

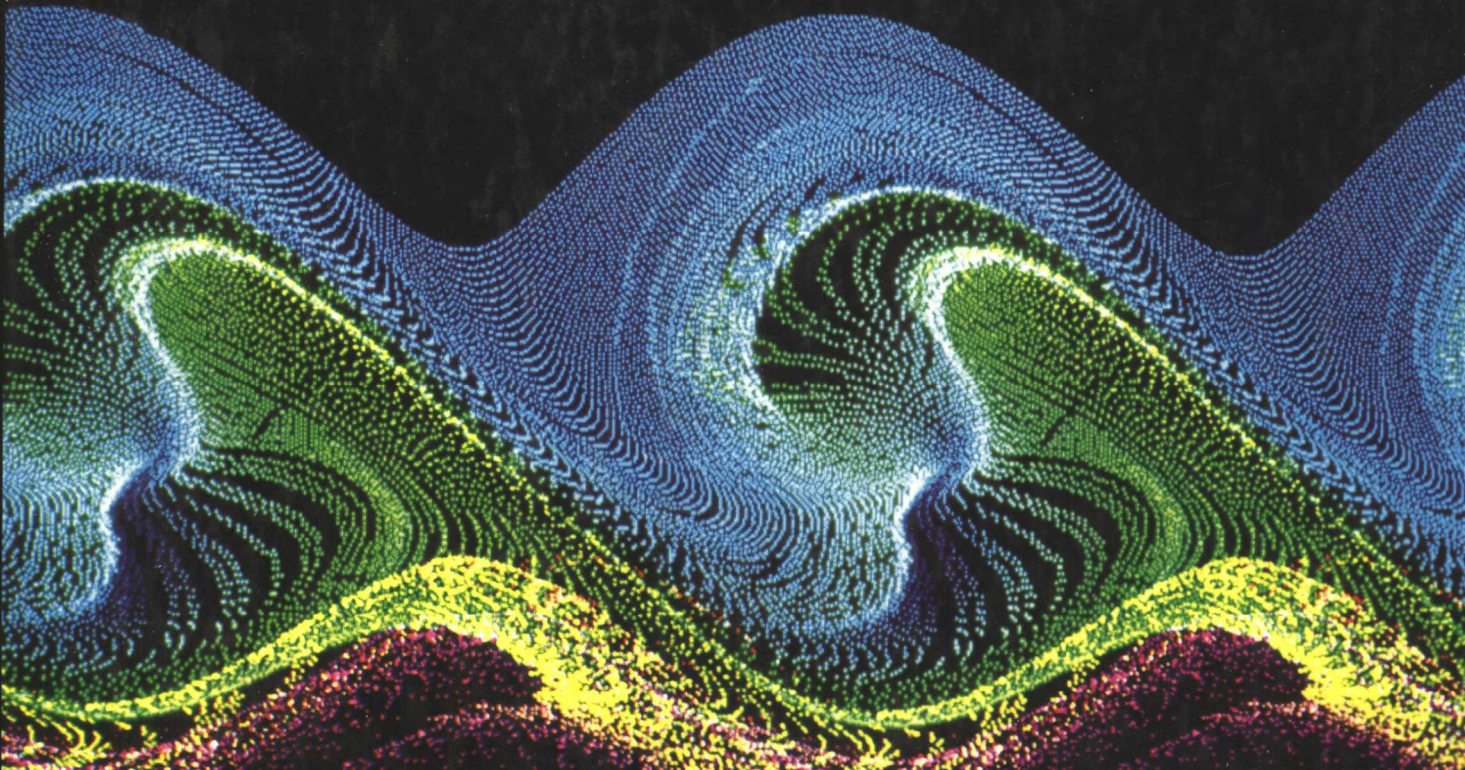
**Frank M. White**  
**Fluid Mechanics**

THIRD EDITION

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# **Fluid Mechanics**

**— T H I R D   E D I T I O N —**

**Frank M. White**

UNIVERSITY OF RHODE ISLAND

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## **FLUID MECHANICS**

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## About the Author

**Frank M. White**, a native of Augusta, Georgia, went to undergraduate school at Georgia Tech and received a B.M.E. degree in 1954. He then attended the Massachusetts Institute of Technology for an S.M. degree in 1956, returning to Georgia Tech to earn a Ph.D. in mechanical engineering in 1959. He began teaching aerospace engineering at Georgia Tech in 1957 and went to the University of Rhode Island in 1964, where he continues to serve as professor of mechanical and ocean engineering.

