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Vorwort

Der X. Internationale Astronautische Kongreß, eine Jubiläumsveranstaltung der International Astronautical Federation (I.A.F.), wurde in besonders eindrucksvoller Weise in der Woche zwischen dem 30. August und dem 5. September 1959 von der British Interplanetary Society in London durchgeführt, wo schon 1951 der zweite dieser Kongresse stattgefunden hatte. Die Vorträge von 1959 zeigen den fast unglaublichen Fortschritt dieser acht Jahre, den wohl kaum einer der Teilnehmer jenes historischen Kongresses erwartet hätte, der die Gründung der I.A.F. brachte. Die astronautischen Forscher und Techniker dürfen berechtigte Genugtuung über die vielfache Bestätigung eines großen Teiles ihrer damaligen Gedanken und Projekte empfinden.

Die beiden vorliegenden Berichtsbände enthalten die beim X. I.A.F.-Kongreß gehaltenen Vorträge, soweit deren Manuskripte von den Verfassern vor oder nach dem Kongreß zur Verfügung gestellt wurden. Von fast sämtlichen übrigen Vorträgen liegen wenigstens mehrsprachige Zusammenfassungen vor, außerdem Kurzfassungen von vier Arbeiten, die in den „Astronautica Acta" erschienen sind. Der British Interplanetary Society sei für die Mühe, möglichst alles Vortragsmaterial zu beschaffen, wärmstens gedankt.

Wie im Vorjahr wurden die einzelnen Beiträge zum Zweck schnellerer Veröffentlichung nicht in alphabetischer Reihenfolge nach den Namen der Verfasser, sondern je nach der Fertigstellung der Korrekturen und Abbildungen angeordnet. Dem ersten Band wurden wieder ein thematisch geordnetes Inhaltsverzeichnis und ein alphabetisches Verzeichnis aller Mitarbeiter vorangestellt.

Der Herausgeber spricht den Übersetzern der meisten Zusammenfassungen der einzelnen Artikel ins Französische beziehungsweise Englische, den Herren Professor Ing. Baudouin Fraeijs de Veubeke (Liège), Dr. Wolfgang B. Klemperer (Santa Monica, Calif.) und Mr. Frederick I. Ordway, III (Washington, D.C.) für ihre überaus wertvolle Hilfe den besten Dank aus. Auch dem Springer-Verlag gebührt besonderer Dank für die gewohnt gute Herstellung und technische Ausstattung der beiden Berichtsbände.

Friedrich Hecht, Wien

Determination of Air Density and the Earth's Gravitational Field from the Orbits of Artificial Satellites

By

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(With 9 Figures)

(Received July 9, 1959)

Abstract — Zusammenfassung — Résumé

Determination of Air Density and the Earth's Gravitational Field from the Orbits of Artificial Satellites. Methods for evaluating air density and scale height from the changes in the orbits of satellites are presented, and are used to determine air density at heights between 200 and 400 Km., and to trace the variation of density with time. The methods take account of the oblateness of the earth and atmosphere, the tumbling of the satellites, and the rotation of the atmosphere.

The evaluation of the successive terms in the earth's gravitational potential with the aid of satellites is discussed, and results so far obtained are outlined.

Bestimmung der atmosphärischen Dichte und des Erdschwerefeldes aus den Bahnen künstlicher Satelliten. Es werden Methoden zur Auswertung der Luftdichte und der Höhenstufen aus den Bahnänderungen von Satelliten angegeben. Sie werden benützt, um die Luftdichte in Höhen zwischen 200 und 400 km zu bestimmen und die zeitliche Dichtenänderung zu verfolgen. Die angewendeten Verfahren setzen die Abplattung der Erde und der Atmosphäre, die sich überschlagende Bewegung der Satelliten und die Rotation der Atmosphäre in Rechnung.

Die Auswertung der aufeinander folgenden Terme im Gravitationspotential der Erde mit Hilfe von Satelliten wird erörtert; die bis jetzt erzielten Ergebnisse werden skizziert.

Détermination de la masse volumique de l'air et du champ de gravitation par analyse des orbites de satellites artificiels. Des méthodes sont présentées pour évaluer la masse volumique de l'air à partir des altérations observées dans les orbites de satellites. Elles sont appliquées au calcul à des altitudes comprises entre 200 et 400 km. et permettent de retracer la variation de la densité avec le temps. L'aplatissement de la terre et de l'atmosphère et sa rotation ainsi que la rotation transversale du satellite sont pris en considération.

On discute l'évaluation des termes successifs du potentiel de gravitation à l'aide des satellites et les résultats obtenus jusqu'à présent sont esquissés.

I. Introduction

The main perturbations to earth-satellite orbits, those due to the atmosphere and the oblateness of the earth, are of quite different types, and, to a first approximation, can be treated separately. Because of this fortunate separation of the main effects, the properties of the atmosphere and the gravitational field can be

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analysed in a surprisingly detailed manner by observing how the orbit of an actual satellite departs from the fixed ellipse which represents the orbit over spherical earth in vacuo. The present paper reviews some of the work on these topics done during the past year at the Royal Aircraft Establishment, Farnborough.

The main effects of the earth's atmosphere and oblateness on satellite orbits of appreciable eccentricity may be summed up as follows. The atmospheric drag reduces the eccentricity and the length of the major axis, (and consequently the period of revolution). For a given satellite, the rate of change of all these quantities depends primarily on the air density near perigee, and the rates of change gradually increase as the perigee height slowly decreases. The main effects of the earth's oblateness are quite different: a rotation of the plane of the orbit, about the earth's axis; a rotation of the major axis of the orbit, in the orbital plane, but with no significant change in its length; and a slight distortion of the orbital ellipse, but with no significant change in eccentricity.

Though the main effects of the atmosphere and the non-spherical components of the earth's gravitational field are distinct, it proves necessary to take account of some secondary effects which do interact: for example, the side-force created by the rotation of the atmosphere slightly alters the rate of rotation of the orbital plane; and the third harmonic in the earth's gravitational field (which expresses asymmetry about the equator) causes a small oscillation in perigee distance, which has to be taken into account in studies of the air density.

