-fourth electron of each atom is not constrained to remain attached to its parent atom like the other 73 but instead goes wandering off freely through the space between atoms. Since this is true of all the atoms, there are as many free electrons as there are atoms, and these swarm about inside the metal much as the molecules of a gas swarm about, as visualized in the kinetic theory of gases.

This internal "gas" of free electrons is what distinguishes a metal, like copper, from a nonmetal, like sulfur. When an electric current is sent through a wire, these free electrons slowly migrate through the framework of atoms composing the wire. In fact, it is this slow migration of the electrons which *is* the current in the wire. As an example, if 6 amperes from a battery is passed through a piece of No. 17 copper wire, the free electrons in the wire are migrating along it at a rate of somewhat over a centimeter per minute, or let us say an inch every two minutes.

One may visualize the enormous number of collisions these electrons will make with each other and with the atoms composing the wire as they drift through 1 in. of it under the urging influence of the electric potential difference supplied by the battery. The energy supplied by the battery in driving the electrons through the wire is used to increase the average velocity of their rapid random thermal motion (which is much faster than that of a rifle bullet), just as the molecules of air in a room speed up when the temperature rises. Part of this is handed on to the atoms by elastic collisions, causing the atoms to vibrate more and more violently about their fixed equilibrium positions in the wire; in other words, the wire gets hot.

If enough energy is supplied, the random thermal motion of the free electrons becomes great enough, as the wire heats up, so that a small fraction of them acquire sufficient energy to fly completely out of the metal, by overcoming the restraining electric potential, which ordinarily holds the free electrons within the metal. One may see, in a qualitative way, that the free electrons, as a whole, must be firmly bound to the positive ions among which they circulate, and the kinetic energy which is required to enable an electron to escape against this restraining influence is called the "thermionic work function," or sometimes the "heat

[1] This figure is not to be taken too literally. The number of free electrons is certainly of the same order of magnitude as the number of atoms, however. For further details, see U. Dehlinger, *Z. Elektrochem.*, **38**, 150 (1932).

of evaporation of electrons," in analogy with the evaporation of water, for example. Thus the electrons "evaporate" from a hot metal much as water evaporates from a wet cloth in the sunshine.

The theory of this process, which is called "thermionic emission," was first worked out by O. W. Richardson,[1] who derived the equation

$$I = AT^{\frac{1}{2}}e^{-\frac{b}{T}} \tag{2-1}$$

from theoretical considerations. At present, it is generally conceded that the correct equation is more probably

$$I = AT^{2}e^{-\frac{b}{T}} \tag{2-2}$$

which was later derived by Dushman.[2] Actually, the choice between the two equations is of theoretical interest only; for $e^{-\frac{b}{T}}$ is such a powerful factor that it overwhelms the $T^{\frac{1}{2}}$ or T^2 factor, and consequently the experimental data fit either equation equally well. In these equations,

$$A = \frac{4\pi mek^2}{h^3}S \tag{2-3}$$

where S is the area of the emitting surface, m and e are the mass and charge of an electron [e in (2-1) and (2-2) is of course Napierian log base], k is Boltzmann's constant,[3] h is Planck's constant,[3] T is the absolute temperature, and

$$b = \frac{10^7 E_w e}{k} \tag{2-4}$$

where E_w is the thermionic net work function, or "latent heat of evaporation of the electrons" (in ergs per electron).

The operator of a Coolidge tube soon learns that no appreciable emission of electrons from the filament occurs until it is hot enough to glow brightly. Once the temperature is reached at which a measurable current passes between the electrodes, a very slight increase in the filament heating current causes a very great increase in the high-voltage "tube" current carried by the emitted electrons. This rapid variation is represented by the $e^{-\frac{b}{T}}$ term in equation (2-2).

The tube current and voltage in the Coolidge tube are thus regulated by means of the filament heating current, much as these factors are controlled in a gas tube by means of a leak valve. Enough has been

[1] O. W. Richardson, *Phil. Mag.*, **28**, 633 (1914).

[2] S. Dushman, *Phys. Rev.*, **21**, 623 (1923); see also L. A. DuBridge, *Am. J. Phys. (Am. Phys. Teacher)*, **7**, 357 (1939).

[3] See Appendix I.