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# Kinematics and Dynamics of Machinery

HARPER • INTERNATIONAL • EDITION

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1817

**HARPER & ROW, PUBLISHERS, New York**
Cambridge, Philadelphia, San Francisco,
London, Mexico City, São Paulo, Sydney

*Photo Credit:* Figure 5.30 on p. 320. Courtesy of D. P. Naismith

Sponsoring Editor: Cliff Robichaud
Project Editor: Eleanor Castellano
Production Manager: Marion A. Palen
Compositor: Science Press
Printer and Binder: Maple Press
Art Studio: Fine Line, Inc.

Portions of this book have been previously published in *Mechanism: Design-Oriented Kinematics.*

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Library of Congress Cataloging in Publication Data
Wilson, Charles E.
Kinematics and dynamics of machinery.
Includes index.
1. Machinery, Kinematics of. 2. Machinery,
Dynamics of. I. Sadler, J. Peter. II. Michels, Walter J.
III. Title.
TJ175.W758 1983 621.8'11 83-123
ISBN 0-06-044437-1

*There is no expedient to which a man will not resort to avoid the real labor of thinking.*

SIR JOSHUA REYNOLDS

It is the authors' sincere hope that users of this book will be encouraged to think and to explore designs and methods not explicitly described herein. Although worked out examples have been used extensively to illustrate applications to engineering problems, the intent is to build a capacity for solving new problems that one may encounter in the future.

*This book is dedicated to our parents.*

# Preface

Unlike some subjects, the theoretical concepts associated with kinematics and dynamics of machinery have immediate application to practical problems. As a result, most students of mechanical engineering have a great interest in mechanisms and machines. The authors have made extensive use of practical examples to maintain this interest while developing the analytical and graphical skills necessary to analyze linkages, cams, gears, and other mechanisms, and providing a foundation for the study of machine design.

Parts of this book are based on *Mechanism: Design-Oriented Kinematics.* The earlier book, which was well received in the United States and abroad, was heavily oriented toward applications and toward graphical techniques. While retaining these features, the new book has been almost entirely rewritten to reflect technological advances. Emphasis is now placed on analytical techniques suitable for machine calculations. In addition, two chapters on statics and dynamics of machines have been added.

Analytical vector methods and complex number methods are utilized for solving representative linkage problems. These methods are readily programmable, allowing analysis of a complete cycle of motion, using a computer or programmable calculator. Examples of machine calculations are given for specific problems. Program listings are not shown because the selection of a language will depend on the user's experience and the hardware and software available.

Graphical methods are also used extensively for linkage analysis. If a linkage is to be analyzed for a single position, construction of velocity and acceleration polygons provides a reasonably quick solution. An added advantage of the polygon method, utilizing the image principle, is that the polar plots so generated are an aid to visualizing linkage motion. When machine calculations are necessitated by the requirement that a linkage be studied over a wide range of motion, graphical methods may be used to spot-check machine results.

This book begins with basic concepts including linkage classification by motion characteristics, degrees-of-freedom of planar and spatial joints and linkages, limiting positions, transmission angle, and linkage interference. Modeling and computer graphics are illustrated. A number of mechanisms for specific applications are described including linkage combinations, fixed- and variable-speed power trains, industrial robots, and the rotating combustion engine. Both the international system of units (SI) and conventional English units are used.

Analytical vector methods, complex number methods, and graphical methods are described and applied to the displacement, velocity, and acceleration analysis of planar linkages. A spatial linkage is analyzed by analytical and graphical methods, and the synthesis of a function generator is described.

The design and analysis of cams is treated by graphical and analytical methods for various types of followers. The design and analysis of spur, helical, worm, and bevel gears is also treated, including shaft loading. Fixed- and variable-ratio gear trains, as well as other drive trains, are analyzed. Both the tabular and formula methods are applied to analysis of epicyclic trains.

Mechanisms are analyzed statically, and d'Alembert's principle is used to obtain dynamic analyses. Graphical and analytical methods are used. Balancing of rotors and reciprocating machinery is treated as well.

For text use, most of the topics in this book may be covered in a three-credit-hour course given to undergraduate engineering students who have completed a course in statics and dynamics of rigid bodies. If more time is available, students may be challenged by emphasis on some of the more difficult topics, such as spatial linkages, synthesis, friction in mechanisms, balancing of multicylinder machines, and the theory of envelopes applied to cam design. Instructors will be able to select among a variety of methods demonstrated, according to their goals and preferences. For example, either analytical vector methods or complex number methods may be used as a basis for writing computer programs to solve planar linkages. However, if analysis of spatial linkages is to follow analysis of planar linkages, then vector methods might be used with both.

