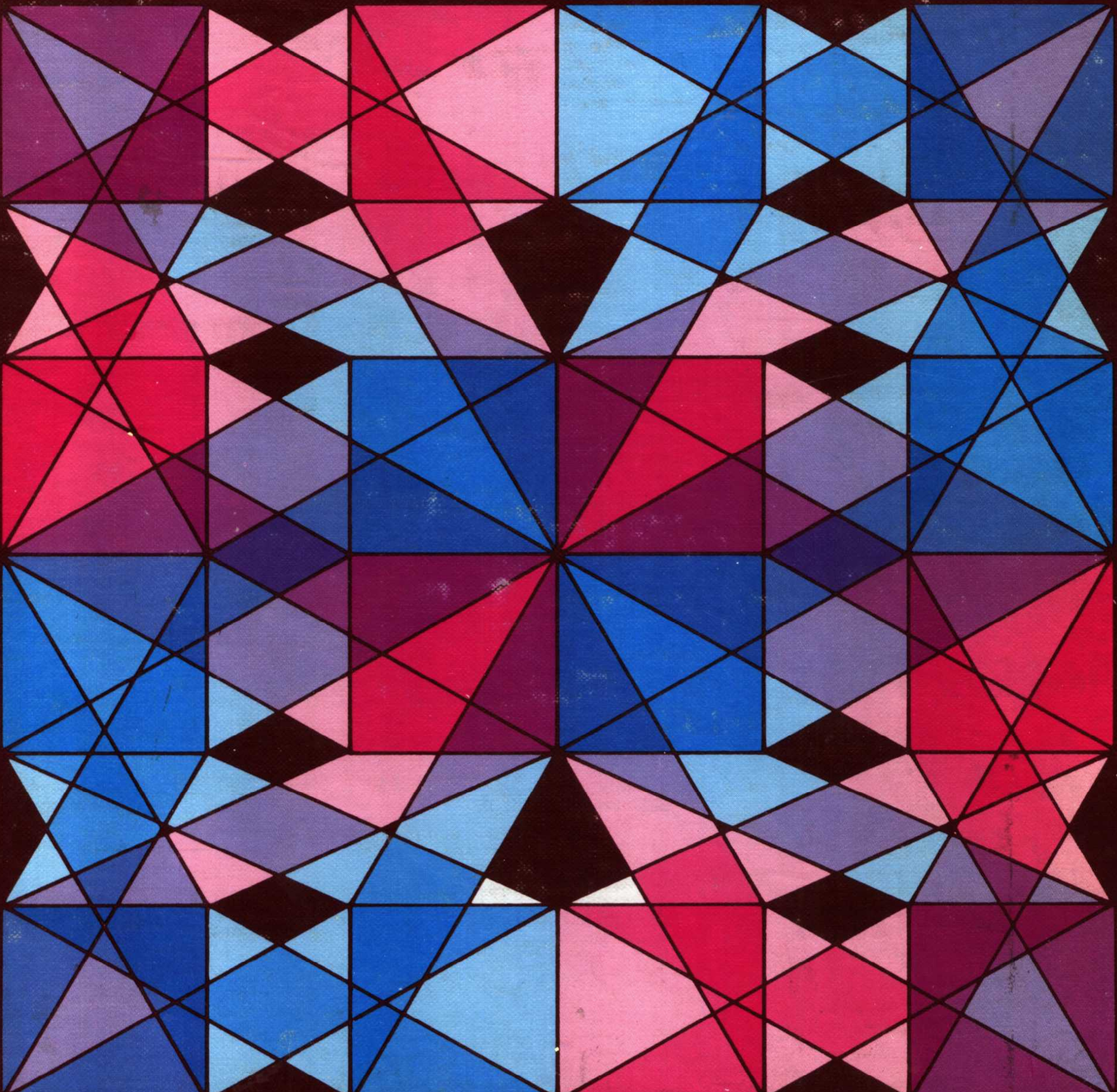
Engineering Mechanics 2nd edition

DYNAMICS

R.C. Hibbeler



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Preface

The purpose of this book is to provide the student with a clear and thorough presentation of the theory and application of the principles of engineering mechanics. Emphasis is placed on developing the student's ability to analyze problems—a most important skill for any engineer. Furthermore, the Système International or SI system of units is used for numerical work since this system is intended in time to become the worldwide standard for measurement.