There are many minor perturbations to the orbit, the most important being those due to the gravitational attractions of the moon and the sun. Most of the other perturbations, e.g. electromagnetic and relativity effects, have proved too small to be worth considering at present.

II. Determination of Average Air Density, at Heights between 200 and 400 Km.

The action of atmospheric drag makes the orbit of an earth satellite slowly contract, and its period of revolution T decrease. When the eccentricity of the orbit exceeds about 0.02, the drag at perigee is much greater than at apogee, and the rate of decrease of T is determined by the total loss in velocity due to the integrated effect of drag in the region near perigee. If the mass and dimensions of the satellite are known, and estimates are made of its aerodynamic drag, values of air density at heights near perigee can be obtained.

Such estimates of air density have been made in a number of previous papers [1—11]. Since the effective cross-sectional areas and drag coefficients of the satellites are difficult to determine exactly, the various authors have inevitably made different estimates of these quantities. Also most of the previous papers refer to only one or two satellites. In this section of the present paper, which is a revised version of [13], values of density are derived from all the ten satellites launched before the end of 1958 whose orbits are known, the satellites are treated consistently, and a more accurate method of determining density is used.

1. Previous Estimates

The results from the previous papers [1—11], and values [12] obtained from instruments aboard Sputnik 3 (1958 δ 2), are collected in Fig. 1, together with two proposed interim model atmospheres [6, 14] and the A.R.D.C. model atmosphere [15], which was probably the most widely-used standard atmosphere in the year prior to satellite launchings. As is well known, the results from satellites show the density at heights between 200 and 400 Km. to be 5—15 times greater than indicated by the A.R.D.C. model. In Fig. 1, the abscissa is the density rela-

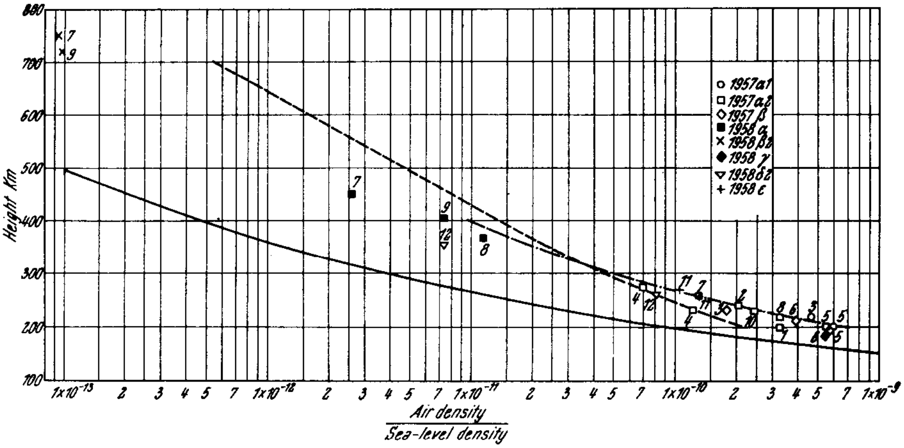

Fig. 1. Values of air density obtained by means of satellites [1—12], with proposed "standard atmospheres" [6, 14, and 15]. Reference numbers are shown beside the plotted points
- - - - - - N.R.L. atmosphere (1958) [14]
—·—·—·— Smithsonian atmosphere (1957—2) [6]
———— A.R.D.C. atmosphere (1956) [15]

tive to sea level, the sea-level density being taken as 1.23 Kg./m³ or 0.0765 lb./cu. ft. The scatter of the points in Fig. 1 is not significant, since it could be due to the different methods and assumptions of the different investigators.

2. The Present Method of Analysis

The drag D encountered by a satellite moving through air of density ϱ is expressed in the usual way in terms of a drag coefficient C_D, as

$$D = \frac{1}{2}\varrho V^2 S C_D, \qquad (1)$$

where S is a reference area, which is taken as the mean cross-section of the satellite perpendicular to the direction of motion, and V is the velocity of the satellite relative to the ambient air. Because of the rotation of the atmosphere, V differs slightly from the velocity v of the satellite relative to the earth's centre, and, if the atmosphere is assumed to rotate with the same angular velocity as the earth, it can be shown [16] that the drag is given with adequate accuracy by

$$D = \frac{1}{2}\varrho v^2 F S C_D, \qquad (2)$$

where

$$F = \left(1 - \frac{r_{po} w}{v_{po}} \cos i_o\right)^2, \qquad (3)$$

r_{po} = initial distance of perigee from earth's centre,
v_{po} = velocity of satellite relative to earth's centre at initial perigee,
w = angular velocity of earth,
i_o = initial orbital inclination.

For most of the satellites so far launched, the value of F lies between 0.90 and 0.95.

If a satellite of mass m is in an orbit with semi-major axis a and eccentricity e between 0.015 and 0.15, the air density at perigee, ϱ_p, can be expressed [5, 17] as

$$\varrho_p = -\frac{dT}{dt}\frac{1}{3\delta}\sqrt{\frac{2e}{\pi a H}}\left\{1-2e-\frac{H}{8ae}+0\left(e^2,\frac{H^2}{a^2e^2}\right)\right\}, \tag{4}$$

where dT/dt is the rate of change of the period of revolution T, which can be determined accurately from observations, and $\delta = F\,SC_D/m$. In deriving eq. (4) it is assumed that, above perigee height y_p, air density varies exponentially with height, the density ϱ at height y ($> y_p$) being given by

$$\varrho = \varrho_p \exp\left\{-\frac{y-y_p}{H}\right\}, \tag{5}$$

where H is constant. Since the value of H is not accurately known at heights of 200—400 Km., eq. (4), though it determines $\varrho_p\sqrt{H}$ with little error, does not yield an accurate value of ϱ_p.