We wish to acknowledge the suggestions and comments of our students and colleagues at the New Jersey Institute of Technology and the University of North Dakota. In addition, we acknowledge the help of Ferdinand Freudenstein of Columbia University, Robert Williams of the CAD/CAM Systems Division of Control Data Corporation, Kenneth Waldron of Ohio State University, and William Park of Pennsylvania State University, who reviewed the entire manuscript and made many substantive suggestions. We are grateful for the help of our editors, Carl McNair and Cliff Robichaud. We note with deep regret the sad and untimely death in 1979 of our coauthor Walter Michels.

CHARLES E. WILSON
J. PETER SADLER

# Symbols

Vectors shown in **boldface**, scalar magnitudes in lightface.

| | |
|---|---|
| $\mathbf{A} \cdot \mathbf{B}$ | Dot (scalar) product of vectors $\mathbf{A}$ and $\mathbf{B}$ |
| $a$ | Gear tooth addendum |
| $a$, $\mathbf{a}$ | Acceleration |
| $a^c$, $\mathbf{a}^c$ | Coriolis acceleration |
| $a^n$, $\mathbf{a}^n$ | Normal acceleration |
| $a^t$, $\mathbf{a}^t$ | Tangential acceleration |
| $\mathbf{bc}$ | Velocity of $C$ relative to $B$ (velocity difference) |
| C | Cylinder pair; planet carrier |
| $\mathbf{C}$ | Force couple |
| $\mathbf{C}_i$ | Inertia couple or inertia torque |
| $c$ | Center distance |
| CAD | Computer-aided design |
| $d$ | Diameter of pitch circle |
| $d_b$ | Diameter of base circle |
| $d_e$ | Pitch diameter of equivalent spur gear |
| DF | Degrees-of-freedom |
| $e$ | Instantaneous efficiency; cam-follower offset; piston offset; eccentricity |
| $e^{j\theta}$ | Polar form of a complex number |
| $\mathbf{F}$ | Force |
| $F_a$ | Axial or thrust gear tooth force component |
| $\mathbf{F}_e$ | External force |
| $\mathbf{F}_i$ | Inertia force |
| $\mathbf{F}_{ij}$ | Force exerted by a member $i$ on member $j$ |
| $\mathbf{F}_n$ | Normal gear tooth force |
| $F_r$ | Radial gear tooth force component |
| $\mathbf{F}_s$ | Shaking force |
| $F_t$ | Tangential gear tooth force component |
| $f_i$ | Joint connectivity |
| $G$ | Center of mass |
| $H$ | Helix pair |
| $h$ | Cam follower lift |
| $hp$ | Horsepower |
| $I$ | Mass moment of inertia |
| $j$ | Cam follower jerk; $\sqrt{-1}$, the imaginary unit used to represent quantities on the complex plane |
| $\mathbf{i}, \mathbf{j}, \mathbf{k}$ | Cartesian unit vectors |
| $L$ | Link length |
| $L_d$ | Length of diagonal (of linkage polygon) |
| $l$ | Lead of worm |
| $\ell_i$ | Length of link $i$ |
| $\mathbf{M}_s$ | Shaking moment |

| | |
|---|---|
| $m$ | Mass; module; slope |
| $m^n$ | Normal module |
| $N$ | Number of gear teeth |
| $\mathbf{N}$ | Normal force |
| $N_f$ | Formative number of gear teeth |
| $n$ | Rotational speed (revolutions per minute) |
| $n_c$ | Number of constraints |
| $n_L$ | Number of links |
| $O_1$ | Fixed bearing on link 1 |
| $\mathbf{ob}$ | Absolute velocity of point $B$ |
| $P$ | Prism pair; planet gear; power; diametral pitch |
| $\mathbf{P}$ | Piston force |
| $P^n$ | Normal diametral pitch |
| $p$ | Transverse circular pitch |
| $p_b$ | Base pitch |
| $p^n$ | Normal circular pitch |
| $p_w$ | Axial pitch of worm |
| $R$ | Revolute pair; ring gear; length of crank |
| $\mathbf{R}$ | Position vector |
| $r$ | Radius of pitch circle |
| $\mathbf{r}$ | Position vector; vector representing a link |
| $\dot{\mathbf{r}}$ | Derivative of $\mathbf{r}$ with respect to time |
| $r^\circ$ | Train value (speed ratio) for a planetary train relative to the carrier |
| $r_a$ | Length of cam-follower arm; radius of addendum circle |
| $r_b$ | Base circle radius; radius of back cone element |
| $r_c$ | Center distance between cam and follower pivots; pitch radius of equivalent spur gear |
| $r_f$ | Radius of cam-follower roller; radius of friction circle |
| $r_m$ | Mean pitch radius |
| $r_v$ | Velocity ratio |
| $\mathbf{r}^u$ | Unit vector |
| $\mathbf{r}_x$ | $x$ component of vector $\mathbf{r}$ |
| $S$ | Sphere pair; sun gear |
| $\mathbf{s}$ | Displacement |
| $s$ | Cam follower displacement |
| $\dot{s}$ | Slider velocity |
| $s_n$ | Axial spacing of engine cranks and cylinders |
| $\mathbf{T}$, $T$ | Torque |
| $\mathbf{T}_e$ | External torque |
| $t$ | Time; gear tooth thickness |
| $\mathbf{v}$ | Velocity |
| $v_p$ | Pitch line velocity |
| $W$ | Work |
| $w$ | Gear tooth width; weight |
| $x$, $y$, $z$ | Cartesian coordinates |
| $\times$ | Cross product |
| $\alpha$, $\boldsymbol{\alpha}$ | Angular acceleration |
| $\alpha$ | Cam rotation angle; angle of approach |
| $\beta$ | Angle of recess |
| $\Gamma$ | Pitch angle |

