SIGNALS, SYSTEMS; AND TRANSFORMS

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Preface

In very general terms, a *system* is a mechanism that operates upon *signals* (a form of information) to produce other signals. As examples, a stereo system takes a low-level audio signal and produces a high-level sound signal from the system's speakers, a radar tracking system takes radar return signals and produces estimates of where a target will be at the next radar return, and an economical model takes available economic data and predicts future economic behavior. In these diverse situations, the system concept is central and plays an important role in our increasingly quantitative-oriented world. Understanding the *system theory approach* is therefore becoming indispensable in such disciplines as engineering, economics, computer science, modeling, mathematics, and science.

This textbook is concerned with presenting the fundamental aspects of the system theory approach. It is written at a level that is comprehensible to students who have had a course in calculus, have an ability to manipulate complex numbers, and have had some exposure to differential equations. For most electrical engineering students, it should be possible to master the ideas in this text at the junior-year level. Students in other disciplines may be appropriately introduced to the system theory approach upon completion of the aforementioned prerequisites.

Although many of the motivating examples that appear throughout this text are oriented toward electrical engineering, a conscientious effort has been made to incorporate examples from other disciplines as well. The reasoning behind this approach is twofold. First, it is strongly felt that electrical engineering students should be made to appreciate that the tools they use in studying circuits, communication networks, and control systems are directly applicable to a far wider class of interesting applications.

Second, in recent years other quantitative-oriented disciplines are increasingly being exposed to concepts that have been standard to the electrical engineering profession. With this in mind, this textbook seeks to expose the important aspects of system theory from a general viewpoint and to demonstrate their applicability to electrical engineering and other disciplines by means of selected examples and problems.

It is generally possible to classify a given system as being either *discrete-time*, *continuous-time*, or a combination of discrete- and continuous-time. It is widely appreciated that the basic concepts central to discrete-time systems are more easily understood than are their continuous-time counterparts. On the other hand, it can be generally said that to each discrete-time concept there exists an identifiable continuous-time analogy. This being the case, we have here made the pedagogical decision to first develop a discrete-time idea and then immediately follow it with the analogous continuous-time idea.

In the introductory chapter, a philosophical treatment of the notions of signals and systems is undertaken. This includes the essential features distinguishing a discrete- and continuous-time signal and system; the conversion operation of continuous- to discrete-time signals; and motivational applications that include digital filtering, circuit analysis, and numerical integration and differentiation.

The formal development of signal theory is begun in Chapter 2 where discrete-time signals are examined in detail. It is there shown that a discrete-time signal may be viewed as a sequence of numbers. A procedure for making changes in the discrete-time variable is then studied. Furthermore, elementary operations on signals and fundamental signals such as the unit-impulse, unit-step, and sinusoid are explored. The chapter concludes with relevant signal operator applications. A similar treatment of continous-time signals is made in Chapter 3. The analogy between discrete- and continuous-time signals is here emphasized.

Chapter 4 develops the fundamental notion of linear signal operators. Linear signal operators are important due to their widespread usage in various practical applications and to the fact that a rather thorough analysis of such operators is possible. A parallel treatment of linear discrete- and continous-time signal operators is here given in which the similarity between analogous concepts is emphasized. Attention is directed toward the homogeniety and additivity properties of linear operators. In addition, such fundamental notions as operator time-invariance, stability, and transfer function are examined.

In contemporary system theory, the use of signal transformation theory is pervasive. For continous-time signals, the Laplace and Fourier transforms are preeminent. The basic properties and applications of the Laplace transform are studied in Chapter 5. The Laplace transform is shown to be an important tool for the study of signals and the linear operations on signals.

In Chapter 6 the techniques of transformation theory are developed for discrete-time signals. The z-transform is closely related to the Laplace transform; its function is to reduce linear difference equations or equivalently linear discrete-time systems to an algebraic form just as the Laplace transform reduces differential equations to algebraic forms. The applications of both Laplace and z-transform theory is presented in Chapter 7. Here the transfer function concept is used in several analysis contexts: modeling, relating interconnected systems, stability analysis, and others.

Chapter 8 presents the fundamental elements of Fourier series expansions and the corresponding signal approximation techniques leading to the development of Fourier transforms (particularly the computational aspects thereof) in Chapter 9.

The material presented is suitable for junior-level engineering students. A preliminary course in network theory is helpful for understanding the applications in Chapter 7. The book may also be used in a self-study mode for engineers and scientists desiring an introduction to the area of signal analysis.