This limitation can be avoided however. Let H^* be the best estimate of H. Then the density at height $y_p + 0.5\,H^*$ may, by means of (5) and (4), be expressed as

$$\varrho_{p+0.5H^*} = \varrho_p \exp\left(-\frac{H^*}{2H}\right) =$$

$$= -\frac{dT}{dt}\frac{1}{3\delta}\sqrt{\frac{2e}{\pi a H^*}}\left[\sqrt{\frac{H^*}{H}}\exp\left(-\frac{H^*}{2H}\right)\right]\left\{1-2e-\frac{H^*}{8ae}+0\left(e^2,\frac{H^2}{a^2e^2}\right)\right\}. \tag{6}$$

The function in square brackets is insensitive to the value of H^*/H, and may be taken as 0.593, with error less than 2.5 %, if the estimated value H^* does not differ from the true value H by a factor of more than 1.5. Since H is usually known to within a factor of 1.5, and since the likely error in m/SC_D is as a rule appreciably greater than 2.5 %, it is legitimate to replace the term in square brackets in (6) by this constant, so that (6) becomes

$$\varrho_{p+0.5H^*} = -\frac{0.158}{\delta}\frac{dT}{dt}\sqrt{\frac{e}{aH^*}}\left\{1-2e-\frac{H^*}{8ae}+0\left(e^2,\frac{H^2}{a^2e^2}\right)\right\}. \tag{7}$$

The error arising from neglect of terms in e^2 and H^2/a^2e^2 is less than 3.5 % if $0.015 < e < 0.15$. The use of eq. (7) to obtain the density at height $0.5\,H^*$ above perigee should not lead to errors of more than 4.5 % if H^* is not in error by a factor of more than 1.5. If H^* is in error by a factor of 2, the error in (7) rises to 12 %.

3. Evaluation of δ

In applying eq. (7) to actual satellites, the chief difficulty is the evaluation of $\delta = FSC_D/m$. The value of SC_D depends on the shape of the satellite, the manner in which it is rotating, and the mechanism of reflexion of the air molecules. Diffuse reflexion, in which the air molecules are re-emitted in a random manner after striking the surface (or, more strictly, according to the KNUDSEN cosine law), has been chosen as the most likely mechanism, in preference to specular reflexion, in which the molecules are reflected as if from a mirror. Though there is scope for argument about the speed of re-emission of the molecules in diffuse reflexion, the drag coefficients obtained under the various possible assumptions do not differ by more than about 5 %. It has been assumed here that the re-emission speed has a Maxwellian distribution about the speed appropriate to the satellite's temperature, and drag from electrical forces has been ignored.

The value of SC_D is most difficult to estimate for the near-cylindrical satellites, Explorers 1, 3 and 4 and Atlas (1958 α, γ, ε and ζ). It appears very probable, both on general dynamical principles and from observations [7], that each of these cylindrical satellites, and the rockets of the Russian satellites [10, 18], have rotated about their axis of maximum moment of inertia, i.e. an axis perpendicular to their length, though the angle between this axis of rotation and the direction of motion has varied. If this picture is correct, the two extreme modes of rotation are (a) travelling exactly like an aeroplane propeller, and (b) tumbling end-over-end: in (a) the axis of spin and the direction of motion are parallel; in (b) they are at right angles; and in practice the angle may be anywhere between these extremes. A recent study [19] has shown that, under the assumptions of diffuse reflexion, a rotating cylinder of length l and diameter d ($d \sim 0.1\,l$) has a value of SC_D of about 2.2 ld under regime (a), and about 1.5 ld under regime (b). For any motion between these extremes SC_D lies between 1.5 ld and 2.2 ld. The near-cylindrical satellites have been treated here as cylinders, with SC_D taken as 1.85 ld, the mean of the values under regimes (a) and (b). If each satellite has rotated about its axis of maximum of inertia, this value of SC_D will not be in error by more than 19 %.

The spherical satellites Sputnik 1 and Vanguard 1 (1957 α 2 and 1958 β 2) both have cylindrical antennae, and their drag coefficient C_D has been taken as 2.2, with S as the mean cross-section during one rotation. This value should not be in error by more than 5 %, since $C_D \simeq 2.1$ for a sphere and $C_D \simeq 2.2$ for a rotating cylinder. For the conical Sputnik 3 (1958 δ 2), S has been taken as the mean of the cross-sections in modes of rotation (a) and (b), with $C_D = 2.3$. For Sputnik 2 (1957 β), no weights and dimensions are available, but the value of δ can be inferred from Sputnik 1. Since the perigee heights of Sputniks 1 and 2 were virtually the same [10], eq. (4) gives

$$\delta_2 = \delta_1 \sqrt{\frac{a_1 e_2}{a_2 e_1}} \left(\frac{dT}{dt}\right)_2 \Big/ \left(\frac{dT}{dt}\right)_1 \tag{8}$$

where suffixes 1 and 2 refer to Sputniks 1 and 2 respectively. Since the factors on the right hand side of (8) are known, δ_2 can be evaluated. Values of δ for the rockets of Sputniks 1 and 3 (1957 α 1 and 1958 δ 1) can be found similarly. This indirect method should be reliable if, as is believed [20, 21], each of the satellites 1957 α 1, 1957 β and 1958 δ 1 retained a virtually constant value of δ until the last day of its life.

The values of m/SC_D and δ obtained by these various methods for the satellites with known orbits launched in 1957 and 1958 are listed in Table I. If the assump-

Table I. *Values of Mass m, m/SC_D and δ for Satellites 1957 α—1958 ζ*

Satellite		Mass m Kg.	m/SC_D Kg./sq. m.	δ sq. m./Kg.
Sputnik 1	1957 α 2	83	110	0.0088
Sputnik 1 rocket	1957 α 1	—	62	0.015
Sputnik 2	1957 β	—	58	0.016
Explorer 1	1958 α	14	23	0.039
Vanguard 1	1958 β 2	1.5	23	0.040
Explorer 3	1958 γ	14	23	0.039
Sputnik 3	1958 δ 2	1327	190	0.0049
Sputnik 3 rocket	1958 δ 1	—	59	0.016
Explorer 4	1958 ε	17.5	29	0.032
Atlas	1958 ζ	3960	28	0.032

tions already stated are justified, the error (standard deviation) in the tabulated values of m/SC_D and δ will probably be about 10%. It is interesting to note that the values of m/SC_D for Sputnik 1 rocket, Sputnik 2 and Sputnik 3 rocket are very similar: this tends to confirm the speculation that a similar rocket was used for all three.