| | |
|---|---|
| $\gamma$ | Cam follower rotation; pitch angle |
| $\delta\phi$, $\delta x$ | Virtual displacements |
| $\theta$ | Angular position of link; cam angle; angle of action; connecting rod angle |
| $\theta_i$ | Angular position of link $i$ |
| $\theta_n$ | Angular spacing of engine cylinders |
| $\lambda$ | Lead angle of worm |
| $\mu$ | Coefficient of sliding friction |
| $\rho$ | Radius of curvature |
| $\rho_p$ | Radius of curvature of pitch curve |
| $\Sigma$ | Angle between shafts |
| $\phi$ | Transmission angle; pressure angle; transverse pressure angle; friction angle |
| $\phi_i$ | Angular position of link $i$ |
| $\phi^n$ | Normal pressure angle |
| $\psi$ | Involute angle; helix angle |
| $\psi_n$ | Angular spacing of engine cranks |
| $\omega$, $\boldsymbol{\omega}$ | Angular velocity |

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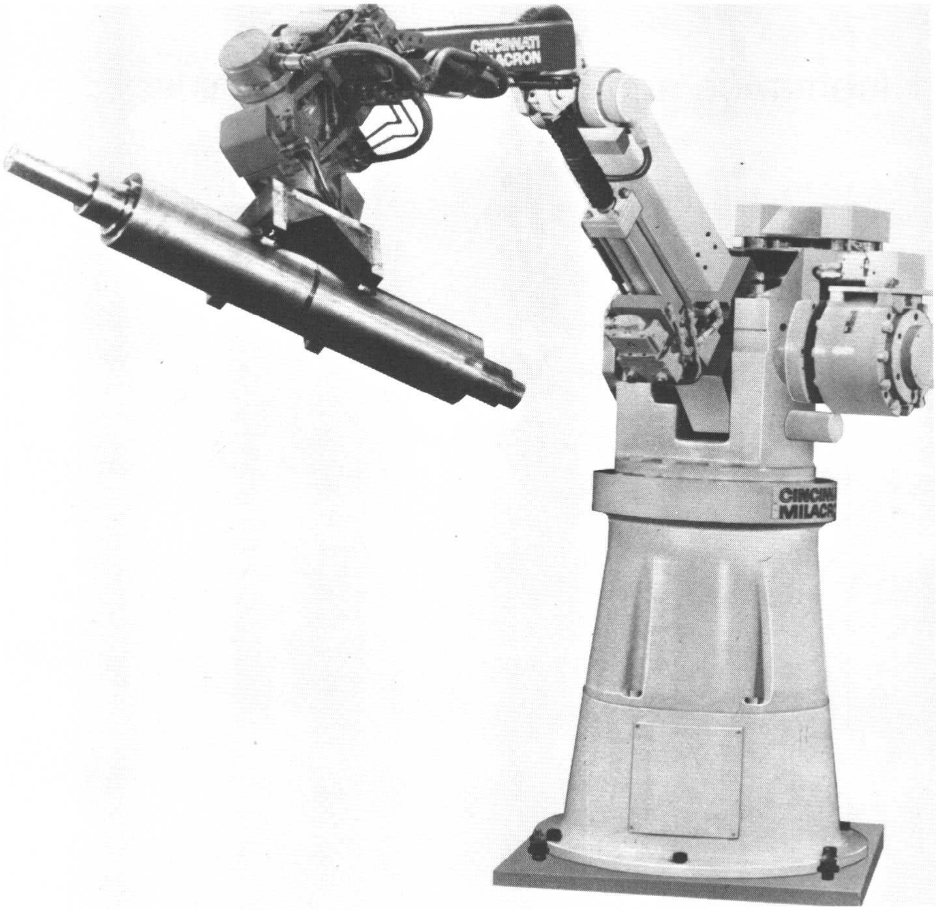


An industrial robot. SOURCE: Cincinnati Milacron.

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