At the University of Rhode Island, he became interested in oceanographic and coastal flow problems and in 1966 helped found the first department of ocean engineering in the United States. His research interests have always been in shear layers and convection heat transfer. Known primarily as a teacher and writer, he has received the ASEE Westinghouse Teaching Excellence Award in addition to six University of Rhode Island teaching awards. His research accomplishments include some 80 technical papers and reports and the 1973 ASME Lewis F. Moody Research Award in Fluids Engineering. He has written three undergraduate textbooks: *Fluid Mechanics*, *Heat Transfer*, and *Heat and Mass Transfer*.

During 1979–1990 he was editor in chief of the *ASME Journal of Fluids Engineering* and now serves as chairman of the ASME Publications Committee. In 1986 he was named a Fellow of ASME and in 1991 received the ASME Fluids Engineering Award. He lives with his wife, Jeanne, in Narragansett, Rhode Island.

# Preface

The third edition of this textbook sees additions and deletions but no philosophical change. The basic outline of eleven chapters and five appendixes remains the same. The triad of integral, differential, and experimental approaches is retained. The informal, student-oriented style is retained. On the other hand, many problem exercises and some fully worked examples have been changed, and a number of new photographs and figures have been added.

The total number of problem exercises continues to increase, from 1089 in the first edition, to 1169 in the second, to 1392 in this third edition. About 750, or 53 percent, of the present problems are new. Also new to this edition are ninety-one word problems, or “thought exercises,” and, also a first, there are nine design projects: more extensive, open-ended exercises which challenge the student to obtain parameter studies and flow optimizations without knowing any obvious exact “answers.”

There are some revisions in every chapter, with the most extensive being in Chapters 3, 4, 7, 8, and 10. Chapter 1—which is purely introductory and could be assigned as reading—continues to be toned down from earlier editions. I am trying to accept the constant reviewer suggestions that this chapter can do without such detailed discussions.

Chapter 2 is improved by a better discussion of the stability of floating bodies, with a simpler procedure for computing the metacentric height. Coverage is confined to static fluids and rigid-body motions.

Chapter 3 has been rearranged so that Bernoulli’s equation comes *last*, after control-volume mass, linear momentum, angular momentum, and energy studies. I know that some texts have an entire chapter on the Bernoulli equation, but it is a dangerously restrictive relation which is often misused by both students and graduate engineers.

Chapter 4 used to be confined solely to the basic partial differential equations of fluid mechanics, but now a few *solutions* are included also, for both potential flow and viscous flow. This meant bringing forward some viscous material from Chapter 6 and some inviscid flow from Chapter 8. Both reviewers and students complained that they wanted to see some applied results right after covering the equations themselves. If you disagree with this, just hold back and treat Sections 4.10 and 4.11 later in your course.

Chapter 5 has been slightly modified to approach dimensional analysis from the point of view of selecting *scaling variables* before using the pi theorem. Students have always complained that the pi theorem is too ambiguous, leading to a multitude of different parameter groups. By deciding in advance how to scale and present the data, the ambiguity is reduced or eliminated. Section 5.6, on “inventive use of the data,” has been eliminated, made unnecessary by scaling concepts.

In Chapter 6, Section 6.5 on “alternate forms of the Moody chart” has been dropped and replaced by a standard presentation of the three basic pipe-flow computations: pressure drop, flow rate, and pipe sizing. I liked the “alternate forms,” obviously, but no one else did. I have also introduced some newer types of flow meters for further enrichment.