The contents of each chapter are organized into well-defined sections. Selected groups of sections contain the development and explanation of specific topics, illustrative example problems, and a set of problems designed to test the student's ability to apply the theory. Many of the problems depict realistic situations encountered in engineering practice. It is hoped that this realism will both stimulate the student's interest in engineering mechanics and provide a means for developing the skill to reduce any such problem from its physical description to a model or symbolic representation to which the principles of mechanics may be applied. In any set, the problems are arranged in order of increasing difficulty. Furthermore, the answers to all but every fourth problem, which is indicated by an asterisk, are listed in the back of the book. SI units are used in all the numerical examples and problems; however, for the convenience of some instructors, every fifth problem is stated *twice,* once in SI units and again in FPS units.

Besides a change from FPS to SI units and the addition of many new problems, this book differs from the author's first edition: *Engineering Mechanics: Dynamics* in many respects. Most of the text material has been completely rewritten so that topics within each section are categorized into subgroups, defined by bold face titles. The purpose of this is to present a structured method for introducing each new definition or concept and to provide a convenient means for later reference or review of the material.

Another unique feature used throughout this book is the "Procedure for Analysis." This guide to problem solving, which was initially presented in Sec. 9-3 of the first edition of *Engineering Mechanics: Statics*, is essentially a step-by-step set of instructions which provide the student with a logical and orderly method to follow when applying the theory. As in the first edition, the example problems are solved using this outlined method for solution in order to clarify application of the steps.

Since mathematics provides a systematic means of applying the principles of mechanics, the student is expected to have prior knowledge of algebra, geometry, trigonometry, and some calculus. Vector analysis is introduced at points where it is most applicable. Its use often provides a convenient means for presenting concise derivations of the theory, and it makes possible a simple and systematic solution of many complicated three-dimensional problems. Occasionally, the example problems are solved using several different methods of analysis so that the student develops the ability to use mathematics as a tool, whereby the solution of any problem may be carried out in the most direct and effective manner.

The contents of this book are presented in 11 chapters.* In particular, the kinematics of a particle is discussed in Chapter 12,† followed by a discussion of particle kinetics in Chapter 13 (equations of motion), Chapter 14 (work and energy), and Chapter 15 (impulse and momentum). A similar sequence of presentation is given for the planar motion of a rigid body: Chapter 16 (planar kinematics), Chapter 17 (equations of motion), Chapter 18 (work and energy), and Chapter 19 (impulse and momentum). If desired, it is possible to cover Chapters 12 through 19 in the following order with no loss in continuity: Chapters 12 and 16 (kinematics), Chapters 13 and 17 (equations of motion), Chapters 14 and 18 (work and energy), and Chapters 15 and 19 (impulse and momentum).

Time permitting, some of the material involving spatial rigid-body motion may be included in the course. The kinematics and kinetics of this motion are discussed in Chapters 20 and 21, respectively. Chapter 22 (vibrations) may be included if the student has the necessary mathematical background. Sections of the book which are considered to be beyond the scope of the basic dynamics course are indicated by a star and may be omitted. Note, however, that this more advanced material provides a suitable reference for basic principles when it is covered in more advanced courses.

The author has endeavored to write this book so that it will appeal to both the student and the instructor. Many people helped in its development. I wish to acknowledge the valuable suggestions and comments

*A discussion of units and a review of vector analysis is given in Appendixes A and B, respectively.

†The first 11 chapters of this sequence form the contents of *Engineering Mechanics: Statics*.