James A. Cadzow
Hugh F. Van Landingham

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1

Introduction to Signals and Systems

1.1 INTRODUCTION TO SIGNALS

Contemporary *system theory* has found application in virtually all quantitative disciplines and even in disciplines that were heretofore conceived of as being nonquantitative in nature. Central to the system theory philosophy is the concept of *signal*. In a most fundamental sense, the word *signal* connotes the process of conveying information in some format. This interpretation holds for the most primitive form of information transmittal such as the smoke signal system employed by early-day American Indians to the most sophisticated form of modern-day communication theory.

For our purposes, we use the expression signal to denote a measurement or observation that contains information describing some phenomenon. In order to give our study mathematical structure, we designate signals by means of symbols such as the letters u, x, or y and refer to them as *the signals* u, x, or y. Thus, in a particular situation, the signal x might denote a particular time segment of an audio voltage waveform, the time history of an economic process, the time history of the neurological activity of a muscle system, and so forth.

Information by its very nature implies the notion of being variable or changeable. This is readily demonstrated by the ordinary process of conversation in which information is transferred by auditory signals (words). Thus, the prehistoric cave dweller, who could emit only a series of gruntlike sounds, was able to transmit far less information than is his or her twentieth-century counterpart, who is able to use a complex time sequence of sounds (words and sentences).

A signal in which the information characteristics can change, or fluctuate, is said to be dependent on another variable, which has been classically called time. This independent variable is referred to as time, since, in a large variety of signal theory applications, the underlying independent variable is intrinsically time. For example, the electrocardiogram signal that is displayed on a hospital monitor is seen to have an amplitude (voltage corresponding to the heart's electrical activity) that changes as a function of time. It must be mentioned, however, that there are many situations in which the signal is dependent on a variable other than time (for instance, distance, temperature, or frequency). Thus, when studying the behavior of a vibrating string, the independent variable is a distance measure. The nature of the independent variable is, of course, contingent on the particular measurement or observation being studied. We suffer no real loss in generality, however, by referring to the independent variable as time.

A signal is then very simply an ordinary function of an independent variable. Thus, the value of the signal x at the time instant t is denoted by the symbol $x(t)$. The reader must be careful in distinguishing the difference between the symbol x, which denotes the entire time history of the signal, and the symbol $x(t)$, which specifies the value of the signal at the time instant t. Although this distinction is of importance from a precise mathematical viewpoint, we often use the symbol $x(t)$ to denote a signal whenever there is no danger of misinterpretation. This practice is common in much of signal theory literature.

Continuous- and Discrete-Time Signals

As indicated above, a signal denotes a measurement or observation that contains information relevant to some phenomenon. Generally, the measurement's amplitude changes as time evolves. The manner in which the time variable evolves plays a most profound role in the resultant signal analysis. In many practical situations, the given measurement can change at any instant of time. These signals are called *continuous-time signals*, to reflect the continuous dependence of the signal on time. On the other hand, there exists an important class of processes in which the relevant signals can change value (or are defined) only at specific instants of time. These signals are said to be *discrete-time signals*. These rather abstract concepts are best illustrated by examples.

A sketch of the temperature fluctuation in a room might appear as shown in Fig. 1.1a. Here, signal x specifies the time history of the room temperature with $x(t)$ denoting its value at the specific time instant t. Since the room's temperature is capable of changing at any instant of time, this is clearly a continuous-time signal. In point of fact, many of nature's phenomena are modeled by relationships (differential equations) that are explicitly dependent on continuous-time signals. This is exemplified by Newton's laws of motion, voltage-current relationships in electrical networks, thermodynamic laws, and so forth.

On the other hand, there exists a class of dynamical phenomena that, typically, are man-made in origin and are characterized by discrete-time signals. Examples of this type of signal are abundant in the fields of econometrics, numerical analysis (algorithms), social sciences, operations research, computer sciences, etc. As an illus-