4. Evaluation of Air Density

If values of δ are available, the air density at height 0.5 H^* above perigee can be found from eq. (7). For the Russian satellites, values of dT/dt, a and e have been taken from orbital determinations made in Britain [2, 22—25]. For the U.S. satellites, values have been obtained from the orbital data regularly issued as part of their prediction services by the Smithsonian Astrophysical Observatory, Cambridge, Mass., and Project Space Track, Bedford, Mass. The values of H^* chosen are consistent with values of H given later in this paper.

The resulting values of density, from all the satellites in Table I, are plotted in Fig. 2, and a curve has been drawn through the points to indicate likely average values for density. It is usual when constructing a 'model atmosphere' to make

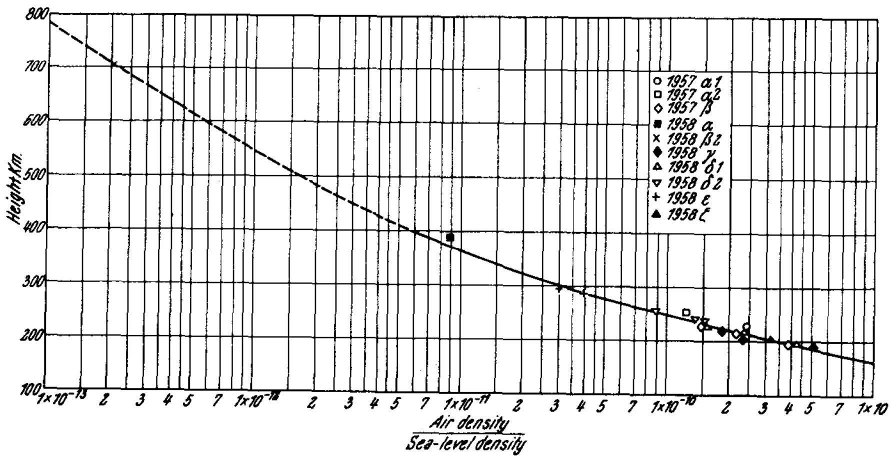

Fig. 2. Values of air density obtained from satellites 1957α—1958ζ, allowing for rotation of the atmosphere

a plausible assumption about the variation of temperature and air composition with height, and thence derive density. Since the primary measurements are of density, however, it seems more logical to take the density as the basic parameter and then to deduce temperature from it. This procedure has the further merit that it shows the large errors in temperature which can arise, even when a good fit is achieved for the density.

Several features of Fig. 2 are worth noting.

(1) The 10 different satellites give surprisingly consistent values of air density. None of the 15 plotted points in the cluster between 190 and 260 Km. differs from the curve by a factor greater than 1.4, and the average factor of difference is about 1.2. Probably, therefore, the assumptions made were not grossly in error, as they would have been if, for example, one of the cylindrical satellites had flown like an arrow instead of rotating.

(2) Even if the method of analysis were perfectly accurate, the points in Fig. 2 would not all lie on the same curve, since the effective air density is known to

vary from day to day and month to month, by up to 30 % at 200—250 Km. height [20, 21] and by a factor of 3 at 700 Km. [26]. The curve in Fig. 2 can therefore only represent average values.

(3) The points in Fig. 2 cover latitudes between 50° N and 35° S; but there is no sign of any regular variation with latitude, and the spread of the points suggests that, between 50° N and 35° S, density does not depart from its average value by a factor of more than about 1.5. This is in contrast with results from rocket experiments [27], which suggest a variation by a factor of up to 5 between 59° N and 35° N on a summer day. The accuracy of initial lifetime-estimates for the Russian satellites, whose perigees moved from an initial latitude near 50° N towards the equator, also strongly suggests that density does not vary greatly within this range of latitude.

(4) The result from Vanguard 1 in Fig. 2, at a height of 700 Km., should be treated with caution, partly because it is unsupported, and partly because the effect of charged drag [28] is likely to be important at this height, where over 5 % of the atoms may be ionized.

Table II lists values of average density from the curve in Fig. 2, for heights between 200 and 400 Km., where the results are most reliable.

Table II. *Average Air Density at Heights between 200 and 400 Km.*

Height Km.	Air Density / Sea-Level Density	Density (gm/c. c.)
200	3.5×10^{-10}	4.3×10^{-13}
220	2.1×10^{-10}	2.5×10^{-13}
240	1.2×10^{-10}	1.5×10^{-13}
260	7.6×10^{-11}	9.3×10^{-14}
280	4.7×10^{-11}	5.8×10^{-14}
300	3.1×10^{-11}	3.8×10^{-14}
320	2.1×10^{-11}	2.6×10^{-14}
340	1.5×10^{-11}	1.8×10^{-14}
360	1.1×10^{-11}	1.3×10^{-14}
380	7.8×10^{-12}	9.6×10^{-15}
400	5.8×10^{-12}	7.1×10^{-15}

III. Variations in Air Density

1. Method

During 1958 the Royal Aircraft Establishment, Farnborough, provided a prediction service for satellites which passed over or near Britain, and simple methods of predicting the times of transit and the geometry of the orbit were developed [29], which relied on a judicious mixture of observation and theory, and required only a desk calculating machine for computation. Sputniks 2 and 3 and the rocket of Sputnik 3 were the satellites of most interest in Britain, since none of the early U.S. satellites (except Explorer 4) reached our latitudes. The prediction service relied almost entirely on simple visual observations relative to the stars, sent in by volunteer observers. Most of these observations were accurate to 1° in direction and 2 seconds in time, and they were used to estimate the time at which the satellites passed through apex, the point of maximum latitude north. The r.m.s. error in determining apex time was usually not more than 3 seconds. From two apex times approximately a day apart, the mean nodal period of revolution T in the interval could be determined with r.m.s. error usually not more than 0.3 sec., or 1 part in 20,000. From this series of values of T, a series of 'observed values' of dT/dt can be obtained with adequate accuracy.

