FILTER THEORY AND DESIGN: ACTIVE AND PASSIVE

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PREFACE

The purpose of this book is to present a unified and combined treatment of the two phases of filter design; namely, Approximation and Realization. It is written both for the student and the design engineer. Although a prior course in network analysis and synthesis, of the classical type, would be beneficial it is not essential.

Filter design is one of these rare areas that require a deep understanding of topics that range from the highly theoretical to the extremely practical. Although there are texts on various aspects of the filter design problem such as approximation theory, network synthesis, active networks etc. we felt that the need existed for a book that "tells the complete story". Our objective therefore was to take the reader through the entire process starting from the Specification Phase, where practical system design considerations play a major role, through the Approximation Phase, which is almost entirely a mathematical exercise, and finally to the Realization Phase, which relies both on highly developed theories of network synthesis as well as on practical knowledge of "real life" components and their imperfections.

In addition to the obvious usefulness of this approach to the design engineer we have found it of equal benefit to the student. At present the book serves as a text for a sequence of two one-term courses at the University of Toronto. The first course is offered to both senior-year and graduate students while the second course is taken primarily by graduate students. Graduate students in these courses come from a variety of research areas such as circuit theory, communications, biomedical engineering, etc., as well as engineers from industry.

In order to achieve our goal, as stated in the above, and still keep the book a reasonable size, a number of decisions were made. The treatment of the approximation problem is quite general, however attention is focussed on the approximation for filters realized using passive LC and active RC technologies. A short section outlines the application of approximation methods to discrete-time filters. Also, although the formulation is general, the approximation problem is solved only for filters which are specified to meet attenuation requirements.

Concerning the realization technologies, it was decided to concentrate on Active-RC, especially those circuits that use the Integrated Circuit (IC) Operational Amplifier (Op Amp). However, a concise and practical treatment of LC design methods was deemed beneficial since many LC filters continue to be designed and manufactured every day. Also, two of the major methods for active filter design start from a passive LC prototype.

In the active filter area, we consider in detail the design of high-order (greater than two) filters. However, since biquadratic sections are useful in their own right as well as building blocks in higher order filter structures, a thorough treatment of biquads is presented. In all cases, only circuits that have proven practical are considered. This eliminated all methods that do not use op amps.

Throughout the text we make frequent reference to the FILTOR 2 package of filter design programs. This is a set of over 100 Fortran IV routines that have been developed, at the University of Toronto, based on the material presented here, and will shortly be made generally available. Although the book can be used for teaching without the programs, we have found it highly beneficial to assign problems and design projects that require the use of the programs. The programs are described in a separate manual which will become available in the near future.

We now present an overview of the material contained in this book. Chapter 1 introduces the reader to the filter design problem. The problem is defined and background information is presented. This chapter also serves to establish terminology and notation. Chapter 2 is an introduction to the approximation part of the book. Chapters 3 to 5 are concerned with the solution of the approximation problem: Chapter 3 deals with filters having a maximally-flat passband attenuation, while Chapter 4 deals with the equiripple passband case. Chapter 5 completes the approximation phase for attenuation specifications by presenting an iterative stopband approximation method.

Chapter 6 presents various frequency and network transformations. The material presented enables the design of classical filters using closed-form expressions and/or filter tables. This chapter concludes with an important section dealing with the bilinear z-transform. There we show how the material on approximation may be directly applied to the design of discrete-time filters of the recursive type. These include both digital filters and filters realized using Charge Transfer Devices (CTD's).

The design of passive LC ladder filters, as well as a study of their sensitivity properties, is the subject of Chapter 7.

In Chapter 8 we present the basic concepts and necessary background knowledge for the design of active filters. This chapter also serves as a concise overview of the active filter part. Chapter 9 deals with the theory and design of second-order active filters, or biquads. In an introductory filters course, Chapters 8 and 9 contain sufficient information on the active filters area.

Each of the three remaining chapters deals with a particular method for the design of high-order active filters. Chapter 10 is concerned with the cascade method of design. A particular coupled-biquad structure is also presented. Design methods based on the simulation of LC ladders are studied in Chapters 11 and 12. Methods which essentially replace the ladder inductors by simulated inductances are considered in Chapter 11. The other approach to ladder simulation, namely, simulating the internal working or the operation of the ladder network is the subject of Chapter 12.

There are many ways for teaching from this book or for using it for self study. In fact we have attempted to present the material in a manner that allows the reader to study the part that interests him without having to read the entire book. For instance, most of the material on approximation could be skipped by a reader specifically interested in realization. Also, it is not essential to study the details of passive design in order to study active filters.

A specific package which we have found quite successful for a one-term (40 lecture hours) course at the senior/first year graduate level is as follows: Chapters 1, 2, 3, 4 (excluding Sections 4.3 and 4.4), 5 (a light treatment), 6 (excluding Sections 6.7 and 6.8), 8 and 9. The students in this course are assigned problems selected from those given in the book in addition to few design problems requiring the use of the programs. A follow-up course intended for graduate students covers the rest of the book and is highly design oriented. Here again extensive use is made of the programs in design projects.

