compressible-fluid dynamics

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preface

This book introduces the fundamentals of compressible-fluid motion, or gasdynamics. It is intended as a text for senior and graduate students in engineering, physics, and applied mathematics. It may also be useful as a source for applied researchers in various fields.

The first three chapters are introductory in nature and are included to provide a self-contained treatment of fluid mechanics. Chapters 1 and 2 concisely present the relevant continuum mechanics and thermodynamics; Chapter 3 is a description of dimensional reasoning as it applies to compressible-fluid flow. A student who encounters this material for the first time in these chapters will likely find it heavy going: on this account some previous preparation in the form of a first course in fluid mechanics and in thermodynamics is desirable. Those students who already enjoy adequate preparation in these areas may wish to skip this material altogether.

This book contains more material than can reasonably be covered in a normal one-semester or one-quarter course. The selection of material suitable to a particular course can safely be entrusted to the instructor.

There are already several excellent books in the field of gasdynamics, in particular Shapiro's *Compressible Fluid Flow* and Liepmann and Roshko's *Elements of Gasdynamics*. I have enjoyed the considerable advantage of reading these works as a student and as a teacher, and acknowledge my debt to them. Merely to retread the ground already covered by these existing works would, however, be wasteful and self-defeating. What is offered here is not a redigest of some venerable work but a book whose merits and deficiencies are peculiarly its own. They can best be assessed by reading the book itself.

In selecting the material to be covered, I have tried to emphasize fundamental topics such as acoustics, shock waves, and the nature of compressible flow itself. In short, the selection has favored fundamentals over techniques. In choosing specific problems for consideration, I have preferred those which seem to be accessible to ordinary experience: along these lines, no special effort has been made to treat somewhat esoteric subjects such as high-temperature gases or fluids far from thermodynamic equilibrium.

Many individuals have given valuable help along the road to publication. The manuscript has been read and criticized in its entirety by Louis Solomon. It has been read and criticized in its various parts by several of my colleagues at Rensselaer, in particular Steven Ball, Henrik Hagerup, Gerald Kliman, Howard Littman, Charles Muckenfuss, Euan Somerscales, and Hendrik Van Ness. The thorough review of the manuscript by Richard Corlett and the many corrections to it by Howard Cyphers have been especially helpful. Assistance in calculation has been given by E. T. Laskaris, Michael Liu, Dean Nairn, Antonio Artiles, and Pedro Porrello. Editorial help at a very practical level has come from Bruce, Claudia, Stephen, and Jean Thompson. The typing has been most effectively handled by Joanne Margosian. To all these individuals, and several others unnamed, my thanks.

This book could be written only in a situation where individual enterprises of this kind are encouraged: the support of the School of Engineering at Rensselaer Polytechnic Institute, and of Hendrik Van Ness in particular, has been invaluable. Finally, I would like to acknowledge the influence of a zealous advocate of vectors, Kenneth Bisshopp, and of a fine teacher, Ascher Shapiro, who first introduced me to this subject.

Philip A. Thompson

list of symbols[1]

a	Helmholtz function, $a \equiv e - Ts$
A	Cross-sectional area; amplitude coefficient; surface area
$\mathbf{b}$	Boundary or shock-front velocity
B	Constant in Tait equation
c	Sound speed, $c^2 \equiv \left(\frac{\partial P}{\partial \rho}\right)_s$, or phase velocity
c_p	Specific heat at constant pressure, $c_p \equiv \left(\frac{\partial h}{\partial T}\right)_p$
c_v	Specific heat at constant volume, $c_v \equiv \left(\frac{\partial e}{\partial T}\right)_v$
C^+, C^-	Labels for characteristics in xt plane
D_{ik}	Component of rate-of-deformation tensor
e	Base of natural logarithm; specific internal energy
$\mathbf{e}$	Unit vector
E	Acoustic energy per unit volume, $E \equiv \frac{p^2}{2\rho_0 c_0^2} + \frac{\rho_0 u^2}{2}$; voltage (electrical potential)
f	Wave function; general function; number of degrees of freedom; number of fundamental dimensions
F	Wave function; force; thermodynamic function, $F \equiv \int \frac{dP}{\rho c}$
g	Wave function; acceleration of gravity; Gibbs function, $g \equiv h - Ts$

[1] A few minor symbols, which have been used only briefly, are omitted from this list.

$\mathbf{G}$	Specific body-force vector
h	Specific enthalpy, $h \equiv e + Pv$; Planck's constant; liquid depth in shallow-water theory
H	Scale height of the atmosphere, $H \equiv \dfrac{RT_0}{g_0}$; Bernoulli constant, $H \equiv h + \dfrac{u^2}{2} + \Psi$
i	Imaginary unit, $i \equiv \sqrt{-1}$
$\mathbf{i}$	Cartesian unit vector
I	Integral
I_s	Specific impulse
$\mathscr{I}m$	Imaginary part of following expression
$\mathbf{j}$	Cartesian unit vector
J	Mass flux, $J \equiv \rho u$; molecular flux
J^+, J^-	Riemann invariants
k	Wave number, $k \equiv \dfrac{2\pi\nu}{c} = \dfrac{\omega}{c}$; Boltzmann constant
$\mathbf{k}$	Wave-number vector, $\mathbf{k} \equiv k\mathbf{e}$
K	Constant; bulk modulus; spring constant
l	Distance along acoustic ray; spacing distance
L	Length; inductance
m	Particle mass
m^+, m^-	Labels for Mach waves (characteristics); distances along such waves
$\dot{m}$	Rate of mass flow
M	Mach number, $M \equiv \dfrac{u}{c}$; total mass
$\tilde{M}$	Molecular weight
n	Distance normal to a streamline; index of refraction, $n \equiv \dfrac{c_0}{c}$; number density of molecules; number of dimensional variables
$\mathbf{n}$	Unit normal vector
N	Frequency of oscillation in the atmosphere ($N^2 > 0$ for stability)
$\tilde{N}$	Avogadro's number
$\mathscr{N}$	Number of moles

O	Abbreviation for order of magnitude
p	Acoustic (perturbation) pressure, $p \equiv P - P_0$; number of dimensionless variables
P	Pressure (absolute)
Pr	Prandtl number, $\mathrm{Pr} \equiv \mu c_p/\kappa$
q	Traffic flow rate, $q \equiv \rho u$
$\mathbf{q}$	Heat-flux vector
r	Spherical or cylindrical coordinate; number of variables with independent dimensions
R	Radius; specific gas constant, $R \equiv \dfrac{\tilde{R}}{\tilde{M}}$; electrical resistance
$\tilde{R}$	Universal gas constant, $\tilde{R} = \tilde{N}k$
$\mathscr{R}$	Acoustic impedance, $\mathscr{R} \equiv \rho c$
Re	Reynolds number, $\mathrm{Re} \equiv \dfrac{Lu}{\nu}$
$\mathscr{R}e$	Real part of following expression
s	Specific entropy; distance along streamline
S	Condensation, $S \equiv \dfrac{\rho - \rho_0}{\rho_0}$; surface
t	Time
T	Absolute temperature
$\mathbf{T}$	Surface-traction vector
u	Velocity magnitude; velocity component in x direction
$\mathbf{u}$	Vector velocity
U	Velocity; volume velocity $U \equiv uA$
v	Relative velocity component parallel to shock front; velocity component in y direction
$\mathbf{v}$	Molecular velocity
$\mathcal{v}$	Specific volume, $\mathcal{v} \equiv \dfrac{1}{\rho}$
V	Volume; peculiar molecular velocity
w	Relative velocity component normal to shock front
$\mathbf{w}$	Wind velocity
$\mathbf{x}$	Vector space coordinate
X	General variable

x, y, z	Cartesian space coordinates
z	Cylindrical coordinate; altitude above sea level
α	Attenuation coefficient; constant; amplitude function; thermal diffusivity, $\alpha \equiv \kappa/\rho c_p$
β	Bulk coefficient of thermal expansion, $\beta \equiv \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_p$; shock-front angle; coefficient in linearized equation, $\beta = \lvert\sqrt{M_\infty{}^2 - 1}\rvert$
γ	Ratio of specific heats, $\gamma \equiv \frac{c_p}{c_v}$; exponent in Tait equation
Γ	Dimensionless thermodynamic variable, $\Gamma \equiv \frac{\rho^3 c^4}{2}\left(\frac{\partial^2 v}{\partial P^2}\right)_s$
Γ_c	Circulation, $\Gamma_c \equiv \oint \mathbf{u} \cdot d\mathbf{l}$
δ	Small change; diffusivity, $\delta \equiv \frac{4\mu/3 + \mu_v}{\rho} + \frac{\gamma - 1}{\mathrm{Pr}}\frac{\mu}{\rho}$; ratio of specific heats, $\delta \equiv \frac{c_{pp}}{c_{pg}}$; boundary-layer thickness
δ_{ik}	Kronecker delta
Δ	Change; decibel level; shock-front thickness
ε	Small quantity; deviation of shock-front angle from Mach angle, $\varepsilon \equiv \beta - \mu$; energy per molecule
ε_{ijk}	Levi-Civita triple-index tensor
η	Similarity variable; mass fraction
θ	Flow angle; temperature
κ	Thermal conductivity
λ	Wavelength
Λ	Molecular mean free path
μ	Viscosity (ordinary shear viscosity); Mach angle, $\mu \equiv \sin^{-1}\frac{1}{M}$
μ_v	Bulk viscosity
ν	Kinematic viscosity, $\nu \equiv \frac{\mu}{\rho}$; frequency
ξ	Fluid-particle displacement; mass displacement
Ξ	Entropy flux
π	Pi, $\pi = 3.14159\ldots$

Π	Dimensionless pressure jump, $\Pi \equiv \frac{[P]}{\rho_1 c_1^2}$; general dimensionless quantity
Π_{ik}	Molecular momentum flux
ρ	Density
σ	Molecular diameter
σ_{ik}	Component of stress tensor
Σ	Sum; function of spatial coordinates
Σ_{ik}	Component of viscous stress tensor
τ	Relaxation time; dimensionless time
Υ	Dissipation function, $\Upsilon \equiv \Sigma_{ik} D_{ik}$
ϕ	Velocity potential
φ	Intermolecular potential
Φ_e	Acoustic energy flux (intensity)
Φ_m	Acoustic momentum flux
χ	Mole fraction
ψ	Polar coördinate; stream function
Ψ	Force potential
ω	Angular frequency, $\omega = 2\pi\nu$; Prandtl-Meyer function
$\mathbf{\Omega}$	Vorticity vector, $\mathbf{\Omega} \equiv \nabla \times \mathbf{u}$
Ω_{ik}	Component of spin tensor

contents

one
descriptions of fluid motion

1.1 Introduction

Fluid mechanics is the study of the motions of gases and liquids. *Gasdynamics*, or *compressible-fluid flow*, is the study of those motions for which changes in fluid density play an essential role.

Although there is some density change in every physical flow, it is often possible to neglect such changes and to treat the flow according to the idealization that the fluid is incompressible. This approximation may be applicable to gases, e.g., in low-speed flow around an airplane or flow through a vacuum cleaner, as well as to liquids.

On the other hand, the very small density changes associated with acoustic motions in liquids and gases cannot be neglected. Discussion of the precise conditions under which density changes must be considered will require the development of a little analytical apparatus and will be postponed to Chap. 3. Anticipating the results of that discussion, we simply set down the main practical categories of motion for which fluid compressibility plays a crucial role:

1. Wave propagation within the fluid
2. Steady flow in which the fluid speed is of the same order of magnitude as the speed of sound
3. Convection driven by body forces, e.g., gravity, acting on fluid subject to thermal expansion
4. Large-scale convection of gases in the presence of body forces

This list is not necessarily exhaustive, and some motions may fit into more than one category. This book is devoted mostly to the first two categories of motion, which have in common a relation to the fluid sound speed.

The principal equations of motion are developed in fairly general form in this chapter. This level of generality is not required for many of the applications, e.g., viscous forces can be neglected in many problems, but will permit us to arrive rationally at the various simplifying approximations. Some of the detailed steps will be omitted from the derivations given in this chapter. For a more complete treatment, see *Batchelor* [1967, chaps. 1–3] or *Aris* [1962, chaps. 1–6].

The equations of motion are developed from the concept that the fluid is a *continuum*; i.e., the fluid is considered to be matter which exhibits no structure, however finely it may be divided. This model makes it possible to treat fluid properties (such as density, temperature, and velocity) at a *point* in space and mathematically as continuous functions of space and time. The application of such a model to fluid motion is due principally to Leonhard Euler (1755). Treatments of fluid mechanics and solid mechanics from the continuum viewpoint have much in common (many of the equations in this chapter are applicable to fluids and solids indifferently, in fact), and the subjects taken together are called *continuum mechanics.*

There is an alternative way of proceeding, which begins with the particulate view of matter and by averaging over large numbers of molecules arrives finally at the continuum equations. This method, while perhaps more general in principle, is limited by practical difficulties to the Boltzmann equation applied to a dilute gas and will not be pursued here.

The continuum model may be expected to fail when the size of the fluid region of interest is of the same order as a characteristic dimension of the molecular structure. A suitable characteristic dimension for gases is the mean free path Λ (of the order of 10^{-7} m for air at standard conditions). For liquids, a corresponding characteristic molecular dimension is not clearly defined but may be taken to be a distance equal to several intermolecular spacings (for water, the intermolecular spacing L is of the order of 10^{-10} m). These dimensions are so small that the continuum model is violated only in extreme cases; two examples are the motion of dust or smoke particles of very small diameter d in the atmosphere (with $d \sim \Lambda$) and the propagation of high-frequency sound of wavelength λ in gases (with $\lambda \sim \Lambda$) or even in liquids (with $\lambda \sim L$). On the other hand, the mean free path Λ varies inversely with the density of a gas, so that under conditions of very low pressure, e.g., at high altitudes or in a vacuum chamber, even relatively large fluid regions cannot be described by the continuum model. We will be concerned only slightly with such extreme cases and adopt the continuum view henceforth. Occasional use will be made, however, of results from the elementary kinetic theory of gases, i.e., the *particulate* view, where such results enhance our physical insight for the problem of fluid motions.

1.2 Dynamical laws of motion

A natural concept within the continuum model is that of the *material volume*. This is an arbitrary collection of matter of fixed identity enclosed by a *material surface* (or boundary) every point of which moves with the local fluid velocity. This surface is purely hypothetical and in general does not correspond to any physical boundary in the flow: it may be helpful to imagine it as a perfectly flexible and extensible membrane of zero mass. As the material volume moves through space, it is deformed in shape and changed in volume, as sketched in Fig. 1.1. We will refer to the material volume as $V(t)$ and the material surface as $S(t)$. If the volume $V(t)$ is shrunk to a point, the resulting material point is called a *fluid particle*.

By definition, the surface $S(t)$ is impenetrable to matter: interdiffusion of chemical species thus cannot be accounted for by this particular model.

The dynamical laws of motion, from which most of the equations in this book are deduced, are stated for a material volume as follows:

1 *Conservation of mass* (*continuity*): The mass of a material volume is constant.
2 *Balance of linear momentum* (*Newton's second law*): The rate of change of the material-volume momentum is equal to the sum of the surface forces (due to pressure and viscous stresses) and body forces (such as gravity) acting upon it.

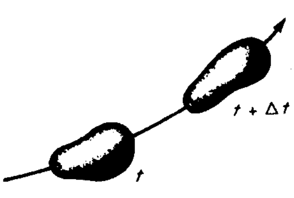

Figure 1.1
Material volume at time t and at time $t + \Delta t$.