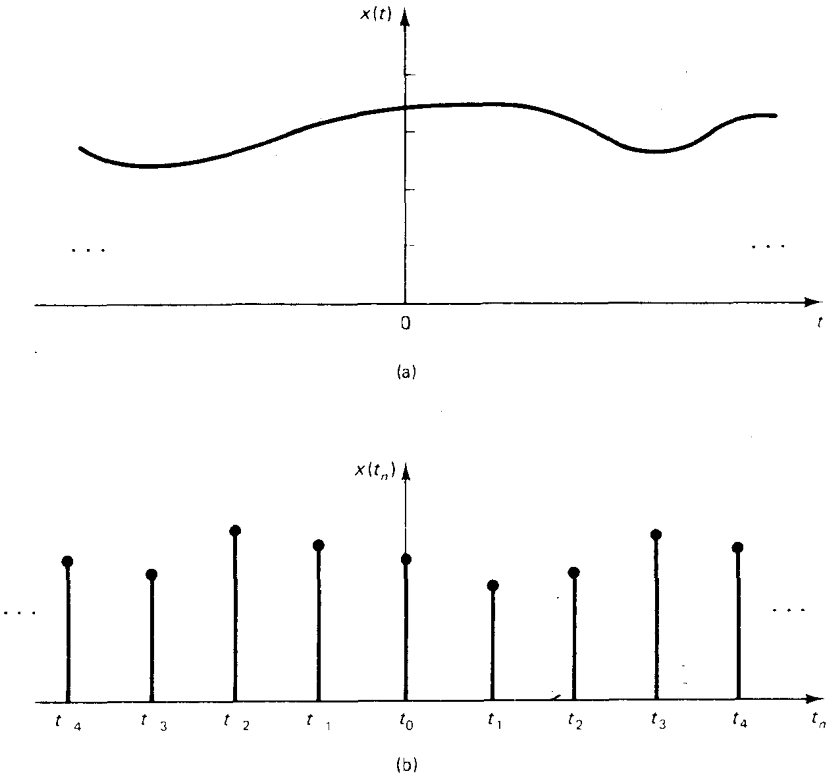

Figure 1.1 Sketch of typical signals that are functions of time: (a) room temperature versus t, and (b) gross national product versus t_n.

tration, consider the determination of our nation's gross national product (GNP). The gross national product is calculated at the end of specific three-month intervals, at which time the various economical components that constitute GNP are determined. A typical plot of GNP might appear as shown in Fig. 1.1b where t_n denotes the end of the given nth three-month period at which time the GNP is to be evaluated. This signal is obviously discrete-time in nature. It is noted that the abscissa axis in this plot is drawn in a continuous manner even though the signal itself has meaning only at the specific time instants t_n. It is for illustrative purposes that we have so displayed the abscissa axis.

Before proceeding further, let us give a more general interpretation to a discrete-time signal. In essence, a discrete-time signal is an ordered set of numbers

$$\ldots, x(t_{-2}), x(t_{-1}), x(t_0), x(t_1), x(t_2), \ldots \tag{1.1a}$$

where the discrete-time variable t_n indicates in which position the number $x(t_n)$ appears in the set of numbers. The three dots to the left of $x(t_{-2})$ and to the right of $x(t_2)$ indicate that the set of numbers continues indefinitely to the left and right, respectively. With this interpretation, it follows that we can think of a discrete-time signal as being a sequence of numbers. For notational convenience, it is desirable to suppress the t in the independent variable t_n and express $x(t_n)$ as $x(n)$ with n being an integer. Therefore, we hereafter interpret a discrete-time signal as being a sequence of numbers in which $x(n)$ denotes the nth member of the sequence. Thus, the discrete-time signal (1.1a) will be hereafter more compactly represented as

$$\ldots, x(-2), x(-1), x(0), x(1), x(2), \ldots \tag{1.1b}$$

In using this shorthand notation, however, it is important to always keep in mind the implicit concept that the integer argument n designates the time instant t_n at which the measure $x(t_n)$ becomes known.

There has recently been a great deal of interest devoted to the study of discrete-time signals. This is obviously a byproduct of the digital computer's development and utilization. The digital computer is a device typically employed to carry out some form of data processing in a rapid manner. Since the computer can essentially only add, subtract, multiply, and divide numbers, the data upon which it operates must be in the format of a sequence of numbers (recorded on magnetic tape, disks, cards, etc.). Therefore, the digital computer is typically used to perform some systematic processing of data which are in the form of a discrete-time signal. Hopefully, this will serve as an adequate motivational stimulus for the further study of discrete-time signals.

The signals displayed in Fig. 1.1 are obviously different in nature. In Fig. 1.1a, the time variable t takes on a continuum of values (that is, values in an interval), and it is for this reason that the corresponding signal is said to be a continuous-time signal. On the other hand, the time variable for the signal displayed in Fig. 1.1b is defined only at discrete-time instants, which results in such signals being referred to as discrete-time signals. Most signals can be classified as being either continuous- or discrete-time in nature as exemplified in Table 1.1.

Since continuous- and discrete-time signals are basically different, it is only natural that different methods have evolved for analyzing their characteristics. Thus we treat these two important classes of signals separately. Wherever possible, however, we point out the many common characteristics shared by each. In the next two chapters, we study some basic properties of discrete-time signals and then extend these concepts to continuous-time signals. This order of presentation reflects the fact that discrete-time signals are inherently easier to characterize and study.

TABLE 1.1 EXAMPLES OF DISCRETE- AND CONTINUOUS-TIME SIGNALS

Signal Description	Signal Type
Monthly new house sales in U.S.A.	Discrete-time
Hourly traffic flow at a highway intersection	Discrete-time
Weekly hotel occupancies	Discrete-time
Daily room temperature at 8:00 A.M.	Continuous-time
Voltage waveform at an amplifier's output terminal	Continuous-time
Speed of a launched rocket	Continuous-time
Electrocardiogram recording	Continuous-time

1.2 CONTINUOUS TO DISCRETE-TIME SIGNAL CONVERSION

In many applications, the underlying descriptive signal(s) being investigated (or used) is inherently continuous-time in nature. If we are to employ the considerable powers of the digital computer for the processing of such signals, however, it is necessary to convert these signals into a format that is compatible with digital computation. Namely, it is necessary to transform the continuous-time signal into a sequence of numbers that may then be manipulated by a digital computer algorithm. This transformation process is commonly referred to as *analog-to-digital (A-to-D) conversion.*

The operation of A-to-D conversion may be conveniently depicted as a switch closing instantaneously at the sample instants t_n. This conceptual model is depicted in Fig. 1.2, where the continuous-time signal $x(t)$ appears at the switch's input terminal and the associated sampled elements $x(t_n)$ appear at the output terminal. A-to-D converters are commonly available hardware items that appear in a variety of computer-based systems as typified by digital controllers and signal processors.

It is possible to provide a rather thorough analysis of the sampling operation. This is particularly true in the case where the sampling instants are equidistant, that is,

$$t_n = nT \qquad \text{for } n = 0, \pm 1, \pm 2, \ldots \tag{1.2}$$

in which T is a fixed time interval specifying the *sampling period.* For this uniform sampling scheme, it is readily shown that no information is lost through the sampling process provided that (1) the continuous-time signal is bandlimited and (2) the sampling period T is selected to be smaller than the reciprocal of the highest frequency component of the continuous-time signal. This is a rather startling result since it implies that in such cases, the entire continuous-time signal can be equivalently represented by its sampled values. (See Fig. 1.2.)

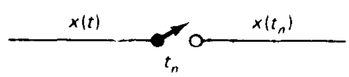

Figure 1.2 Continous-time to discrete-time signal conversion.

Example 1.1

Determine the number sequence generated when the continuous-time signal

$$x(t) = \begin{cases} 1 - |t| & \text{for } -1 \le t \le 1 \\ 0 & \text{for all other values of } t \end{cases}$$

is uniformly sampled with sampling period (1) $T = \frac{1}{4}$ s (second), (2) $T = \frac{1}{2}$ s, and (3) $T = 1$ s.

It is beneficial to make a plot of $x(t)$ versus t as shown in Fig. 1.3a in order to visualize the sampling operation. In the following, we shall drop the explicit appearance of sampling period T and write $x(n)$ instead of $x(nT)$ for the sampled signal. Thus, the reader must interpret the sampled signal as a sequence of numbers spaced by T-second intervals, where T is the underlying sampling period.

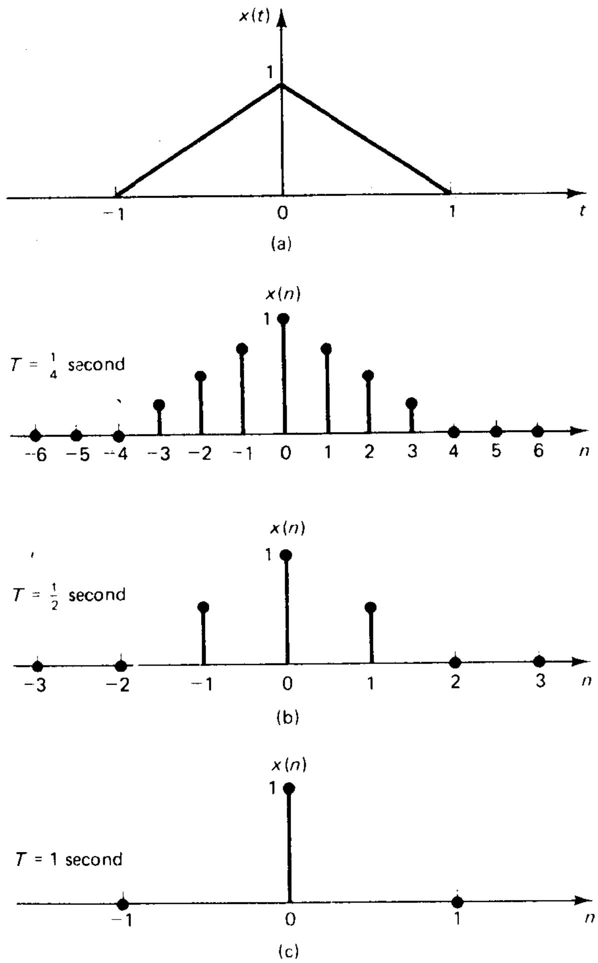

Figure 1.3 Process of uniform sampling with different sampling periods: (a) unsampled waveform (effectively, $T = 0$ s); (b) $T = \frac{1}{4}$ s; (c) $T = \frac{1}{2}$ s; and (d) $T = 1$ s.

Figure 1.3b–d gives a plot of the resultant sampled sequences generated for the three specified sampling periods. Although the same function $x(t)$ is being sampled, it is clear that the sampled sequence obtained depends very critically on the sampling period T. For example, all essential information can be lost by selecting T too large, as is evident in this case for $T = 1$ second.

Example 1.2

Determine the number sequence generated when the continuous-time function

$$x(t) = \begin{cases} 0 & \text{for } t < 0 \\ t + e^t & \text{for } t > 0 \end{cases}$$

is uniformly sampled with sampling period T.

In contrast to the approach taken in Example 1.1, we determine the resultant sampled signal using analytical means. Specifically, the sampled number $x(nT)$ is simply obtained by evaluating the function $x(t)$ at the time instant $t = nT$. For the function above, we then have

$$x(nT) = \begin{cases} 0 & \text{for } n = -1, -2, -3, \ldots \\ nT + e^{nT} & \text{for } n = 0, 1, 2, \ldots \end{cases}$$

and, as is our practice in the remainder of this text, we now drop the explicit appearance of T in the argument of $x(nT)$ to obtain

$$x(n) = \begin{cases} 0 & \text{for } n = -1, -2, -3, \ldots \\ nT + e^{nT} & \text{for } n = 0, 1, 2, \ldots \end{cases}$$

From this expression, it is apparent that the sequence generated depends strongly on the sampling period T.

1.3 INTRODUCTION TO SYSTEMS

Although the study of continuous- and discrete-time signals is important within its own right, we are primarily concerned with investigating procedures whereby a given signal x is changed (transformed) into another signal y in some systematic manner. This transformation procedure is represented by the mathematical notation

$$y = \boldsymbol{T}x \tag{1.3}$$

where $\boldsymbol{T}$ represents some well-defined rule by which the signal x is changed into the signal y. Relationship (1.3) defines a "system" characterization and is depicted as shown in Fig. 1.4. The arrows on the lines leading into and out of the box indicate the direction of signal flow.

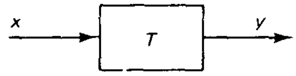

Figure 1.4 Block diagram of system representation.

In this representation, we interpret x as being the system's input signal (or excitation) and y as the system's corresponding output signal (or response). Thus, the excitation signal x is said to generate the response signal y through the characteristic rule $\boldsymbol{T}$. The rule $\boldsymbol{T}$ within the box completely defines the operational characteristic of the system. We are mainly concerned with those situations in which this rule takes the form of a linear differential equation or a linear difference equation. Typical examples of this system's viewpoint now follow.

1. The system is an automobile, the excitation is the accelerator pedal position, and the response is the automobile's velocity.
2. The system is the U.S. economy, the excitation is the prime interest rate, and the response is the inflation rate.
3. The system is an FM stereo receiver, the excitation is an RF signal (to which