Chapter 8 now picks up from the sample plane potential flows of Section 4.10 and plunges right into inviscid-flow analysis. The method of *boundary elements* is now introduced briefly because it is a simple and powerful technique which is very popular in industry.

In the study of gradually varied open-channel flow, Section 10.6, the older tabular method of computation has been replaced by a direct numerical integration. Students are now quite familiar with numerical techniques such as Runge-Kutta and need not use a table or spreadsheet.

Some additional fluid properties and correlation formulas have been included in the appendices, which are otherwise much the same.

This text now includes some elegant software, prepared by Password, Inc., of Baltimore, Maryland, to help the student learn fluid mechanics and to illustrate certain topics. A diskette is included in the back of the book and is available for both Macintosh and PC Windows systems. The software is divided into seven numbered sections, or scenarios, as follows:

1. Viscosity and density of gases at various temperatures.
2. Viscosity and density of liquids at various temperatures.
3. Hydrostatic pressure and force on immersed plates.
4. Force required to support a fluid jet turning through an angle.
5. Three types of pipe flow calculations using the Moody chart:
  - a. Pressure drop for known flow rate and pipe size.
  - b. Flow rate for known pressure drop and pipe size.
  - c. Pipe size for known flow rate and pressure drop.
6. Plane potential flow patterns illustrated in detail:
  - a. A source near a plane wall.
  - b. Two equal vortices of like sense.
  - c. Two equal vortices of unlike sense.
  - d. The Rankine oval of arbitrary aspect ratio.
7. Compressible flow of an ideal gas:
  - a. Isentropic nozzle flow with area changes.
  - b. The normal shock wave.
  - c. Constant-area duct flow with friction.
  - d. Constant-area frictionless duct flow with heat transfer.
  - e. Oblique shock waves.
  - f. Prandtl-Meyer supersonic expansion waves.

When one of these topics appears in the text, a numbered computer-disk icon will appear in the text margin to key the reader toward the software. I have found this software to be very helpful and educational, and recommend that it be installed and used in your course. It is quite easy to use.

So many people have helped me that I cannot remember or list them all. I would like especially to express my appreciation to the reviewers and correspondents who gave detailed suggestions: A. J. Baker, University of Tennessee; John R. Biddle, California Polytechnic University; David Caughey, Cornell University; John M. Cimbala, Pennsylvania State University; M. E. Clark, University of Illinois; Martin Crawford, University of Alabama at Birmingham; Robert M. Hartman, Widener University; Jacques Lewalle, Syracuse University; George C. Lindauer, University of Louisville; D. E. Richards, Rose-Hulman; Richard F. Salant, Georgia Tech; Alexander J. Smits, Princeton University; and R. J. Sobey, University of California at Berkeley. Finally, I continue to enjoy the support of my wife and family in these writing efforts.

*Frank M. White*

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# Chapter 1

## Introduction

### 1.1 Preliminary Remarks

Fluid mechanics is the study of fluids either in motion (fluid *dynamics*) or at rest (fluid *statics*) and the subsequent effects of the fluid on the boundaries, which may be either solid surfaces or other fluids. Both gases and liquids are classified as fluids, and the number of fluids engineering applications is enormous: breathing, blood flow, swimming, pumps, fans, turbines, airplanes, ships, rivers, windmills, pipes, missiles, icebergs, engines, filters, jets, and sprinklers, to name a few. When you think about it, almost everything on this planet either is a fluid or moves with respect to a fluid.

The essence of the subject of fluid flow is a judicious compromise between theory and experiment. Since fluid flow is a branch of mechanics, it satisfies a set of well-documented basic conservation laws, and thus a great deal of theoretical treatment is available. The theory is often frustrating, however, because it applies mainly to certain idealized situations which may be invalid in practical problems. The two chief obstacles to a workable theory are geometry and viscosity. The general theory of fluid motion (Chap. 4) is too difficult to enable the user to attack arbitrary geometric configurations, so that most textbooks concentrate on flat plates, circular pipes, and other easy geometries. It is possible to apply numerical techniques to arbitrary geometries, and specialized textbooks are now appearing which explain these digital-computer approximations [1, 2].<sup>1</sup> This book will present many theoretical results while keeping their limitations in mind.