If the air density at a given height and the effective cross-sectional area S of the satellite both remained constant from day to day, the daily change in period would increase smoothly as the satellite's orbit slowly sank lower into the atmosphere: all the terms in eq. (4) would either change slowly and smoothly or remain constant. In reality, for every satellite so far launched, the rate of decrease of T has been irregular, thus implying irregularities in drag, which could be caused by changes in either the effective atmospheric density near perigee or the effective cross-section S. Most of the satellites so far launched have probably rotated about their axis of maximum moment of inertia, and, if so, the mean cross-section during a complete rotation should remain constant as long as the mode of rotation of the satellite is unchanged, though it would not be constant if, for example, tumbling end-over-end changed to spinning like an aeroplane propeller. A clue to the mode of rotation is provided by the fluctuation in brightness of a satellite: if this remains almost the same for many months it is unlikely that the mode of rotation has changed appreciably.

For the rocket of Sputnik 3 (1958 δ 1), the rate of decrease of period was particularly erratic, but the brightness fluctuated regularly with a period which increased slowly from 8.5 seconds in July to 9.5 seconds in November. This strongly suggests that the mean cross-section did not vary significantly and that irregularities in the rate of decrease of period can be ascribed to variations in atmospheric density. Similar conclusions apply [20] to Sputnik 2 (1957β). For both these satellites the perigee height decreased from 226 Km. initially to about 180 Km. ten days before the end of the satellite's life, and any conclusions about air density relate to this height band.

2. Results

Over 1,000 observations of Sputnik 2 and the rocket of Sputnik 3 have been analysed in the manner described in section III.1, to obtain the rate of change of

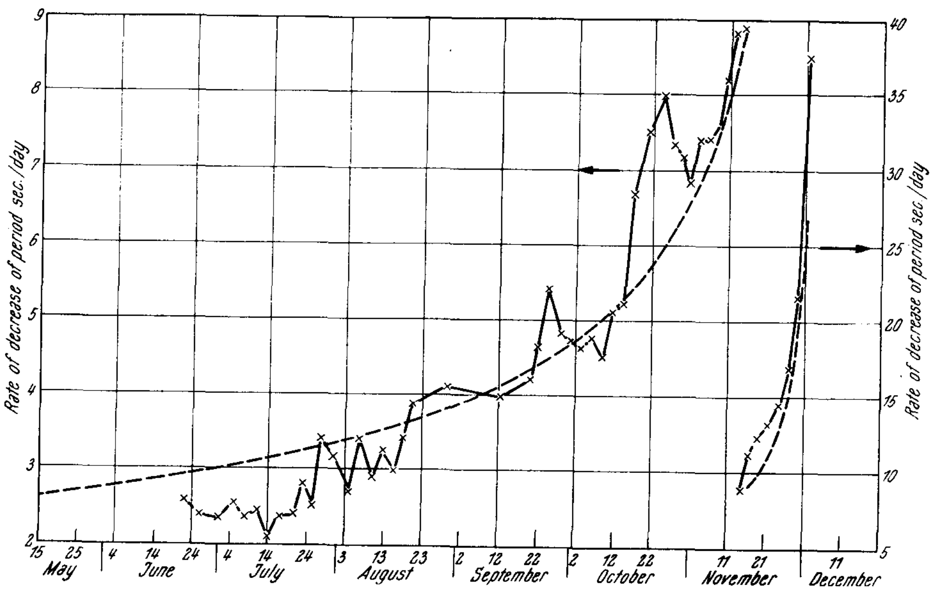

Fig. 3. Rate of change of period for Sputnik 3 rocket (1958 δ 1)
------ Theoretical curve
—x—x— Observed values

nodal period at intervals of 3 or 4 days. The results for Sputnik 3 rocket are shown in Fig. 3, in which the great majority of the observational values plotted are believed to be in error by less than 0.1 sec./day. For comparison, a theoretical curve, calculated on the assumption that density at a given height is constant, is also shown in Fig. 3. This curve is derived from the simplest theory, which gives [20]

$$\frac{dT}{dt} = -\frac{3\,e_0\,T_0}{4\,t_L\sqrt{1 - t/t_L}}\,,$$

where t is time after launch, t_L is the total lifetime, e_0 the initial eccentricity, and T_0 the initial period. In assessing irregularities in dT/dt, no advantage is gained by using a subtler theory.

Two main features stand out in Fig. 3, and even more in Fig. 4, where the observed values have been divided by the theoretical to give a curve which represents, in effect, the ratio $\frac{\text{air density}}{\text{average air density}}$. First, there is the general impression of irregularity. The value of density on any particular day is a poor guide to the likely value 3 days later: for instance, on 17th October the density

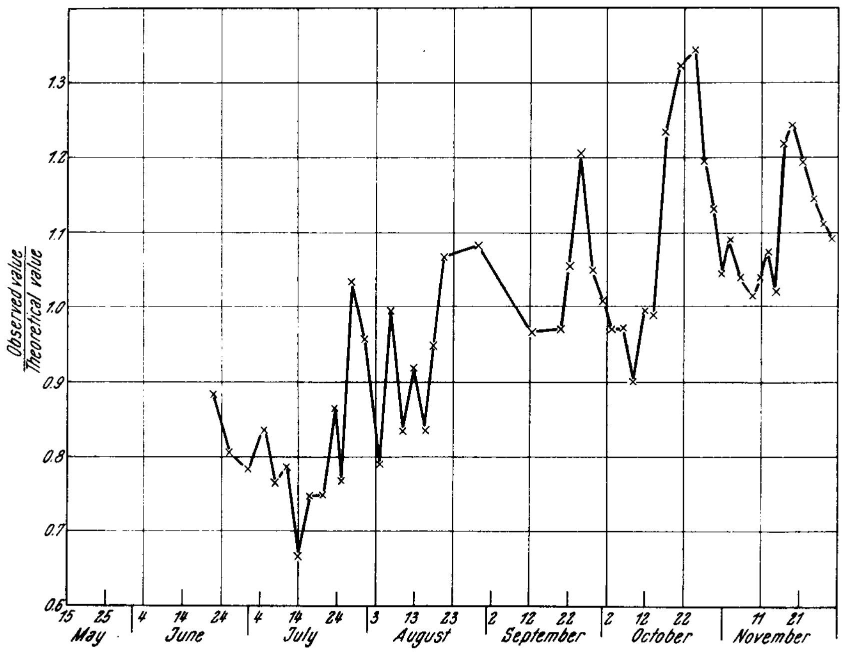

Fig. 4. Rate of change of period of Sputnik 3 rocket (1958 δ 1). Ratio of observed values to theoretical