We would like to express our sincere thanks and gratitude to a number of people who contributed in a variety of ways to the development of this work. Professor K. C. Smith provided encouragement throughout the course of this undertaking. Martin Snelgrove has been the principal contributor to the development of the filter design programs. He has been assisted in this task by Moez Dharssi. Ken Martin's research strongly influenced the material of the last two chapters. Les Brown's work provided some of the material on two-amplifier biquads in Chapter 9. Needless to say, the research of almost all of the graduate students who worked with the first author during the last few years helped in the development of the material on active filters. The second author is grateful to the management of ESE Ltd., Rexdale, Ont. for providing him with the opportunity to work on filter design during his tenure as chief development engineer. Elisa Krissilas, Maggie Byrne and Sarah Cherian participated in typing the various drafts. The final copy was most ably typed by Joan Lidkie who deserves a special note of thanks. Our research in the filter design area has been supported by the National Research Council of Canada through research grants (A.S.) and fellowships (P.B.). Last, but not least, special thanks go to our families for their patience and understanding.

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CHAPTER 1

INTRODUCTION

This book is mainly concerned with the design of a certain class of linear systems which are usually referred to as Electric Wave Filters. In the broadest sense, a filter may be considered to be a signal processing device which operates on an input signal to produce an output signal bearing a prescribed relationship to the input signal. Design of a linear system to satisfy such an objective is not, generally, an easy task. This book is devoted to the design of a special class of such devices, for which the input-output relationship may be described in terms of the ability of the system to "pass" a certain class of signals having a specified spectrum, and to "stop" a certain class of signals having a different (and usually disjoint) spectrum. Such filters are frequently called "frequency selective networks". We shall make these ideas more precise in the following sections.

Few proofs are included in this introductory chapter, and the interested reader is referred to the selected references listed at the end of the chapter.

1.1 LINEAR SYSTEMS

It is assumed that the reader has a background in network analysis and a working knowledge of elementary linear system theory. The following material is presented as a brief review, and for the purpose of establishing notation. Fig. 1.1 illustrates the "black box" concept of a linear system. The system is characterized in the time domain by its impulse response g(t), which is the output signal y(t) produced in response to a unit impulse $\delta(t)$, applied at the input. For an arbitrary input signal u(t), the output signal y(t) is given by the well known convolution integral

$$y(t) = \int_{-\infty}^{\infty} g(t - \tau)\, u(\tau)\, d\tau \; . \qquad (1.1)$$

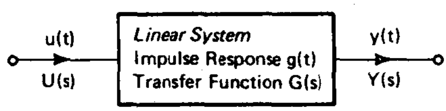

Figure 1.1 Black box representation of a linear system.

In the usual manner, we denote the Laplace transforms of the time signals by the use of upper case letters. Thus we have the system transfer function G(s), which is the transform of g(t), defined by

$$G(s) = \int_{0}^{\infty} g(t)\, e^{-st}\, dt,$$

with the inverse transform given by

$$g(t) = \frac{1}{2\pi j} \int_{-\infty}^{\infty} G(s)\, e^{st}\, ds$$

where $s = \sigma + j\omega$ is the complex frequency variable. Under the Laplace transformation (1.1) becomes simply

$$Y(s) = G(s)\, U(s)$$

so that the transfer function G(s) is the ratio of the output variable to the input variable,

$$G(s) = \frac{Y(s)}{U(s)} = \frac{\text{output variable}}{\text{input variable}} \tag{1.2}$$

The most general types of linear systems we shall consider are those consisting of a finite number of lumped, linear and time-invariant elements. The system is characterized by an Nth order ordinary linear differential equation which results, in the most general case, in a transfer function G(s) which is a real rational polynomial function of the complex variable s. Thus we may write G(s) in general as

$$G(s) = \frac{P(s)}{E(s)}$$

$$= \frac{p_M s^M + p_{M-1} s^{M-1} + \ldots + p_o}{e_N s^N + e_{N-1} s^{N-1} + \ldots + e_o} = \frac{\sum_{k=o}^{M} p_k s^k}{\sum_{k=o}^{N} e_k s^k} \tag{1.3}$$

where p_k and e_k are real numbers so that G(s) is real for real s, and the roots of the polynomials P(s) and E(s) must either be real or occur in conjugate pairs.

In general, for the kinds of systems we shall consider, the degree of the numerator (deg. $[P(s)] = M$) is less than or equal to the degree of the denominator (deg. $[E(s)] = N$), and E(s) is a strictly Hurwitz polynomial (all roots in the open left-half s-plane)†. The denominator polynomial E(s) is variously known as

†A "closed" half of the s-plane includes the jω-axis while an "open" half excludes the jω-axis.