The second obstacle to a workable theory is the action of viscosity, which can be neglected only in certain idealized flows (Chap. 8). First, viscosity increases the difficulty of the basic equations, although the boundary-layer approximation found by Ludwig Prandtl in 1904 (Chap. 7) has greatly simplified viscous-flow analyses. Second, viscosity has a destabilizing effect on all fluids, giving rise, at frustratingly small velocities, to a disorderly, random phenomenon called *turbulence*. The theory of turbulent flow is crude and heavily backed up by experiment (Chap. 6), yet it can be quite serviceable as an engineering estimate. Textbooks are beginning to present digital-computer techniques for turbulent-flow analysis [3], but they are based strictly upon empirical assumptions regarding the time mean of the turbulent stress field.

Thus there is theory available for fluid-flow problems, but in all cases it should be

<sup>1</sup> Numbered references appear at the end of each chapter.

backed up by experiment. Often the experimental data provide the main source of information about specific flows, such as the drag and lift of immersed bodies (Chap. 7). Fortunately, fluid mechanics is a highly visual subject, with good instrumentation [4, 5], and the use of dimensional analysis and modeling concepts (Chap. 5) is widespread. Thus experimentation provides a natural and easy complement to the theory. Appendix C lists a variety of interesting films which have been prepared to visualize fluid-flow phenomena. You should keep in mind that theory and experiment should go hand in hand in all studies of fluid mechanics.