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Russell C. Hibbeler

Contents

12

Kinematics of a Particle

12–1. Introductory Remarks, Kinematics of Particles

Engineering mechanics consists of a study of both statics and dynamics. *Statics* deals with the equilibrium of bodies at rest or moving with constant velocity, whereas *dynamics* deals with bodies having accelerated motion. In general, dynamics is more complicated than statics, since the forces acting on the body must be related to the body's acceleration. The subject of dynamics is usually divided into two parts: (1) *kinematics* is concerned with the geometrical aspects of motion, and (2) *kinetics* is concerned with the analysis of the forces causing the motion. For simplicity in presenting the theory of both kinematics and kinetics, particle dynamics will be discussed first, followed by topics in rigid-body dynamics.

Particle Motion. Recall that a *particle* is defined as a small portion of matter such that its dimension or size is of no consequence in the analysis of a physical problem. In most problems encountered, one is interested in bodies of a finite size, such as rockets, projectiles, or vehicles. Such objects may be considered as particles, provided motion of the body is characterized by motion of its mass center and any rotation of the body can be neglected.

In general, the "kinematics" of a particle is characterized by specifying the particle's displacement, velocity, and acceleration. This chapter begins with the study of the *absolute motion* of a particle, which is motion measured with respect to a *fixed coordinate system.* In this regard, motion along a straight line will be studied before introducing the more general motion along a curved path. Afterwards, the *relative motion* between two particles will be considered, using a translating coordinate system.

12–2. Rectilinear Velocity and Acceleration of a Particle

The simplest motion of a particle is motion occurring along a straight-line path, called *rectilinear motion.*

Position. Consider the particle at point P shown in Fig. 12–1. The coordinate s which is measured from the fixed origin O, is used to define the *position* of the particle at any given instant. If s is positive, the particle is located to the right of the origin; if s is negative, the particle is located to the left. Ordinarily, this position is measured in metres (m).

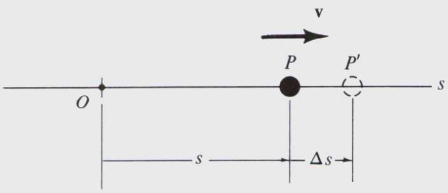

Fig. 12–1

Displacement. The *displacement* of the particle is defined as the *change* in its *position.* This is represented by the symbol Δs. When the particle's final position P' is to the right of its initial position P, Δs is positive, Fig. 12–1; when the displacement is to the left, Δs is negative.

The displacement of a particle must be distinguished from the distance the particle travels. Specifically, the *distance* traveled is defined as the *total length of path* traversed by the particle—*which is always positive.*

Velocity. Consider now that the particle moves through a positive displacement Δs from P to P' during the time interval Δt, Fig. 12–1. The *average velocity* of the particle during this time interval is defined as

$$v_{\text{avg}} = \frac{\Delta s}{\Delta t} \tag{12–1}$$

By taking smaller and smaller values of Δt, and consequently smaller and smaller values of Δs, we obtain the *instantaneous velocity,* defined as

$$v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t}$$

or

$$v = \frac{ds}{dt} \tag{12–2}$$

For both the average velocity and instantaneous velocity, the *direction* is either positive or negative depending upon whether the displacement is positive or negative. For example, if the particle is moving to the right as shown in Fig. 12–1, the velocity is positive. The *magnitude* of the velocity is known as the *speed.* If the displacement is expressed in metres (m) and the time in seconds (s), the speed is expressed as m/s.

Occasionally the term "average speed" is used. The *average speed,* $(v_{sp})_{avg}$, is defined as the total distance of the path traveled by a particle, s_T, divided by the elapsed time Δt, i.e.,

$$(v_{sp})_{avg} = \frac{s_T}{\Delta t} \tag{12–3}$$

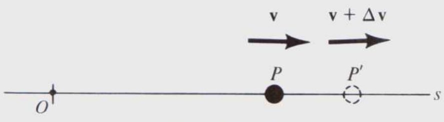

Fig. 12–2

Acceleration. Provided the instantaneous velocities for the particle are known at the two points P and P', the *average acceleration* for the particle during the time interval Δt is defined as

$$a_{avg} = \frac{\Delta v}{\Delta t} \tag{12–4}$$

where Δv represents the difference in the velocities during the time interval Δt, Fig. 12–2.

