ADVANCED PHYSICS

Keith Gibbs

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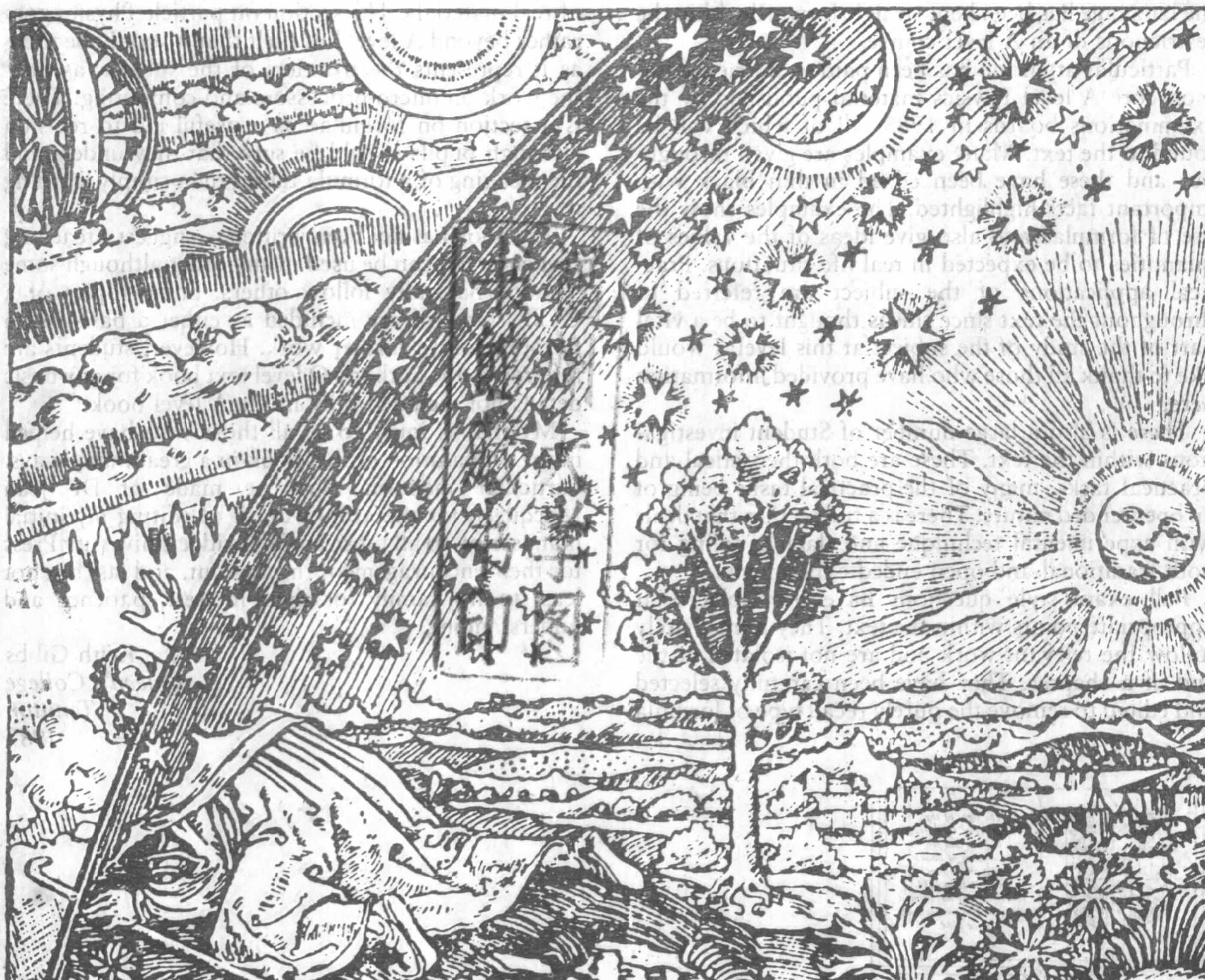

A medieval conception of the universe

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Preface

This is the morning I would not forget
For then we stood in awe
And saw the world created in a day *F.S.Bourne*

This book is a full A level text covering the A level syllabi of all the major examining boards. The double page layout has been followed wherever possible and the language has been kept straightforward within the restrictions of an A level course.

Particular attention has been paid to the new common core A level Physics material published by the examinations boards in 1983, all of which will be found in the text. Many examples are given throughout and these have been edged in thin black with important facts highlighted. The examples show the use of formulae and also give ideas of the values of quantities to be expected in real life situations. Practical applications of the subject are referred to throughout the text since this is thought to be a vital part of the study of the subject at this level. I would like to thank all those who have provided information here.

There is also a large number of Student investigations within the text. These are both theoretical and practical tasks, many of the practical tasks being of an open-ended nature. There is a full section that deals with experimental technique and data handling for both traditional and open ended experiments.

Full exam style questions have been placed at appropriate points within the text. They immediately follow the relevant work and are not isolated at the end of a chapter. They have been carefully selected and edited to remove the purely recall type of formula quotation and experiment description. Since this would simply mean copying out the text teachers have been left to devise these for themselves. There is also a section on the comprehension of Physics texts. I must thank all the examining boards who have given me permission to reproduce these questions.

Particular care has been paid to the layout of the book so that it is easy to follow and attractive to look at. Many diagrams have been included as an aid to understanding and also to make the book more attractive. Tables of useful values appear at relevant places in the text to give the student an idea of the properties of real materials. The section on particle Physics goes rather beyond A level but I felt that it should be there as it represents the frontiers of the subject, as does the work on micro-processors and computing. There is a section on formulae as a useful aid to revision although pupils should be sure that they understand the meaning of a formula rather than simply learning it.

The book has not been written to suggest a teaching sequence and can be used in any order although some sections naturally follow others. Some topics of a GCSE standard are included as either a basis or an introduction to further work. However, students are referred to the author's O level text book for any basic details not included fully in the A level book.

My thanks must go to all those who have helped in the preparation of this book in a great many ways. Particular mention must be made of Dr Jean Macqueen for the arduous task of editing my initial manuscript, to all those at Cambridge University Press for their encouragement throughout, and last but not least to my family for their interest, patience and understanding.

Keith Gibbs
Queen's College
Taunton
1987

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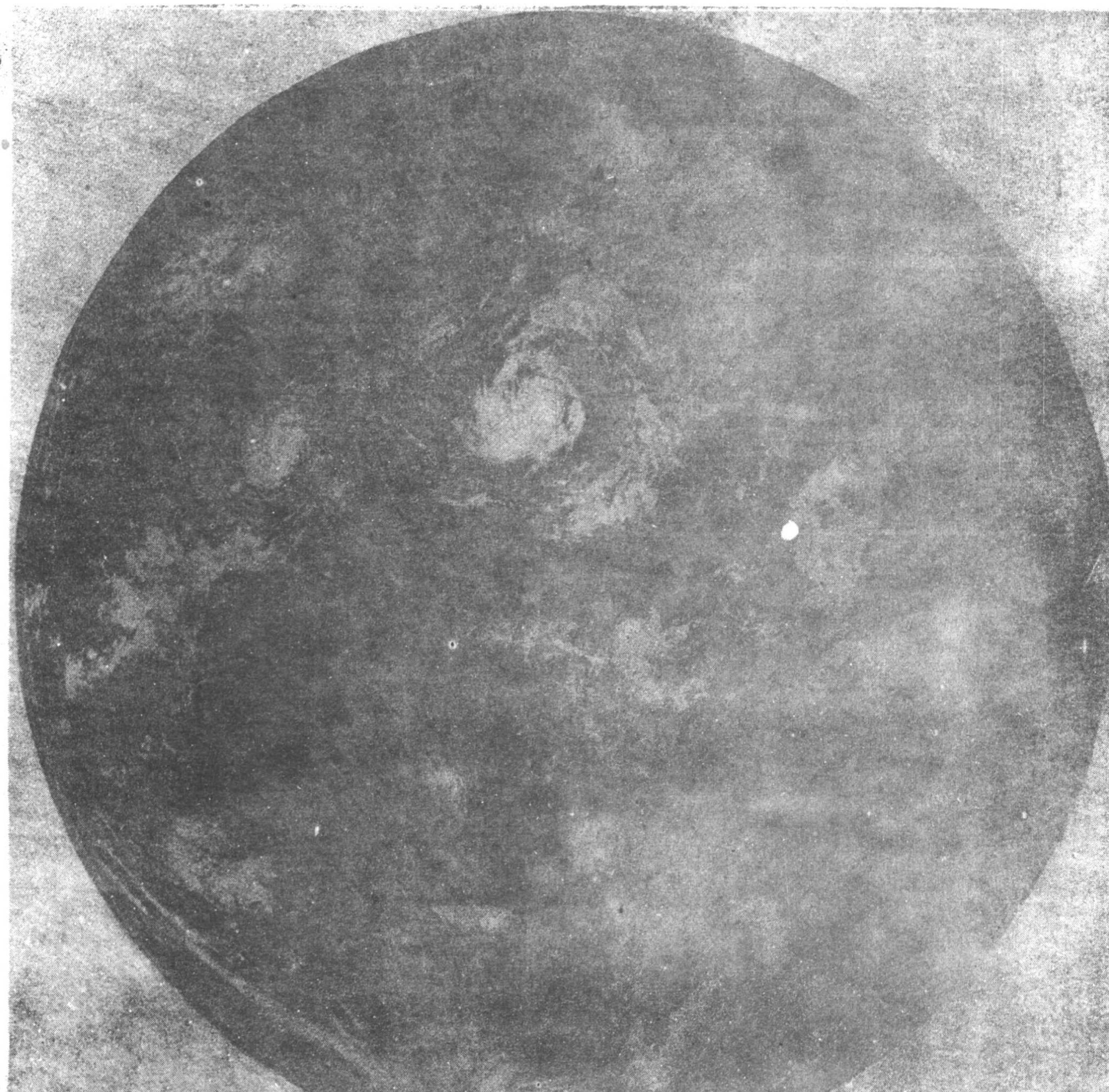


A view of the earth from a weather satellite showing cyclones in the North Pacific (Japan Weather Association)

1 · Physics and physicists

Physics has an impact on our lives in a great number of ways and I will start this book by looking at some of these applications. I do not think that a study of Physics should ever be separated from a study of the real world around us.

If you become a physicist you must be prepared to investigate, to observe, to carry out experiments and then record your results. You must then be able to explain these results to others and discuss your ideas and their opinions. You must be prepared to be adaptable in the rapidly changing world of technology and you must have sufficient mathematical ability to express your results and ideas in precise terms rather than broad generalisations.

Physicists may work in many fields and the list below shows some of these. (I am most grateful to the Institute of Physics for permission to print these.)

None of these careers may be undertaken without knowledge and it is the study of this knowledge and its application that will concern us in the rest of the book. When you are trying to grasp a formula or solve a problem, try not to forget the wider applications of the subject.

Knowledge of Physics is needed:

to set up satellite communications
to investigate 'black holes'
to take scans of the human body
to construct a computer
to detect flaws in structures
to make new materials
to study pollution of air, land and water
to reduce the noise in vehicles
to harness energy of all kinds
to solve crimes

Physics provides answers to questions such as:

why is the sky blue but sunsets red?
how can we save premature babies from dying?
what makes glass transparent?
what holds parts of the atom together?
how can we predict earthquakes?

and many more – this list can only give you just a small insight into the possibilities when working as a physicist.

Careers in Physics

Medical Physics
Health Service
Instrumentation
Health physics
Physics for the handicapped

Education
Schools
Colleges
Universities
Polytechnics

Geophysics
Mineralogy
Petrology
Prospecting
Mineral processing

Environmental Physics
Radiation protection
Conservation
Noise control
Pollution control

Computing
Computer design
System design
Computer aided design
Robotics
Microprocessor control

Meteorology
Oceanography
Weather forecasts
Radio
Travel

Alternative energy
Geothermal
Solar
Wave
Wind

Engineering
Chemical
Civil
Control
Electrical
Mechanical

Scientific Civil Service
Defence
Energy and resources
Patents
Research labs.
Science policy
Standards

Materials science
Metallurgy
New materials
Thin films

Communications
Fibre optics
Satellites
Telecommunications

Industry
Aerospace
Chemical
Electronics
Food
Petroleum
Semiconductor

2 · Basic measurements in Physics

Physics is a science of observation of the world around us. It aims to give an understanding of this world both by observation and by prediction of the way in which objects will behave. It is a science of measurement, but before any measurements can be made we must define the units on which our measurements are made.

The units used in this book are the **International System of Units** (SI) based on the seven base units defined below.

Base units

The **metre** is the length equal to 1 650 763.73 wavelengths in a vacuum corresponding to the transition between two levels in the krypton-86 atom.

The **kilogram** is the mass equal to that of the international prototype kilogram kept at the Bureau International des Poids et Mesures at Sèvres, France.

The **second** is the duration of 9 192 631 770 periods of the radiation corresponding to the transition between two hyperfine levels of the ground state of the caesium-137 atom.

The **ampere** is that constant current which, if maintained in two parallel straight conductors of infinite length and of negligible circular cross-section placed 1 metre apart in a vacuum, would produce a force between them of 2×10^{-7} N per metre of length.

The **kelvin** is 1/273.16 of the thermodynamic temperature of the triple point of water.

The **candela** is the luminous intensity in a given direction of a source that emits monochromatic radiation of frequency 540×10^{12} Hz that has a radiant intensity of 1/683 watt per steradian.

The **mole** is the amount of substance of a system that contains as many elementary particles as there are in 0.012 kg of carbon-12.

The accurate measurement of length

We shall consider two instruments here, the micrometer and the vernier scale; it is likely, however, that digital and interference methods will become more popular in the years to come.

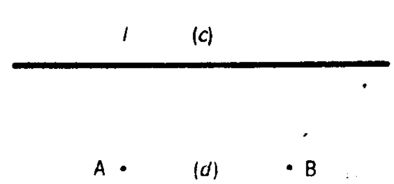
The micrometer (Figure 2.2)

This is a device for the measurement of distances up to a few millimetres with an accuracy of about 0.01 mm. It has an accurately threaded screw fixed to a drum so that when the drum rotates once the screw advances a known distance, usually 0.5 mm, and the jaws close by this amount. The body of the drum is graduated from 0 to 50 so that measurements

Student investigation

Measure the following quantities using what you consider to be the most appropriate measuring device that you have available, and record your results:

(*a*) the volume of the laboratory,
(*b*) the diameter of a marble,
(*c*) the length of the line *l* (Figure 2.1),
(*d*) the separation of the dots A and B,
(*e*) the thickness of one page of this book,
(*f*) the radius of 28 gauge wire,
(*g*) the area of the rectangle PQRS,
(*h*) the mass of one rice grain,
(*i*) the radius of the circle D.



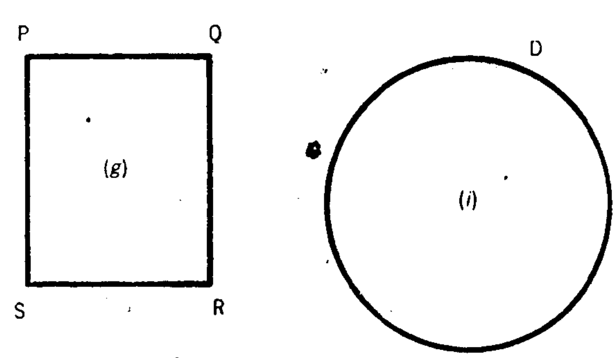


Figure 2.1

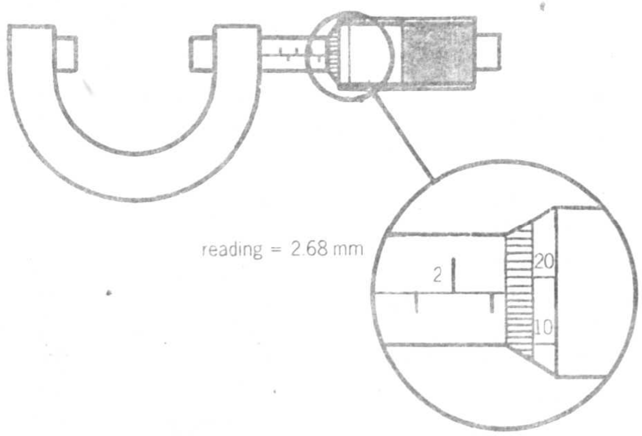


Figure 2.2

of 1/50 of a rotation or 0.01 mm may be made. A ratchet screw is provided so that the object being measured is not squashed and the jaws are not strained.

Before making any measurement it is important to check that the micrometer reads zero when the jaws are closed. If it does not then this zero error must be allowed for when the reading is taken.

The vernier scale (Figure 2.3)

A vernier scale is a useful extension of the main scale and is usually used for lengths of a few centimetres. It is accurate to about 0.1 mm.

The vernier scale is divided into ten parts and is the same length as nine parts on the main scale. This means that if the main scale is graduated in millimetres, each vernier division is 0.9 mm long. The reading on the vernier scale that exactly matches a scale division gives the next decimal place in the measurement.

In Figure 2.3 the reading is 12.7 mm. Vernier scales are frequently found on travelling microscopes and Fortin barometers. An angular vernier scale is used on the table of accurate spectrometers.

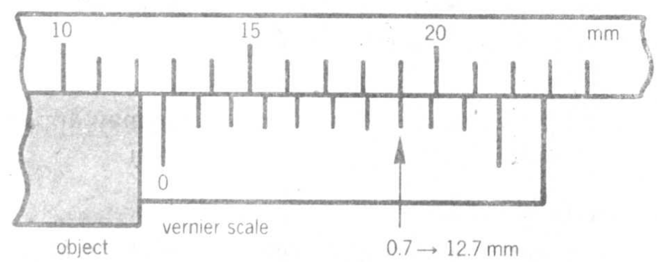


Figure 2.3

1 What are the correct readings shown by Figures 2.4 to 2.7?

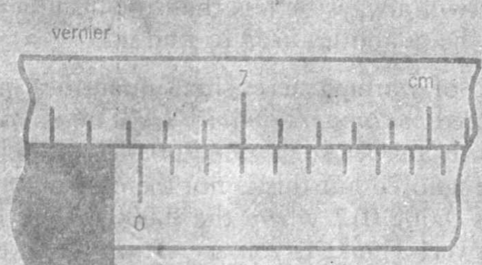


Figure 2.4

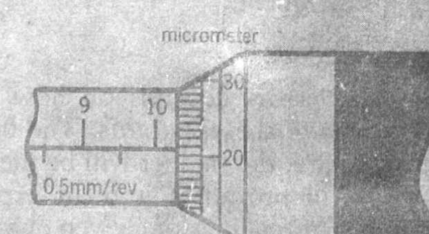


Figure 2.5

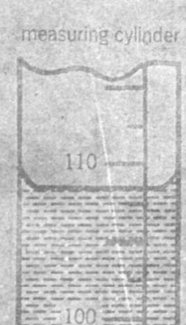


Figure 2.6

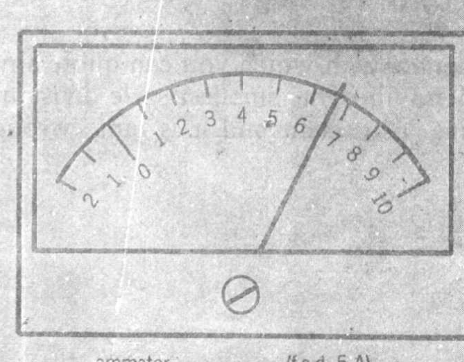


Figure 2.7

3 · Dimensions and errors

Any experiment will involve a series of measurements, and each of these measurements will be made to a certain degree of accuracy. For example, the calculation of a velocity requires the measurement of a time and a distance.

Using a stopwatch you may measure the time to the nearest tenth of a second, and using a metre rule you may find the distance to the nearest millimetre (if it is a fairly small distance measured in the laboratory).

It is very useful to have a rough idea of the kind of result that you might expect before starting an experiment, although of course in research this is not always possible.

There are two basic types of error that may appear in the result.

Systematic errors

These occur due to faulty apparatus such as an incorrectly labelled scale, an incorrect zero mark on a meter or a stopwatch running slowly. Repeating the measurement a number of times will have no effect on this type of error and it may not even be suspected until the final result is calculated. The only way to eliminate this type of error is to change or recalibrate the measuring instrument.

Random errors

The size of these errors depends on how well the experimenter can *use* the apparatus. The better the experimenter you are, the smaller will be the random error that you will introduce into an experiment. Making a number of readings of a given quantity and taking an average will reduce the overall error.

Accuracy of readings

The accuracy with which you can quote any reading will depend upon the smallest scale division on your measuring instrument and it is quite wrong to give results to much greater accuracies than this, especially when the final answer may contain a number of different measurements.

Let us start with a very simple example. If you measure a length with a ruler and get an answer of 6.8 cm, then we assume that you have been able to measure to ±1 mm since that is the last figure in your answer. This means that that reading is accurate to 1 part in 68, i.e., 1.5 %. Now if that reading forms part of an experiment in which there are other measurements then it is useful if the other quantities can be found to the same degree of accuracy. Taking one reading to a very high degree of accuracy is little help if others will be much more inaccurate. In any experiment you should be aware of which readings are the inaccurate ones.

Quoting an answer

When you have made your set of readings, you must be careful when you quote the result. You will probably use a calculator to work out a formula containing perhaps three or four different quantities; your calculator will give an answer to eight places of decimals but do *not* use this as an answer. The accuracy of the answer will always be less than the accuracy of any one of the quantities used to find it.

A word of warning here: small quantities may only be ignored *in comparison with large ones*. For example, in an answer such as 6.700 002 the 0.000 002 may be ignored but this is *not* the case in an answer such as 0.000 012 where the 0.000 002 is 17 % of the answer! Think: if you are sat on by an elephant it does not matter very much if the elephant has a fly sitting on its back (Figure 3.1) – but if an isolated fly is sat on by another fly, then this second fly is important to the first one!

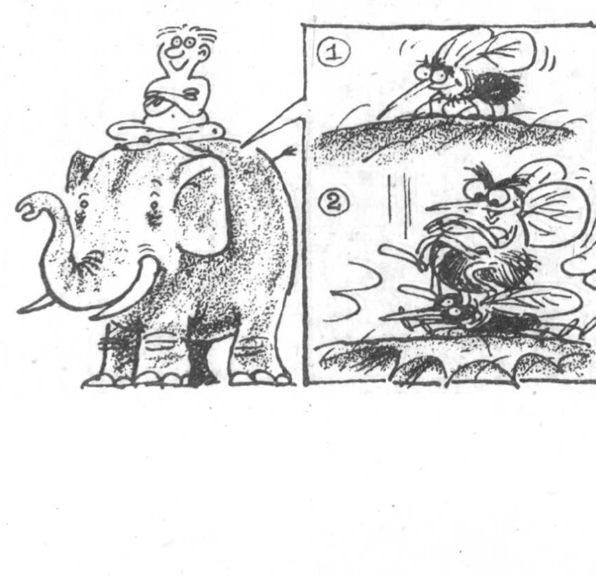


Figure 3.1

Calculating the error

In this section we will imagine that we wish to find the value of a quantity Q that involves the measurement of two other quantities a and b.

(i) A **sum** or **difference** of two quantities, i.e. $Q = a + b$ or $Q = a - b$

Let Q be the length of an object found by taking two readings (a and b) from a ruler (see Figure 3.2).

Therefore $Q = b - a$

Let $a = 16.5$ cm $\pm$ 0.1 cm and $b = 25.4$ cm $\pm$ 0.1 cm.

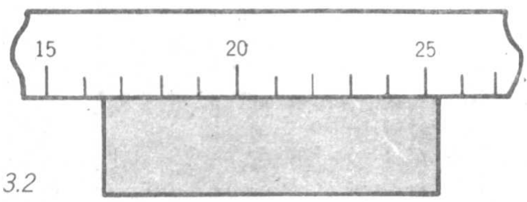


Figure 3.2

Q has its *average* value when both a and b have their average values:

i.e. Q = 25.4 − 16.5 = 8.9 cm

Q has its ***maximum*** value when a has its smallest value and b its largest:

i.e. Q = 25.5 − 16.4 = 9.1 cm

Q has its ***minimum*** value when a has its largest value and b its smallest:

i.e. Q = 25.3 − 16.6 = 8.7 cm

The error in Q is therefore simply the *sum* of the errors in a and b.

We write this as: $\Delta Q = \Delta a + \Delta b$

This formula applies for a sum or difference.

In this example the answer should be written as:

$Q = 9.1 \pm 0.2$ cm

The percentage error in Q would be (0.2/9.1) × 100 = 2.2 %.

(ii) The **product** or **quotient** of two quantities: $Q = ab$ or $Q = a/b$

An example is shown in Figure 3.3.

Let the error in Q be ΔQ
the error in a be Δa
the error in b be Δb

Therefore if $Q = ab$, the maximum value for Q is:

$$Q + \Delta Q = (a + \Delta a)(b + \Delta b) = a\Delta b + b\Delta a + \Delta a\,\Delta b$$

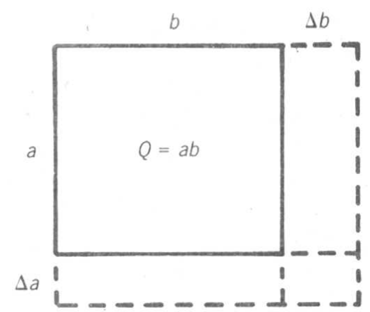


Figure 3.3

Now we can ignore the term $\Delta a \Delta b$, since it is the product of two small quantities and is therefore small *in comparison* with the other terms. Then

$$\Delta Q = a\Delta b + b\Delta a$$

or, expressed as a fractional error:

$$\frac{\Delta Q}{Q} = \frac{\Delta a}{a} + \frac{\Delta b}{b}$$

This will apply to both a product and a quotient. You can also show that if one or more of the quantities (a,b) is raised to a power, say n (i.e., $Q = ab^n$) then:

$$\frac{\Delta Q}{Q} = \frac{\Delta a}{a} + \frac{n\Delta b}{b}$$

Notice that pure numbers have no errors; this can also be assumed for quantities such as π and e.

Example

Find the maximum possible error in the measurement of the force on an object (mass m) travelling at velocity v in a circle of radius r if $m = 3.5$ kg $\pm$ 0.1 kg, $v = 20$ m s^{-1} $\pm$ 1 m s^{-1} and $r = 12.5$ m $\pm$ 0.5 m.

Force (F) = $\dfrac{mv^2}{r}$

Therefore $\dfrac{\Delta F}{F} = \dfrac{\Delta m}{m} + \dfrac{2\Delta v}{v} + \dfrac{\Delta r}{r}$

$$\frac{\Delta F}{F} = \frac{0.1}{3.5} + \frac{2 \times 1}{20} + \frac{0.5}{12.5}$$

$$= 0.03 + 0.1 + 0.08$$

$$= 0.21$$

Therefore $F = 80 \pm 17$ N

The percentage error can be expressed as the sum of the percentage errors in the quantities. In the above example the percentage error in F is 3 % + 10 % + 8 % = 21 %. So the error F is 22 % of 80 = 17 N.

Taking a number of readings

In an experiment taking a number of readings will reduce the error in the final answer. Consider the measurement of the thickness of a hacksaw blade made at a number of places with a micrometer, such that each reading is accurate to ± 0.01 mm.

Suppose that the six readings taken are: 0.65, 0.66, 0.63, 0.66, 0.64 and 0.65 mm. The mean of these will be their sum divided by 6, which is 0.65 mm.

To calculate the final error we work out the difference between each reading and the mean value (without regard to sign) and then divide by the number of readings. Therefore

Final error =

$$\frac{0.00 + 0.01 + 0.02 + 0.01 + 0.01 + 0.00}{6} = 0.008$$

This is the likely error in the answer, and the thickness should therefore be quoted as 0.65 ±0.008 mm.

Errors in graphs

If a graph is plotted then the error in the result is found as follows. Consider the graph shown in Figure 3.4. The best fit line to the points is drawn and its slope found (m). The average value of all the x- and y-coordinates is found; this will give the **centroid** of the line. The lines of greatest and least slope through the centroid are then drawn and their respective slopes found (m_1 and m_2).

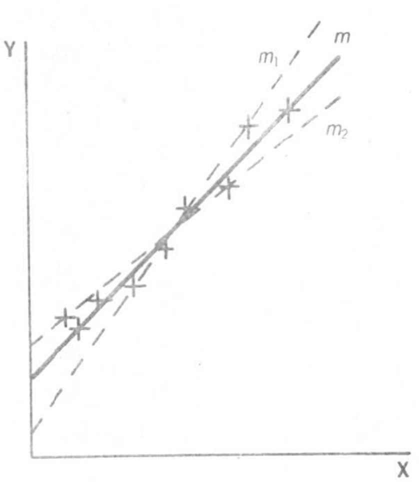


Figure 3.4

The final error in the slope is then given by $\frac{\Delta m}{m}$

where Δm is the difference between either m_1 or m_2 (whichever is the greater) and m.

1 Explain what is meant by
(a) a *random* error,
(b) a *systematic* error.
Give a practical example of each and discuss how they may be reduced or eliminated.

2 A ticker timer giving 50 dots per second is used to measure the velocity of a trolley running down a friction-compensated runway 2 m long.
If the velocity of the trolley is found to be 1.5 m s^{-1}, give an estimate of the accuracy that might be achieved in this result. Show in detail how you arrived at such a figure.

3 The density of a rectangular solid block is found by measuring three of the sides with a ruler which can be read to ± 0.5 mm and then finding its mass from a balance accurate to ± 1 g.
The readings obtained were
Length: 4.56 cm and 2.35 cm Width: 3.52 cm and 1.26 cm
Height: 3.04 cm and 2.61 cm Mass: 32 g
(a) Find
(i) the average volume of the block in mm^3, cm^3 and m^3,
(ii) the maximum and minimum values possible for the volume,
(iii) the average value for the density of the block,
(iv) the maximum fractional error in the volume,
(v) the maximum fractional error in the density,
(vi) the maximum error in the volume in mm^3,
(vii) the maximum error in the density in kg m^{-3}.
(b) Express the volume of the block correctly, showing the error.
(c) Express the density of the block correctly showing the error
(d) Repeat the calculations for the volume but assume that a pair of vernier calipers reading to ± 0.1 mm had been used instead.

Student investigation

Repeat the experiments on page 16 giving an estimate of the accuracy of your results.

Dimensions

The basic quantities in Physics are those of mass, length, time, electric current, temperature, luminous intensity and amount of a substance (see page 4). Other related quantities such as energy, acceleration and so on can be derived from combinations of these basic quantities and are therefore known as **derived** quantities.

The way in which the derived quantity is related to the basic quantity can be shown by the **dimensions** of the quantity. In considering dimensions we will restrict ourselves to those used in mechanics and properties of matter only.

The dimensions of mass are written as [M]
The dimensions of length are written as [L]
The dimensions of time are written as [T]

Note the square brackets round the letter to show that we are dealing with the dimensions of a quantity.

The dimensions of any other quantity will involve one or more of these basic dimensions. For instance, a measurement of volume will involve the product of three lengths and the dimensions of volume are therefore $[L]^3$.

In the same way a measurement of velocity requires a length divided by a time, and so the dimensions of velocity are $[L][T]^{-1}$.

The table below shows the dimensions of various common quantities in mechanics.

Quantity	Dimension
area	$[L]^2$
velocity	$[L][T]^{-1}$
force	$[M][L][T]^{-2}$
energy	$[M][L]^{-2}[T]^{-2}$
power	$[M][L]^{-2}[T]^{-3}$
volume	$[L]^3$
acceleration	$[L][T]^{-2}$
pressure	$[M][L]^{-1}[T]^{-2}$
momentum	$[M][L][T]^{-1}$

Dimensions have two important uses in Physics:

to check equations,
to derive equations.

Use of dimensions to check equations

The dimensions of the quantities of each side of an equation must match: those on the left-hand side must equal those on the right (remember the classic problem of not being able to give the total when five apples are added to three oranges – see Figure 3.5).

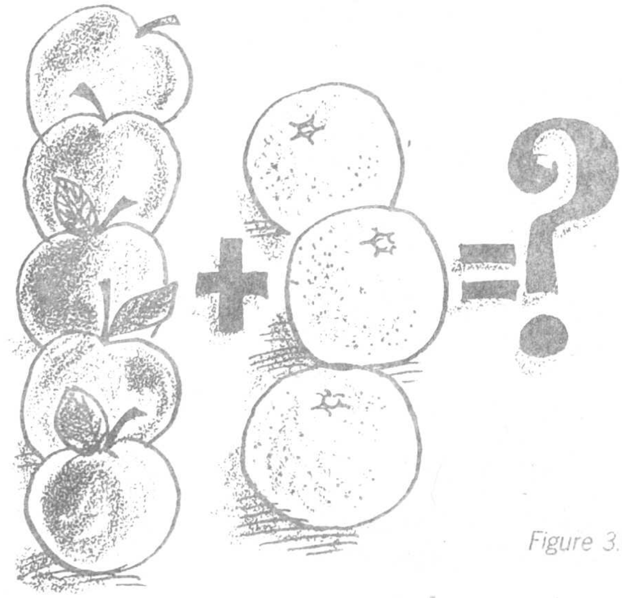


Figure 3.5

For example, consider the equation:

$$s = ut + \tfrac{1}{2}at^2$$

Writing this in dimensional form we have:

$$[L] = [L][T]^{-1}[T] + [L][T]^{-2}[T]^{-2}$$

therefore $[L] = [L] + [L]$

This proves the equation, since the length on the left-hand side of the equation is obtained by adding together the two lengths on the right-hand side.

Notice that $\frac{1}{2}$ is a pure number having no dimensions and is therefore omitted in the dimensional equation.

A further example is shown below.

Example

Show that the equation for impulse $Ft = mv - mu$ is dimensionally correct.

Writing this in dimensional form we have:

$$[M][L][T]^{-2}[T] = [M][L][T]^{-1} + [M][L][T]^{-1}$$

Therefore $[M][L][T]^{-1} = [M][L][T]^{-1}$ and the equation is correct, both sides having the dimensions of momentum.

Use of dimensions to derive equations

If we have some idea upon which quantities a further quantity might depend, then we can use the method of dimensional analysis to obtain an equation relating the relevant variables. You should appreciate that since numbers are dimensionless we cannot use this method to find these in equations, however.

Consider the oscillation of a simple pendulum. We will assume that the period of the pendulum (t) depends in some way on the following quantities:
(i) the mass of the pendulum bob (m)
(ii) the length of the string of the pendulum (l), and
(iii) the gravitational intensity (g).

We therefore write the equation as:

$$t = km^x l^y g^z$$

where x, y and z are unknown powers and k is a dimensionless constant.

Writing this in dimensional form gives:

$$T = M^x L^y L^z T^{-2z}$$

Equating the indices for M, L and T on both sides of the equation we have:

M: $0 = x$
L: $0 = y + z$
T: $1 = -2z$

Therefore:

$$x = 0,\ y = \tfrac{1}{2} \text{ and } z = -\tfrac{1}{2}.$$

The original equation therefore becomes:

$$t = k\left(\frac{l}{g}\right)^{\frac{1}{2}}$$

which is what we would expect for a simple pendulum. Dimensional analysis does not give us the value of the dimensionless constant k which can be shown by other methods to be 2π in this case (see page 78).

Further examples of the use of dimensional analysis to derive equations are found in the discussions later in this book of
(i) viscosity – Stokes's and Poiseuille's laws (pages 124 and 122),
(ii) wave velocity on a stretched string (page 176).

4 Use the method of dimensional analysis to deduce equations for the following:
(*a*) the period of oscillation of a vertical spiral spring,
(*b*) the velocity of waves on a stretched string,
(*c*) the frictional drag on a sphere falling through a liquid,
(*d*) the rate at which liquid flows through a pipe.

5 Use the method of dimensional analysis to check the validity of the following equations:
(*a*) $E = mc^2$, where E is the energy obtainable from a mass m, and c is the velocity of light.
(*b*) Energy stored in a wire $= -\frac{1}{2}\frac{EAe^3}{l}$ where E is Young's modulus, A the cross-sectional area, e the extension and l the original length.
(*c*) Escape velocity from a planet $= 2Rg_0$, where R is the radius of the planet and g_0 is the gravitational intensity at its surface.
(*d*) Period of oscillation of a floating cylinder with length h immersed in a liquid of density d:
$$T = 2\pi\sqrt{\frac{hd}{g}}$$

6 What are the dimensions of the following quantities?
(*a*) work
(*b*) energy
(*c*) power
(*d*) momentum
(*e*) impulse
(*f*) force
(*g*) coefficient of viscosity
(*h*) modulus of elasticity
(*i*) density
(*j*) coefficient of restitution

4 · Experimental work in Physics

Since much of Physics is experimental it is important to know how to perform experiments properly and how to present observations and work out conclusions. How much faith can be put in a theory if it can never be backed up by an experiment? In this chapter we will look at these techniques.

The appendix includes a list of experiments that might form part of a sixth form Physics course although the exact content will depend on the interests and facilities in any particular school (see page 480). Most of the experiments are of a standard nature but the value of open-ended investigations should not be overlooked. After all, the whole purpose of experimental research is that experiments are performed that nobody has done before, and although some are designed to confirm a theoretical prediction many unexpected results are found. For this reason some investigations of this type should form part of the course.

Suggested procedure for practical work

Practical work is of great importance in Physics, and you should treat all experiments carefully no matter how simple they appear to be. A good experimental technique can often be gained from such work.

Read all instructions carefully and plan your work before doing anything. Ask for help if you are in any doubt – this is better than damaging expensive apparatus!

Plan the number and spread of readings that you are going to take. If you have to draw a graph as part of the experiment then be sure to take at least eight readings, and make sure that these cover the full range. Do not attempt to set your readings to particular numbers (e.g. every 10 cm); adjust the variable and then read its value.

Always take more than one measurement if there is time and record the accuracy of each. Results should normally be recorded in table form, the units and accuracy being recorded at the top of each column.

A full and correct conclusion should be written at the end of each experiment, together with a comment on the errors and difficulties and how you would overcome them.

A calculation of the experimental error may be required in some experiments.

Remember that experiments without a mathematical answer are just as important as those that do have a numerical result, and a clearly written conclusion is still required.

Practical investigations

Some experiments are designed to verify a principle and others to measure a numerical quantity. There is, however, a third type: those that serve to examine a property or a situation, and seek to explain it or investigate how things will behave under different sets of circumstances. They are experiments which you would design yourselves, ones where the results are not known to you and which you have not looked up in books. Such experiments are known as **open-ended** – we cannot be sure what we will find! Many such investigations are described elsewhere in the book, but we will mention here some suggestions about how to tackle such investigations.

Before starting an experiment of this type you should be sure of the following:

What am I going to measure or investigate?
Do I need a control experiment?
What is my plan of action?
What apparatus will I need, and is it available?
Are my aims realistic?
Will the readings be taken manually or automatically?
How much time will I be able to allow for the investigation?
How will I present my results?

Having decided on these points you will have a better chance of success. Remember that a null result is not necessarily a wrong result; if you are seeing if something will happen and it doesn't then that is itself a valid experimental result. Don't let yourself be put off if things do not happen in quite the way that you expected.

Graphical methods in Physics

The presentation of experimental results or theories in the form of a graph has two main advantages:

(i) the variation of one quantity with another may be seen easily, and
(ii) the average value of a constant may be determined from the graph.

Before looking at graphs in detail you should realise that certain guidelines should be followed when plotting graphs:

1 The axes should be labelled with both the quantity and units.
2 The graph should be given a title.
3 It should fill the space available on the graph paper or page as far as possible.
4 Suitable scales should be chosen – something like 5 squares to 10 units, *not* 7 to 3!
5 The points should be plotted accurately and clearly.
6 The best fit line to the points should be drawn clearly but finely.

Probably the most useful form of graph is one in the form of a straight line and so we will begin by considering this type.

$y = mx + c$

This is the general equation for a straight line, where y and x are variables and m and c are constants. A general example of the graph produced by such an equation is shown in Figure 4.1. You should notice the following points:

(*a*) When $x = 0$ the intercept on the y-axis is c.
(*b*) When $y = 0$ the intercept on the x-axis is $-c/m$.
(*c*) The slope of the line (the change in y with x (dy/dx) is m.

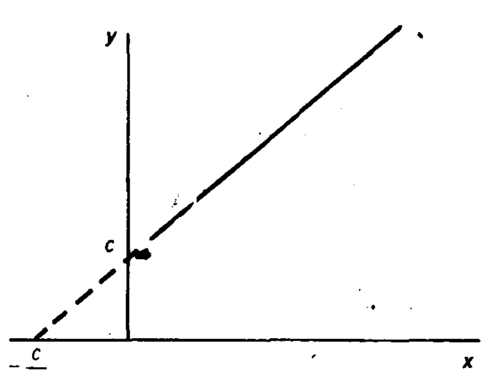


Figure 4.1

There are of course a large number of graphs but we will consider just a few other basic types. The equations and the relevant graphs are shown below.

$y = mx^2 + c$ (Figure 4.2)

This is a basic quadratic; if $c = 0$ the graph passes through the origin. An example of this would be the variation of the kinetic energy of a body with its velocity.

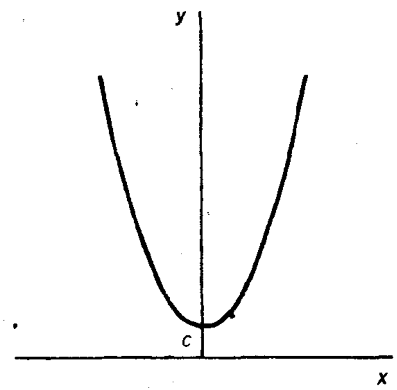


Figure 4.2

$y = ke^x$ (Figure 4.3)

This shows an exponential increase in y with respect to x; k is a constant. An example of this would be the increase in the pressure of air with depth.