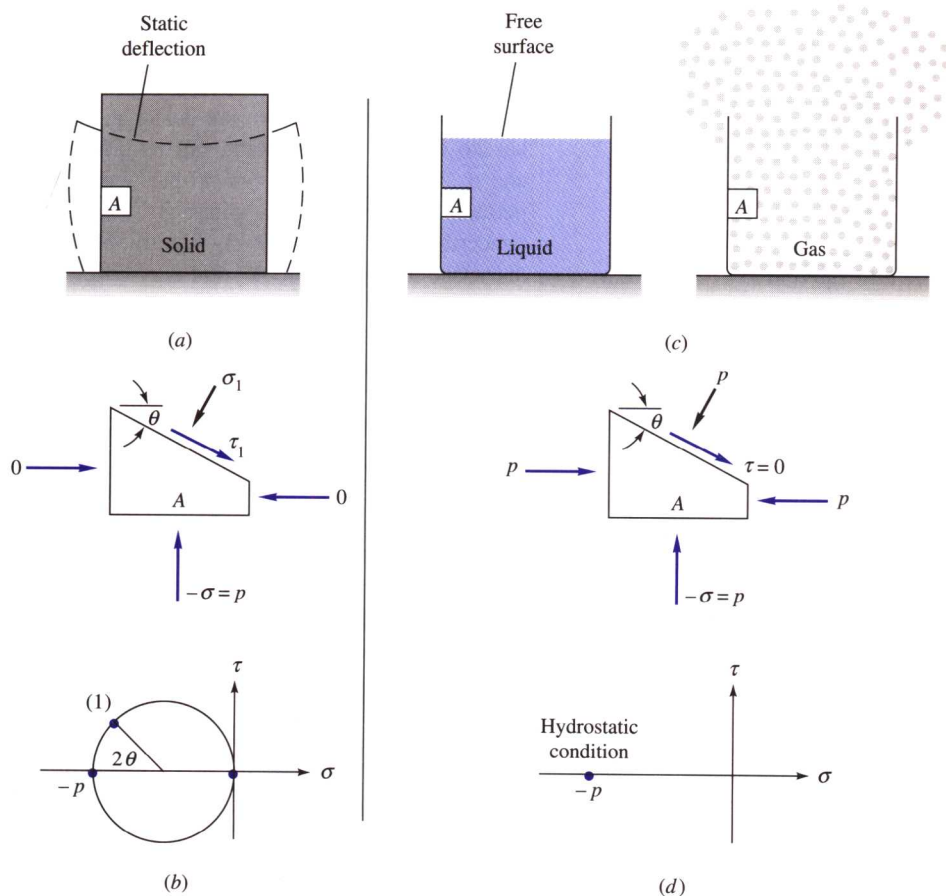
## 1.2 The Concept of a Fluid

From the point of view of fluid mechanics, all matter consists of only two states, fluid and solid. The difference between the two is perfectly obvious to the lay person, and it is an interesting exercise to ask a lay person to put this difference into words. The technical distinction lies with the reaction of the two to an applied shear or tangential stress. *A solid can resist a shear stress by a static deformation; a fluid cannot.* Any shear stress applied to a fluid, no matter how small, will result in motion of that fluid. The fluid moves and deforms continuously as long as the shear stress is applied. As a corollary, we can say that a fluid at rest must be in a state of zero shear stress, a state often called the hydrostatic stress condition in structural analysis. In this condition, Mohr's circle for stress reduces to a point, and there is no shear stress on any plane cut through the element under stress.

Given the definition of a fluid above, every lay person also knows that there are two classes of fluids, *liquids* and *gases*. Again the distinction is a technical one concerning the effect of cohesive forces. A liquid, being composed of relatively close-packed molecules with strong cohesive forces, tends to retain its volume and will form a free surface in a gravitational field if unconfined from above. Free-surface flows are dominated by gravitational effects and are studied in Chaps. 5 and 10. Since gas molecules are widely spaced with negligible cohesive forces, a gas is free to expand until it encounters confining walls. A gas has no definite volume, and when left to itself without confinement, a gas forms an atmosphere which is essentially hydrostatic. The hydrostatic behavior of liquids and gases is taken up in Chap. 2. Gases cannot form a free surface, and thus gas flows are rarely concerned with gravitational effects other than buoyancy.

Figure 1.1 illustrates a solid block resting on a rigid plane and stressed by its own weight. The solid sags into a static deflection, shown as a highly exaggerated dashed line, resisting shear without flow. A free-body diagram of element *A* on the side of the block shows that there is shear in the block along a plane cut at an angle  $\theta$  through *A*. Since the block sides are unsupported, element *A* has zero stress on the left and right sides and compression stress  $\sigma = -p$  on the top and bottom. Mohr's circle does not reduce to a point, and there is nonzero shear stress in the block.

By contrast, the liquid and gas at rest in Fig. 1.1 require the supporting walls in order to eliminate shear stress. The walls exert a compression stress of  $-p$  and reduce Mohr's circle to a point with zero shear everywhere, i.e., the hydrostatic condition. The liquid retains its volume and forms a free surface in the container. If the walls are removed, shear develops in the liquid and a big splash results. If the container is tilted, shear again develops, waves form, and the free surface seeks a horizontal configuration, pouring out over the lip if necessary. Meanwhile, the gas is unrestrained and expands out of the container, filling all available space. Element *A* in the gas is also hydrostatic



**Fig. 1.1** A solid at rest can resist shear. (a) Static deflection of the solid; (b) equilibrium and Mohr's circle for solid element A. A fluid cannot resist shear. (c) Containing walls are needed; (d) equilibrium and Mohr's circle for fluid element A.

and exerts a compression stress  $-p$  on the walls.

In the above discussion, clear decisions could be made about solids, liquids, and gases. Most engineering fluid-mechanics problems deal with these clear cases, i.e., the common liquids, such as water, oil, mercury, gasoline, and alcohol, and the common gases, such as air, helium, hydrogen, and steam, in their common temperature and pressure ranges. There are many borderline cases, however, of which you should be aware. Some apparently "solid" substances such as asphalt and lead resist shear stress for short periods but actually deform slowly and exhibit definite fluid behavior over long periods. Other substances, notably colloid and slurry mixtures, resist small shear stresses but "yield" at large stress and begin to flow as fluids do. Specialized textbooks are devoted to this study of more general deformation and flow, a field called *rheology* [6]. Also, liquids and gases can coexist in two-phase mixtures, such as steam-water mixtures or water with entrapped air bubbles. Specialized textbooks present the analysis of such *two-phase flows* [7]. Finally, there are situations where the distinction between a liquid and a gas blurs. This is the case at temperatures and pressures above the so-called *critical point* of a substance, where only a single phase exists, primarily resembling a gas. As pressure increases far above the critical point, the gaslike substance

becomes so dense that there is some resemblance to a liquid and the usual thermodynamic approximations like the perfect-gas law become inaccurate. The critical temperature and pressure of water are  $T_c = 647\text{ K}$  and  $p_c = 219\text{ atm}$ ,<sup>1</sup> so that typical problems involving water and steam are below the critical point. Air, being a mixture of gases, has no distinct critical point, but its principal component, nitrogen, has  $T_c = 126\text{ K}$  and  $p_c = 34\text{ atm}$ . Thus typical problems involving air are in the range of high temperature and low pressure where air is distinctly and definitely a gas. This text will be concerned solely with clearly identifiable liquids and gases, and the borderline cases discussed above will be beyond our scope.

### 1.3 The Fluid as a Continuum

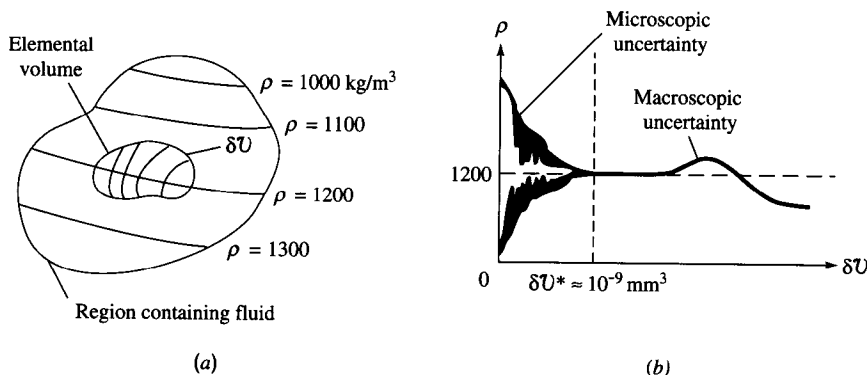
We have already used technical terms such as *fluid pressure* and *density* without a rigorous discussion of their definition. As far as we know, fluids are aggregations of molecules, widely spaced for a gas, closely spaced for a liquid. The distance between molecules is very large compared with the molecular diameter. The molecules are not fixed in a lattice but move about freely relative to each other. Thus fluid density, or mass per unit volume, has no precise meaning because the number of molecules occupying a given volume continually changes. This effect becomes unimportant if the unit volume is large compared with, say, the cube of the molecular spacing, when the number of molecules within the volume will remain nearly constant in spite of the enormous interchange of particles across the boundaries. If, however, the chosen unit volume is too large, there could be a noticeable variation in the bulk aggregation of the particles. This situation is illustrated in Fig. 1.2, where the “density” as calculated from molecular mass  $\delta m$  within a given volume  $\delta V$  is plotted versus the size of the unit volume. There is a limiting volume  $\delta V^*$  below which molecular variations may be important and above which aggregate variations may be important. The *density*  $\rho$  of a fluid is best defined as

$$\rho = \lim_{\delta V \rightarrow \delta V^*} \frac{\delta m}{\delta V} \quad (1.1)$$

The limiting volume  $\delta V^*$  is about  $10^{-9}\text{ mm}^3$  for all liquids and for gases at atmospheric pressure. For example,  $10^{-9}\text{ mm}^3$  of air at standard conditions contains approximately

<sup>1</sup> One atmosphere equals  $2116\text{ lbf/ft}^2 = 101,300\text{ Pa}$ .

**Fig. 1.2** The limit definition of continuum fluid density: (a) an elemental volume in a fluid region of variable continuum density; (b) calculated density versus size of the elemental volume.



$3 \times 10^7$  molecules, which is sufficient to define a nearly constant density according to Eq. (1.1). Most engineering problems are concerned with physical dimensions much larger than this limiting volume, so that density is essentially a point function and fluid properties can be thought of as varying continually in space, as sketched in Fig. 1.2a. Such a fluid is called a *continuum*, which simply means that its variation in properties is so smooth that the differential calculus can be used to analyze the substance. We shall assume that continuum calculus is valid for all the analyses in this book. Again there are borderline cases for gases at such low pressures that molecular spacing and mean free path<sup>1</sup> are comparable to, or larger than, the physical size of the system. This requires that the continuum approximation be dropped in favor of a molecular theory of rarefied-gas flow [8]. In principle, all fluid-mechanics problems can be attacked from the molecular viewpoint, but no such attempt will be made here. Note that the use of continuum calculus does not preclude the possibility of discontinuous jumps in fluid properties across a free surface or fluid interface or across a shock wave in a compressible fluid (Chap. 9). Our calculus in Chap. 4 must be flexible enough to handle discontinuous boundary conditions.

## 1.4 Dimensions and Units

A *dimension* is the measure by which a physical variable is expressed quantitatively. A *unit* is a particular way of attaching a number to the quantitative dimension. Thus length is a dimension associated with such variables as distance, displacement, width, deflection, and height, while centimeters and inches are both numerical units for expressing length. Dimension is a powerful concept about which a splendid tool called *dimensional analysis* has been developed (Chap. 5), while units are the nitty-gritty, the number which the customer wants as the final answer.

Systems of units have always varied widely from country to country, even after international agreements have been reached. Engineers need numbers and therefore unit systems, and the numbers must be accurate because the safety of the public is at stake. You cannot design and build a piping system whose diameter is  $D$  and whose length is  $L$ . And U.S. engineers have persisted too long in clinging to British systems of units. There is too much margin for error in most British systems, and many an engineering student has flunked a test because of a missing or improper conversion factor of 12 or 144 or 32.2 or 60 or 1.8. Practicing engineers can make the same errors. The writer is aware from personal experience of a serious preliminary error in the design of an aircraft due to a missing factor of 32.2 to convert pounds of mass to slugs.

In 1872 an international meeting in France proposed a treaty called the Metric Convention, which was signed in 1875 by 17 countries including the United States. It was an improvement over British systems because its use of base 10 is the foundation of our number system, learned from childhood by all. Problems still remained because even the metric countries differed in their use of kiloponds instead of dynes or newtons, kilograms instead of grams, or calories instead of joules. To standardize the metric system, a General Conference of Weights and Measures attended in 1960 by 40 countries proposed the *International System of Units* (SI). We are now undergoing a painful period of transition to SI, an adjustment which may take the remainder of this century to complete. The professional societies have led the way. Since July 1, 1974, SI units

<sup>1</sup> The mean distance traveled by molecules between collisions.



**Table 1.1** Primary Dimensions in SI and BG Systems

Primary dimension	SI unit	BG unit	Conversion factor
Mass $\{M\}$	Kilogram (kg)	Slug	1 slug = 14.5939 kg
Length $\{L\}$	Meter (m)	Foot (ft)	1 ft = 0.3048 m
Time $\{T\}$	Second (s)	Second (s)	1 s = 1 s
Temperature $\{\Theta\}$	Kelvin (K)	Rankine ( $^{\circ}\text{R}$ )	1 K = 1.8 $^{\circ}\text{R}$

have been required by all papers published by the American Society of Mechanical Engineers, which prepared a useful booklet explaining the SI [9]. The present text will use SI units together with British gravitational (BG) units.

In fluid mechanics there are only four *primary dimensions* from which all other dimensions can be derived: mass, length, time, and temperature.<sup>1</sup> These dimensions and their units in both systems are given in Table 1.1. Note that the kelvin unit uses no degree symbol. The braces around a symbol like  $\{M\}$  mean “the dimension” of mass. All other variables in fluid mechanics can be expressed in terms of  $\{M\}$ ,  $\{L\}$ ,  $\{T\}$ , and  $\{\Theta\}$ . For example, acceleration has the dimensions  $\{LT^{-2}\}$ . The most crucial of these secondary dimensions is force, which is directly related to mass, length, and time by Newton’s second law

$$\mathbf{F} = m\mathbf{a} \tag{1.2}$$

From this we see that, dimensionally,  $\{F\} = \{MLT^{-2}\}$ . A constant of proportionality is avoided by defining the force unit exactly in terms of the primary units. Thus we define the newton and the pound of force

$$\begin{aligned} 1 \text{ newton of force} &= 1 \text{ N} \equiv 1 \text{ kg} \cdot \text{m/s}^2 \\ 1 \text{ pound of force} &= 1 \text{ lbf} \equiv 1 \text{ slug} \cdot \text{ft/s}^2 = 4.4482 \text{ N} \end{aligned} \tag{1.3}$$

In this book the abbreviation *lbf* is used for pound-force and *lb* for pound-mass. If instead one adopts other force units such as the dyne or the poundal or kilopond or adopts other mass units such as the gram or pound-mass, a constant of proportionality called  $g_c$  must be included in Eq. (1.2). We shall not use  $g_c$  in this book since it is not necessary in the SI and BG systems.

<sup>1</sup> If electromagnetic effects are important, a fifth primary dimension must be included, electric current  $\{I\}$ , whose SI unit is the ampere (A).

**Table 1.2** Secondary Dimensions in Fluid Mechanics

Secondary dimension	SI unit	BG unit	Conversion factor
Area $\{L^2\}$	m <sup>2</sup>	ft <sup>2</sup>	1 m <sup>2</sup> = 10.764 ft <sup>2</sup>
Volume $\{L^3\}$	m <sup>3</sup>	ft <sup>3</sup>	1 m <sup>3</sup> = 35.315 ft <sup>3</sup>
Velocity $\{LT^{-1}\}$	m/s	ft/s	1 ft/s = 0.3048 m/s
Acceleration $\{LT^{-2}\}$	m/s <sup>2</sup>	ft/s <sup>2</sup>	1 ft/s <sup>2</sup> = 0.3048 m/s <sup>2</sup>
Pressure or stress $\{ML^{-1}T^{-2}\}$	Pa = N/m <sup>2</sup>	lbf/ft <sup>2</sup>	1 lbf/ft <sup>2</sup> = 47.88 Pa
Angular velocity $\{T^{-1}\}$	s <sup>-1</sup>	s <sup>-1</sup>	1 s <sup>-1</sup> = 1 s <sup>-1</sup>
Energy, heat, work $\{ML^2T^{-2}\}$	J = N · m	ft · lbf	1 ft · lbf = 1.3558 J
Power $\{ML^2T^{-3}\}$	W = J/s	ft · lbf/s	1 ft · lbf/s = 1.3558 W
Density $\{ML^{-3}\}$	kg/m <sup>3</sup>	slugs/ft <sup>3</sup>	1 slug/ft <sup>3</sup> = 515.4 kg/m <sup>3</sup>
Viscosity $\{ML^{-1}T^{-1}\}$	kg/(m · s)	slugs/(ft · s)	1 slug/(ft · s) = 47.88 kg/(m · s)
Specific heat $\{L^2T^{-2}\Theta^{-1}\}$	m <sup>2</sup> /(s <sup>2</sup> · K)	ft <sup>2</sup> /(s <sup>2</sup> · $^{\circ}\text{R}$ )	1 m <sup>2</sup> /(s <sup>2</sup> · K) = 5.980 ft <sup>2</sup> /(s <sup>2</sup> · $^{\circ}\text{R}$ )