The *instantaneous acceleration* at time t is found by taking smaller and smaller values of Δt, and corresponding smaller and smaller values of Δv, so that

$$a = \lim_{\Delta t \to 0} \left(\frac{\Delta v}{\Delta t}\right)$$

or

$$a = \frac{dv}{dt} \tag{12–5}$$

Taking the second time derivative of Eq. 12–2, we can also write

$$a = \frac{d^2 s}{dt^2} \tag{12–6}$$

Both the average and instantaneous acceleration can be either positive or negative. In particular, when the particle is *slowing down,* the velocity change is negative and the particle is said to be *decelerating*. Also, note that when *the velocity is constant, the acceleration is zero*. Units commonly used to express the magnitude of acceleration are m/s^2.

A differential relation involving the displacement, velocity, and acceleration along the path may be obtained by solving for the time differential dt in Eqs. 12–2 and 12–5 and equating, i.e.,

$$dt = \frac{ds}{v} = \frac{dv}{a}$$

so that

$$a\,ds = v\,dv \qquad (12\text{–}7)$$

Constant Acceleration. When the acceleration is constant, $a = a_c$, each of the three kinematic equations $a = dv/dt$, $v = ds/dt$, and $a\,ds = v\,dv$ may be integrated to obtain formulas that relate a_c, v, s, and t.

To determine the *velocity as a function of time,* integrate $a = dv/dt = a_c$, assuming that initially $v = v_1$ at $t = 0$.

$$\int_{v_1}^{v} dv = \int_0^t a_c\,dt$$

$$v - v_1 = a_c(t - 0)$$

$$v = v_1 + a_c t \qquad (12\text{–}8)$$

To determine the *displacement as a function of time,* integrate $v = ds/dt = v_1 + a_c t$, assuming that initially $s = s_1$ at $t = 0$.

$$\int_{s_1}^{s} ds = \int_0^t (v_1 + a_c t)\,dt$$

$$s - s_1 = v_1(t - 0) + a_c(\tfrac{1}{2}t^2 - 0)$$

$$s = s_1 + v_1 t + \tfrac{1}{2}a_c t^2 \qquad (12\text{–}9)$$

To determine the *velocity as a function of displacement,* either solve for t in Eq. 12–8 and substitute into Eq. 12–9, or integrate $v\,dv = a_c\,ds$, assuming that initially $v = v_1$ when $s = s_1$.

$$\int_{v_1}^{v} v\,dv = \int_{s_1}^{s} a_c\,ds$$

$$\tfrac{1}{2}v^2 - \tfrac{1}{2}v_1^2 = a_c(s - s_1)$$

$$v^2 = v_1^2 + 2a_c(s - s_1) \qquad (12\text{–}10)$$

The magnitudes and signs of s_1, v_1, and a_c, used in these equations, are determined from the chosen origin and positive direction of the s axis.

It is important to remember that the above equations are useful *only when the acceleration is constant.* A common example of constant accelerated motion occurs when a body falls freely toward the earth. If air resistance is neglected and the distance of fall is short, then the constant *downward* acceleration of the body is approximately 9.81 m/s².*

*The proof is given in Example 13–3.

PROCEDURE FOR ANALYSIS

When a functional relationship between *any two* of the quantities a, v, s, and t is known, the functional relations describing the other kinematic quantities can be obtained by either the proper differentiation or integration* of the equations $a = dv/dt$, $v = ds/dt$ or $a\,ds = v\,dv$. In attempting to solve a problem, it should be realized that each of these equations relates *three quantities*. Hence when a quantity is known as a function of another quantity, the third quantity is obtained *by choosing the kinematic equation which relates all three*. For example, suppose that the acceleration is known as a function of displacement, $a = f(s)$. The velocity can be determined from $a\,ds = v\,dv$ by substituting $f(s)$ for a, since $f(s)\,ds = v\,dv$ may be integrated.† The velocity *cannot* be obtained by using $a = dv/dt$, since a is not a function of time, i.e., $f(s)\,dt = dv$ *cannot* be integrated. Proceeding on this basis, four common types of problems which are often encountered, and their method for solution, are given as follows:

1. *Acceleration given as a function of time,* $a = f(t)$. To find the velocity as a function of time, substitute into $a = dv/dt$, which yields $dv = f(t)\,dt$, and integrate to obtain $v = h(t)$. The displacement as a function of time is obtained by substituting for v into $v = ds/dt$, which gives $ds = h(t)\,dt$. Integration yields $s = g(t)$.
2. *Acceleration given as a function of velocity,* $a = f(v)$. To find the velocity as a function of time, substitute into $a = dv/dt$, which yields $dv = f(v)\,dt$ or $dv/f(v) = dt$, and integrate to obtain $v = h(t)$. The displacement as a function of time is obtained by substituting for v into $v = ds/dt$, which gives $ds = h(t)\,dt$. Integration yields $s = g(t)$.
3. *Acceleration given as a function of displacement,* $a = f(s)$. To find the velocity as a function of displacement, substitute into $a\,ds = v\,dv$, which yields $f(s)\,ds = v\,dv$, and integrate to obtain $v = h(s)$. The displacement as a function of time is obtained by substituting for v into $v = ds/dt$, which gives $h(s) = ds/dt$ or $ds/h(s) = dt$. Integration yields $s = g(t)$.
4. *Acceleration is constant,* $a = a_c$. Rather than integrating, use one of the appropriate derived equations, 12–8, 12–9, or 12–10.

*Some standard differentiation and integration formulas are given in Appendix C.

†The position s_1 and velocity v_1 must be known at a given instant in order to evaluate either the constant of integration if an indefinite integral is used, or the limits of integration if a definite integral is used.

Example 12-1

A small projectile is fired vertically downward into a fluid medium with an initial velocity of 60 m/s. If fluid resistance causes a deceleration of the projectile which is equal to $a = (-0.4v^3)$ m/s², where v is measured in m/s, determine both the velocity v and position s four seconds after the projectile is fired.

Solution

Since a is given as a function of velocity, $a = (-0.4v^3)$ m/s², to obtain velocity v as a function of time it is necessary to use $a = dv/dt$, since this equation relates v, a, and t. (Why not use Eq. 12-8, $v = v_1 + a_c t$?) If the downward direction is assumed positive, then integrating, with the initial condition that $v = 60$ m/s at $t = 0$, yields*

$$(+\downarrow) \qquad a = \frac{dv}{dt} = -0.4v^3$$

$$\int_{60}^{v} \frac{dv}{-0.4v^3} = \int_0^t dt$$

$$\frac{1}{0.8}\frac{1}{v^2}\bigg|_{60}^{v} = t$$

$$\frac{1}{0.8}\left[\frac{1}{v^2} - \frac{1}{(60)^2}\right] = t$$

$$v = \left\{\left[\frac{1}{(60)^2} + 0.8t\right]^{-1/2}\right\} \text{ m/s}$$

Here the positive root is taken, since the projectile is moving downward. When $t = 4$ s,

$$v = 0.559 \text{ m/s} \qquad \textit{Ans.}$$

Knowing the velocity as a function of time, the position s as a function of time is obtained from $v = ds/dt$, since this equation relates s, v, and t. Using the initial condition $s = 0$ at $t = 0$, we have

$$(+\downarrow) \qquad v = \frac{ds}{dt} = \left[\frac{1}{(60)^2} + 0.8t\right]^{-1/2}$$

$$\int_0^s ds = \int_0^t \left[\frac{1}{(60)^2} + 0.8t\right]^{-1/2} dt$$

$$s = \frac{2}{0.8}\left[\frac{1}{(60)^2} + 0.8t\right]^{1/2}\bigg|_0^t$$

*The *same result* is obtained by evaluating a constant of integration rather than using definite limits on the integral. For example, integrating $dt = dv/-0.4v^3$ yields $t = 1/0.8(1/v^2) + C$. Using the condition that at $t = 0$, $v = 60$ m/s, the constant of integration is $C = -1/0.8[1/(60)^2]$.