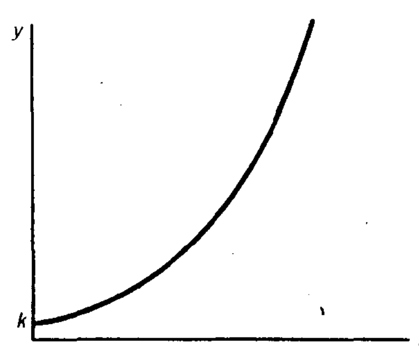


Figure 4.3

$y = ke^{-x}$ (Figure 4.4)

A rather more common form is the exponential decrease of y with respect to x. Once again k is a constant. This equation applies to radioactive decay, the discharge of a capacitor and many other physical phenomena.

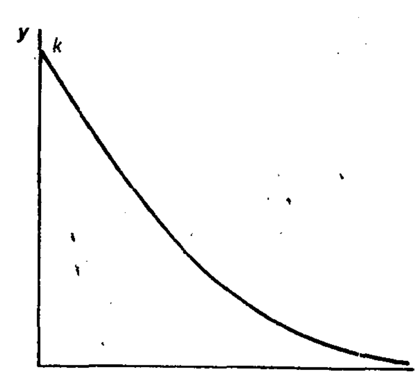


Figure 4.4

It is much more useful to plot the results of an experiment in the form of a straight line and so a means has to be found by which the equations above can be altered to give a linear relation between a function of y and a function of x. This is quite simply done:

For $y = mx^2 + c$: plot y against x^2 (Figure 4.5).

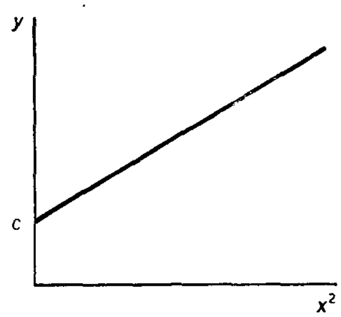


Figure 4.5

For $y = ke^x$: plot y against e^x (Figure 4.6).

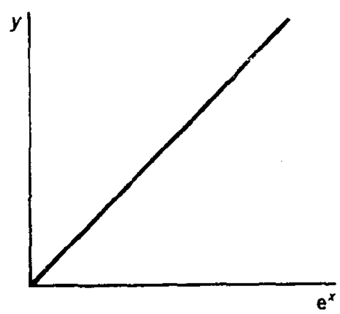


Figure 4.6

For $y = ke^{-x}$: plot y against e^{-x} (Figure 4.7).

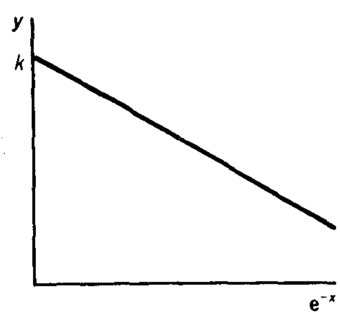


Figure 4.7

$y = ke^{cx}$

Here c is another constant. Taking natural logs gives:

$$\ln y = \ln k + cx$$

Plotting $\ln y$ against x gives a straight line with slope c and intercept on the $\ln y$ axis of $\ln k$ (Figure 4.8).

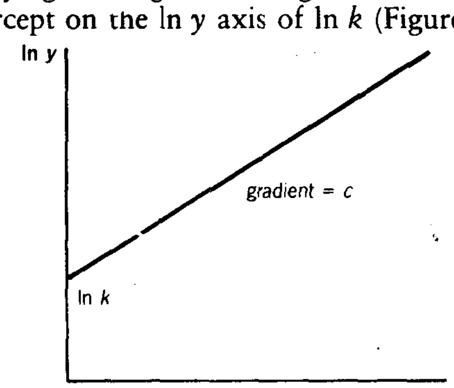


Figure 4.8

Notice that if we have an exponential *decrease*, c is negative.

An alternative method is to take logs of both sides of the equation; this is also useful when one is attempting to derive an unknown equation from a set of experimental results. We will consider first two versions where the equation is known and then one where it is not.

$y = kx^2$

Taking logs gives:

$$\log y = \log k + 2 \log x$$

Plotting $\log y$ against $\log x$ (Figure 4.9) will give a straight line of slope 2, with intercept on the $\log y$ axis of $\log k$.

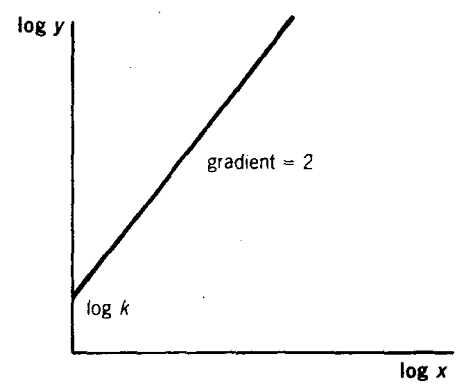


Figure 4.9

$y = ax^b$

Here a and b are constants but both are unknowns. Once again take logs of both sides:

$$\log y = \log a + b \log x$$

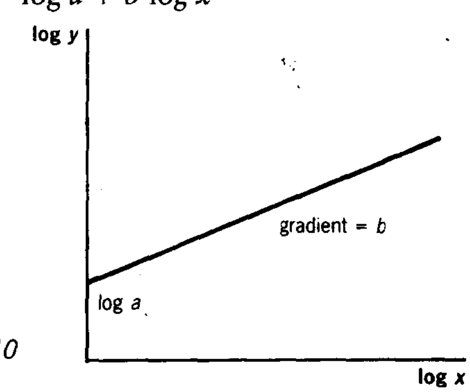


Figure 4.10

Plotting $\log y$ against $\log x$ will give a straight line of slope b and intercept on the $\log y$ axis of $\log a$ (Figure 4.10). Hence both a and b may be found and the form of the equation determined.

An example of the use of these methods may be found on page 75.