was nearly 25 % higher than on the 14th. More detailed analysis shows that this irregularity occurs over even shorter time-intervals: on successive days during November, 1958, density differed from the average by +2 %, +11 %, —1 % and +18 %.

The second main feature of Fig. 4 is more interesting and more fruitful: the maximum values of density, and also the minimum values, show a strong tendency to recur at intervals of about 28 days.

Four possible causes of this 28-day periodicity in air density are worth considering. First, is it caused [10, 30] by the movement of the perigee point from daylight into darkness? The answer seems to be 'no', because perigee took about 3 months to perform a complete cycle of movement from light to darkness and back again. The change in air density from day to night therefore seems to have only a minor effect.

A second possible cause of the 28-day oscillation is variation of density with latitude. Between mid-June and mid-November, 1958, the perigee of Sputnik 3 rocket moved from latitude 40° N to latitude 10° S, changing by roughly 10° every 28 days. The 28-day periodicity in dT/dt might therefore seem to indicate a periodic variation of density with latitude, with maxima at intervals of 10°: is this possible? Again the answer is 'no', for two main reasons. First, the effect of drag is spread over a range of latitudes near perigee: if the density at a given altitude were high at perigee latitude and low at latitudes 5° on either side, the effect would be much the same as low density at perigee latitude and high density

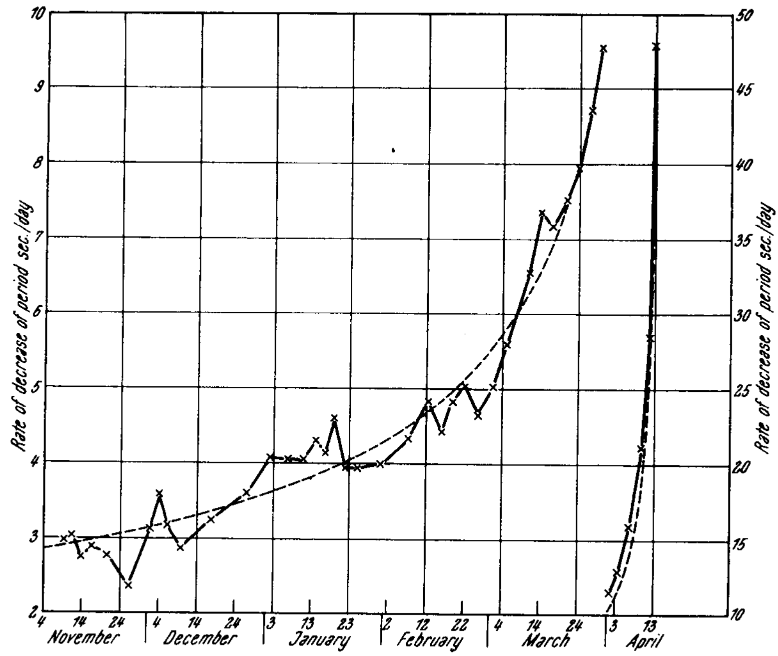

Fig. 5. Rate of change of period for Sputnik 2 (1957 β)

– – – – – – Theoretical curve
—x—x— Observed values

at 5° on either side. Second, other satellites, for which perigee latitude changes at quite different rates, show the 28-day oscillation.

A third possible cause of the oscillations is lunar tides in the atmosphere: any 'tidal bulge' caused by the moon should travel round the earth once every 28 days. But, in 28 days, the right ascension of the perigee of Sputnik 3 rocket moved 90° westwards, chiefly because of the westward rotation of the orbital plane caused by the earth's oblateness. Consequently the period of revolution of the moon, relative to perigee, was only about 22 days. Maximum values of density would therefore be expected to occur at intervals of 22 days or, more probably, 11 days. So lunar tides, though they exist, do not seem to have an important effect on the density.

The fourth and most likely cause of the oscillations is solar disturbances. It has long been known that streams of charged particles shoot out radially from the sun, rather like a jet of water from a revolving sprinkler, and since the sun rotates about its axis, relative to the earth, once every 27 or 28 days[1], the earth tends to pass through these streams at intervals of about 27 days. The impact of these streams of particles on the upper atmosphere gives rise to well-known 27-day periodicities in geomagnetic activity, cosmic rays and the aurora. The evidence from Sputnik 3 rocket suggests that air density at heights between 180 and 220 Km. exhibits similar periodicity.

Do the results from other satellites confirm this suggestion? For Sputnik 1 and its rocket (1957 α 2 and 1) the data available to us are not precise enough for any conclusions to be drawn. For Sputnik 3 (1958 δ 2), increases in drag occurred about 80, 110 and 140 days after launch, but detailed results have not yet been published. For the United States satellite Explorer 1 (1958 α), and for Explorer 3