$$s = \frac{1}{0.4}\left\{\left[\frac{1}{(60)^2} + 0.8t\right]^{1/2} - \frac{1}{60}\right\}\text{m}$$

When $t = 4$ s,

$$s = 4.43 \text{ m} \qquad \textit{Ans.}$$

Example 12-2

A boy tosses a ball in the vertical direction off the side of a cliff, as shown in Fig. 12-3. If the initial velocity of the ball is 15 m/s upward, and the ball is released 40 m from the bottom of the cliff, determine (a) the maximum height s_B reached by the ball and (b) the speed of the ball just before it hits the ground. During the entire time the ball is in motion, it is subjected to a constant downward acceleration of 9.81 m/s² due to gravity. Neglect the effect of air resistance.

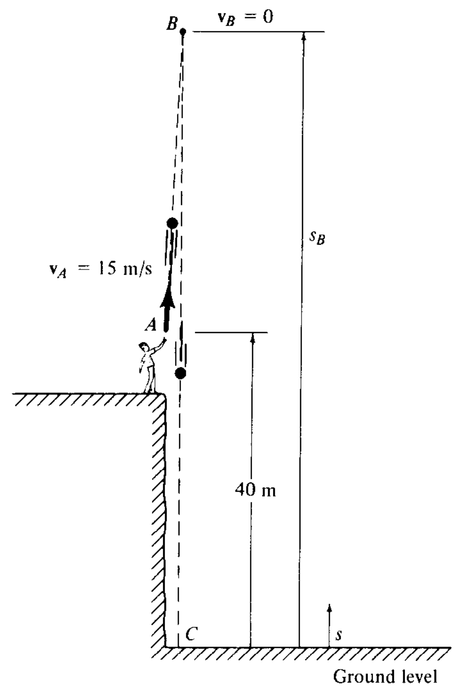

Fig. 12-3

Solution

Part (a). The coordinate axis for position $s = 0$ is taken at the base of the cliff as shown in the figure. At the maximum height s_B, the velocity $v_B = 0$. Furthermore, the ball is thrown from an initial height of $s_A = +40$ m. Since the ball is thrown *upward* at $t = 0$, it is subjected to a velocity of $v_A = +15$ m/s (positive since it is in the same direction as positive displacement). For the entire motion, the acceleration is *constant* such that $a_c = -9.81$ m/s² (negative since it acts in a direction *opposite* to positive velocity or positive displacement). Since a_c is *constant,* throughout the entire motion, the displacement may be related to velocity at points A and B using Eq. 12-10, i.e.,

$$(+\uparrow) \qquad v_B^2 = v_A^2 + 2a_c(s_B - s_A)$$
$$0 = (15)^2 + 2(-9.81)(s_B - 40)$$

so that

$$s_B = 51.5 \text{ m} \qquad \textit{Ans.}$$

Part (b). To obtain the velocity v_C of the ball just before it hits the ground, Eq. 12-10 can be applied between points B and C, Fig. 12-3,

$$(+\uparrow) \qquad v_C^2 = v_B^2 + 2a_c(s_C - s_B)$$
$$= 0 + 2(-9.81)(0 - 51.5)$$
$$v_C = -31.8 \text{ m/s} \qquad \textit{Ans.}$$

The negative root was chosen since the ball is moving *downward.*

Similarly, Eq. 12-10 may also be applied between points A and C, i.e.,

$$(+\uparrow) \qquad v_C^2 = v_A^2 + 2a_c(s_C - s_A)$$
$$= 15^2 + 2(-9.81)(0 - 40)$$
$$v_C = -31.8 \text{ m/s} \qquad \textit{Ans.}$$