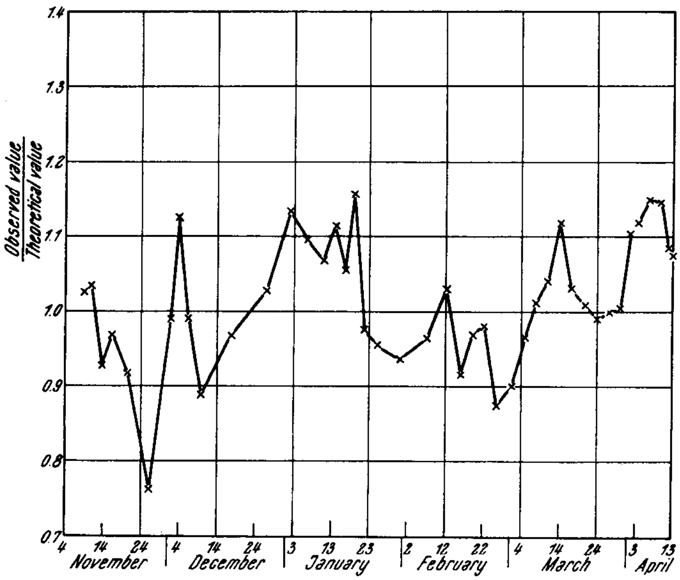

Fig. 6. Rate of change of period of Sputnik 2 (1957 β). Ratio of observed values to theoretical

(1958 γ) except during its last weeks, perigee crossed the equator at intervals of between 24 and 30 days; the oscillations in dT/dt produced by the periodic change in perigee latitude therefore had a period between 24 and 30 days, and, in the present state of knowledge, cannot be reliably distinguished from solar influences of similar period. Our information on Explorer 4 (1958 ε) is limited, but its drag, like that of Sputnik 3 rocket (Fig. 4), increased sharply about 22 August, 1958 and decreased about 8 September. Atlas (1958 ζ) had a lifetime of only one month, too short to yield any conclusive result. For Vanguard 2 (1959 α) and Discoverer 2 (1959 γ) no results are yet available. That leaves Sputnik 2 (1957 β) and Vanguard 1 (1958 β2).

The observations of Sputnik 2 have been analysed in the same way as those of Sputnik 3 rocket to give the results plotted in Figs. 5 and 6. A 28-day oscillation is again discernible, especially during 1958, though it is not so obvious as with Sputnik 3 rocket. The variations have been found by NONWEILER [31] to corre-

[1] It is 27 days at the sun's equator, 28 days at latitude 25°.

spond fairly well with solar flares, and by PRIESTER [32] to show excellent correlation with the sun's 20 cm. radiation.

Results for Vanguard 1, as given by JACCHIA [26], show that this satellite exhibited a similar oscillation to Sputnik 3 rocket, with maxima of density occurring at the same times. The amplitude of the oscillations was however much larger for Vanguard; this is to be expected, since the air density is much lower at its perigee height of 650 Km., and changes are therefore likely to be relatively greater. JACCHIA [33] has further compared the variations in dT/dt for Vanguard 1 with the 10.7 cm. solar radiation, and has found the correspondence to be 'little short of perfect'.

Thus it appears probable that the air density in the upper atmosphere, at heights between 200 and 700 Km., and over a wide range of latitudes, is strongly

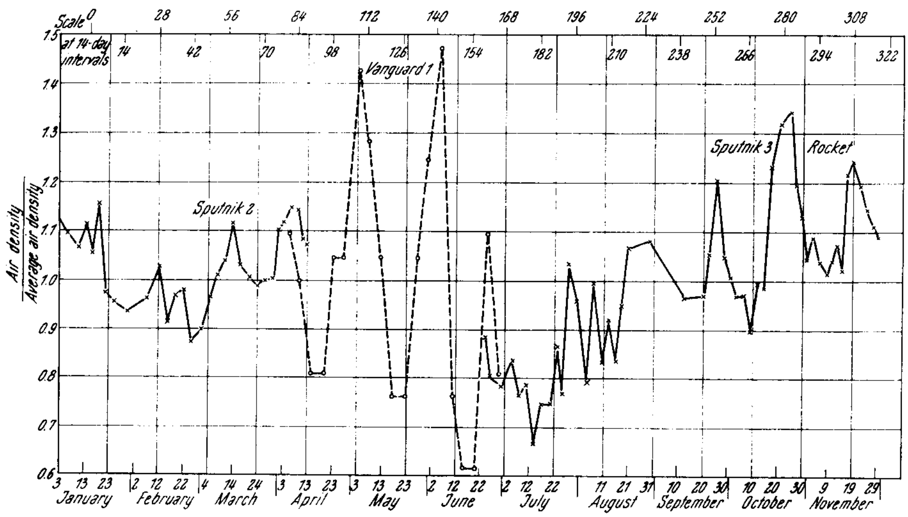

Fig. 7. Variations in air density, as given by the rate of change of orbital period of satellites during 1958: a synthesis of values from Sputnik 2, Vanguard 1 and Sputnik 3 rocket

under solar influence, and exhibits the 27/28-day periodicity which is characteristic of solar effects. Superposed upon the 28-day oscillation, there is an appreciable day-to-day irregularity, which makes exact prediction impossible for satellites which are appreciably affected by air drag.

The persistence of the 28-day oscillation during 1958 is strikingly shown in Fig. 7, in which Figs. 4 and 6 are linked with the results [26] from Vanguard 1: the Vanguard oscillations, though of larger amplitude, fit almost exactly into the pattern, and the 14-day scale at the top of the figure shows how regular the oscillations are.

IV. Scale Height and Air Temperature, at Heights between 200 and 400 Km.

The air temperature in the upper atmosphere may be expressed in terms of the scale height, which is a measure of the rate of change of density with height, and the mean molecular weight. Unfortunately, neither of these quantities is as yet accurately known at heights between 200 and 400 Km., and the values derived are subject to considerable error.

The scale height at any altitude may be defined as the air pressure divided by the rate of decrease of pressure with height. If pressure varies exponentially with altitude, the scale height is equal to the increase in altitude corresponding to a decrease in pressure by a factor e ($=2.718\ldots$). The scale height is closely related to the coefficient H of section II, which is the increase in altitude corresponding to a decrease in density by a factor e. The scale height is given by $H/(1-H')$, where H' is the rate at which H changes with height and is usually small. The air temperature for heights between 200 and 400 Km. may be expressed approximately as

$$T = \frac{1.1\, MH}{1-H'}\ {}^{\circ}\mathrm{K}, \tag{9}$$

if H is in Km.

The value of H should in theory be obtainable from the slope of the curve in Fig. 2; in practice, the accuracy is poor. The values derived in this way rise from $H=37$ Km. at 200 Km. height to $H=71$ Km. at 400 Km. height. But it is possible to link the points in Fig. 2 with other curves, of different slopes. For example, a straight line between 200 and 400 Km. height, with $H=50$ Km., fits the points almost as well.

Of the other methods of finding H, the most direct is to utilize the theoretical equation connecting the eccentricity e with the distance r_p of perigee from the earth's centre. The simplest form of this equation, derived on the assumption of spherically symmetrical earth and atmosphere, is [20]

$$H = \frac{r_{po} - r_p}{\frac{1}{2}\ln\frac{e_o}{e} + 0(e) + 0\left(\frac{H}{ae}\right)}, \tag{10}$$

where zero suffix denotes initial values, and e_0 and e lie between 0.02 and 0.2. Eq. (10) is useful as a first approximation, but a more accurate form is needed which gives the terms in $0(e)$ and $0(H/ae)$, and takes into account the effect of the third harmonic in the earth's gravitational field and the oblateness of the atmosphere. If i is the orbital inclination, ω the argument of perigee, and R the earth's equatorial radius, and ae is written as x, the more accurate form of eq. (10) is found [16] to be

$$H = \frac{r_{po} - r_p + 4.3\sin i\,(\sin\omega_o - \sin\omega)}{\frac{1}{2}\left(1-\frac{3H}{a_o}\right)\ln\frac{x_o}{x} + \ln\frac{x}{x_o}\left(\frac{8x_o - 3H}{8x - 3H}\right) - \frac{x_o - x}{a_o} + \varepsilon R\int_{x_0}^{x}\frac{\cos 2\omega\sin^2 i}{x^2}\,dx}, \tag{11}$$

if distances are measured in Km., $0.02 < e < 0.2$, and the coefficient J_3 of the third harmonic in the earth's gravitational potential (see section II) is taken as -2.2×10^{-6}. The value of ε, which represents the oblateness of the atmosphere, may with adequate accuracy be taken equal to the earth's flattening, 1/298. The last term in the numerator in eq. (11) represents the effect of J_3, and is most important initially when $r_{po}-r_p$ is very small. The last term in the denominator, which depends on the oblateness of the atmosphere, is usually negligible until e falls below 0.04.

Eq. (11) has been applied to all the satellites so far launched for which reasonably good orbital information is available, but the resulting values of H are disappointingly scattered, and it is evident that the observational results are not yet of an accuracy adequate to match the theory. Consequently, it is not worth giving more than a summary of the results, which suggest that H rises from about 45 Km. at 180 Km. height to about 80 Km. at 250 Km. height, with an error factor which might be as large as 1.5. Since Fig. 2 shows that the average value of H between 200 and 400 Km. height is about 50 Km., these figures, if true, would suggest that H, and hence the air temperature, was higher between 200 and 300 Km. height than between 300 and 400 Km., the figures do not, however, appear to be reliable enough to justify this conclusion, which would be in conflict with almost all the 'model atmospheres' proposed for this heightband.

It is probably better to accept the evidence of Fig. 2, which indicates that the average value of H between 200 and 400 Km. height is about 50 Km., and that H tends to increase with height. If a value had to be chosen, H might be assumed to rise from 45 Km. at 200 Km. height to 60 Km. at 400 Km. height.

There is also some difficulty in determining the molecular weight of the air. The oxygen is largely in atomic form above 200 Km. height, but the height-range within which nitrogen changes from molecular to atomic form is still disputed. Above about 300 Km., however, the main constituents of the air are believed to be atomic oxygen and atomic nitrogen, in proportions as yet unknown, [12, 34—36], and the molecular weight may be taken as 15. With the values of H already suggested, the average temperature between 200 and 400 Km. height, as given by (9), would be a little over 1000° K. It should be emphasized however that any values of temperature are far less reliable than those of density.

V. Winds in the Upper Atmosphere

Analysis of kinetheodolite observations of Sputnik 2 [24] showed that the inclination of the orbit to the equator changed from 65.32° initially to about 65.19° at the end of its lifetime. The decrease became much more rapid towards the end of the satellite's life, thus suggesting that the change was due to the atmosphere and increased greatly as perigee came nearer to the earth's surface.

An obvious cause of this change is rotation of the atmosphere, which creates a lateral force on the satellite. This force will tend to reduce the orbital inclination, for a satellite which goes from west to east, and its magnitude will depend on the mean wind speed v_w near perigee height and perigee latitude.

If the angular velocity of the atmosphere in the region near perigee differs from the angular velocity of the earth by a constant factor A, and the orbital eccentricity does not exceed 0.2, the change in the inclination i is found [37] to be

$$\Delta i = \frac{A \sin i}{6} \frac{(2\, I_2 - 4\, e\, I_1) \cos^2 \omega + I_0 - I_2 + 0\,(e^2)}{(1 - T \cos i)\, I_0 + 2\, e\, I_1} \Delta T\,, \tag{12}$$

where T = period of revolution of satellite, expressed as a fraction of a sidereal day, ω = argument of perigee, and the I_n are BESSEL functions of the first kind and imaginary argument, of order n, of argument ae/H. In eq. (12) the change in i is directly related to the change in T, and irregularities in air density do not affect the analysis.

If the BESSEL functions are replaced by their asymptotic expansions, eq. (12) becomes

$$\Delta i = \frac{A \sin i}{3} \frac{\left(1 - \frac{15}{8} k - 2e\right) \cos^2 \omega + k}{(1 - T \cos i)\left(1 + \frac{k}{8}\right) + 2e} \{1 + 0\,(k^2, e^2)\} \Delta T\,, \tag{13}$$

where $k = H/ae$. This form of expansion is most useful, since e and k are of the same order if $0.05 < e < 0.15$. If the terms in k, e and T are ignored, the simpler form derived by BOSANQUET [38] is obtained. Eq. (13) shows that the change in i depends mainly on four factors: