SCHAUM'S OUTLINE OF

THEORY AND PROBLEMS

OF

FEEDBACK and CONTROL SYSTEMS

Second Edition

SCHAUM'S OUTLINE OF

THEORY AND PROBLEMS

OF

FEEDBACK and CONTROL SYSTEMS

Second Edition

CONTINUOUS (ANALOG) AND DISCRETE (DIGITAL)

JOSEPH J. DiSTEFANO, III, Ph.D.
Departments of Computer Science and Medicine
University of California, Los Angeles

ALLEN R. STUBBERUD, Ph.D.
Department of Electrical and Computer Engineering
University of California, Irvine

IVAN J. WILLIAMS, Ph.D.
Space and Technology Group, TRW, Inc.

SCHAUM'S OUTLINE SERIES

McGRAW-HILL, INC.

New York St. Louis San Francisco Auckland Bogotá Caracas Hamburg Lisbon London Madrid Mexico Milan Montreal New Delhi Paris San Juan São Paulo Singapore Sydney Tokyo Toronto

Preface

Feedback processes abound in nature and, over the last few decades, the word feedback, like *computer*, has found its way into our language far more pervasively than most others of technological origin. The conceptual framework for the theory of feedback and that of the discipline in which it is embedded—control systems engineering–have developed only since World War II. When our first edition was published, in 1967, the subject of linear continuous-time (or *analog*) control systems had already attained a high level of maturity, and it was (and remains) often designated *classical control* by the *conoscienti*. This was also the early development period for the digital computer and discrete-time data control processes and applications, during which courses and books in "sampled-data" control systems became more prevalent. Computer-controlled and *digital* control systems are now the terminology of choice for control systems that include digital computers or microprocessors.

In this second edition, as in the first, we present a concise, yet quite comprehensive, treatment of the fundamentals of feedback and control system theory and applications, for engineers, physical, biological and behavioral scientists, economists, mathematicians and students of these disciplines. Knowledge of basic calculus, and some physics are the only prerequisites. The necessary mathematical tools beyond calculus, and the physical and nonphysical principles and models used in applications, are developed throughout the text and in the numerous solved problems.

We have modernized the material in several significant ways in this new edition. We have first of all included discrete-time (digital) data signals, elements and control systems throughout the book, primarily in conjunction with treatments of their continuous-time (analog) counterparts, rather than in separate chapters or sections. In contrast, these subjects have for the most part been maintained pedagogically distinct in most other textbooks. Wherever possible, we have integrated these subjects, at the introductory level, in a *unified* exposition of continuous-time and discrete-time control system concepts. The emphasis remains on continuous-time and linear control systems, particularly in the solved problems, but we believe our approach takes much of the mystique out of the methodologic differences between the analog and digital control system worlds. In addition, we have updated and modernized the nomenclature, introduced state variable representations (models) and used them in a strengthened chapter introducing nonlinear control systems, as well as in a substantially modernized chapter introducing advanced control systems concepts. We have also solved numerous analog and digital control system analysis and design problems using special purpose computer software, illustrating the power and facility of these new tools.

The book is designed for use as a text in a formal course, as a supplement to other textbooks, as a reference or as a self-study manual. The quite comprehensive index and highly structured format should facilitate use by any type of readership. Each new topic is introduced either by section or by chapter, and each chapter concludes with numerous solved problems consisting of extensions and proofs of the theory, and applications from various fields.

Los Angeles, Irvine and
Redondo Beach, California
March, 1990

JOSEPH J. DISTEFANO, III
ALLEN R. STUBBERUD
IVAN J. WILLIAMS

Contents

CONTENTS

Chapter 1

Introduction

1.1 CONTROL SYSTEMS: WHAT THEY ARE

In modern usage the word *system* has many meanings. So let us begin by defining what we mean when we use this word in this book, first abstractly then slightly more specifically in relation to scientific literature.

Definition 1.1a: A **system** is an arrangement, set, or collection of things connected or related in such a manner as to form an entirety or whole.

Definition 1.1b: A **system** is an arrangement of physical components connected or related in such a manner as to form and/or act as an entire unit.

The word **control** is usually taken to mean *regulate*, *direct*, or *command*. Combining the above definitions, we have

Definition 1.2: A **control system** is an arrangement of physical components connected or related in such a manner as to command, direct, or regulate itself or another system.

In the most abstract sense it is possible to consider every physical object a control system. Everything alters its environment in some manner, if not actively then passively—like a mirror *directing* a beam of light shining on it at some acute angle. The mirror (Fig. 1-1) may be considered an elementary control system, controlling the beam of light according to the simple equation "the angle of reflection α equals the angle of incidence α."

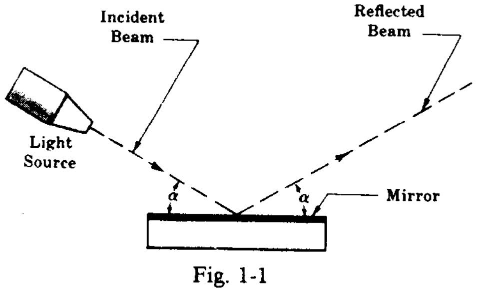

Fig. 1-1

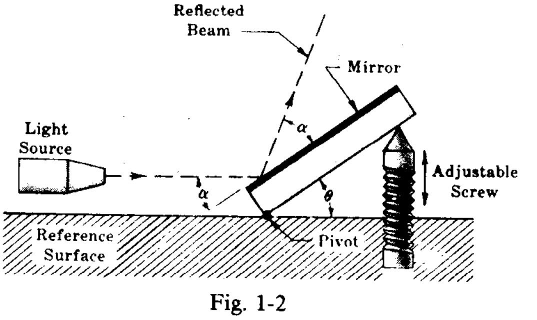

Fig. 1-2

In engineering and science we usually restrict the meaning of control systems to apply to those systems whose major function is to *dynamically* or *actively* command, direct, or regulate. The system shown in Fig. 1-2, consisting of a mirror pivoted at one end and adjusted up and down with a screw at the other end, is properly termed a *control system*. The angle of reflected light is regulated by means of the screw.

It is important to note, however, that control systems of interest for analysis or design purposes include not only those manufactured by humans, but those that normally exist in nature, and control systems with both manufactured and natural components.

1.2 EXAMPLES OF CONTROL SYSTEMS

Control systems abound in our environment. But before exemplifying this, we define two terms: *input* and *output*, which help in identifying, delineating, or defining a control system.

Definition 1.3: The **input** is the stimulus, excitation or command applied *to* a control system, typically from an external energy source, usually in order to produce a specified response *from* the control system.

Definition 1.4: The **output** is the actual response obtained from a control system. It may or may not be equal to the specified response implied by the input.

Inputs and outputs can have many different forms. Inputs, for example, may be physical variables, or more abstract quantities such as *reference*, *setpoint*, or *desired* values for the output of the control system.

The purpose of the control system usually identifies or defines the output and input. If the output and input are given, it is possible to identify, delineate, or define the nature of the system components.

Control systems may have more than one input or output. Often all inputs and outputs are well defined by the system description. But sometimes they are not. For example, an atmospheric electrical storm may intermittently interfere with radio reception, producing an unwanted output from a loudspeaker in the form of static. This "noise" output is part of the total output as defined above, but for the purpose of simply identifying a system, spurious inputs producing undesirable outputs are not normally considered as inputs and outputs in the system description. However, it is usually necessary to carefully consider these extra inputs and outputs when the system is examined in detail.

The terms input and output also may be used in the description of any type of system, whether or not it is a control system, and a control system may be part of a larger system, in which case it is called a **subsystem** or **control subsystem**, and its inputs and outputs may then be internal variables of the larger system.

EXAMPLE 1.1. An *electric switch* is a manufactured control system, controlling the flow of electricity. By definition, the apparatus or person flipping the switch is not a part of this control system.

Flipping the switch on or off may be considered as the input. That is, the input can be in one of two states, on or off. The output is the flow or nonflow (two states) of electricity.

The electric switch is one of the most rudimentary control systems.

EXAMPLE 1.2. A *thermostatically controlled heater or furnace automatically regulating the temperature of a room or enclosure* is a control system. The input to this system is a reference temperature, usually specified by appropriately setting a thermostat. The output is the actual temperature of the room or enclosure.

When the thermostat detects that the output is less than the input, the furnace provides heat until the temperature of the enclosure becomes equal to the reference input. Then the furnace is automatically turned off. When the temperature falls somewhat below the reference temperature, the furnace is turned on again.

EXAMPLE 1.3. The seemingly simple act of *pointing at an object with a finger* requires a biological control system consisting chiefly of the eyes, the arm, hand and finger, and the brain. The input is the precise direction of the object (moving or not) with respect to some reference, and the output is the actual pointed direction with respect to the same reference.

EXAMPLE 1.4. A part of the human temperature control system is the *perspiration system*. When the temperature of the air exterior to the skin becomes too high the sweat glands secrete heavily, inducing cooling of the skin by evaporation. Secretions are reduced when the desired cooling effect is achieved, or when the air temperature falls sufficiently.

The input to this system may be "normal" or comfortable skin temperature, a "setpoint," or the air temperature, a physical variable. The output is the actual skin temperature.

EXAMPLE 1.5. The control system consisting of *a person driving an automobile* has components which are clearly both manufactured and biological. The driver wants to keep the automobile in the appropriate lane of the roadway. He or she accomplishes this by constantly watching the direction of the automobile with respect to the direction of the road. In this case, the direction or heading of the road, represented by the painted guide line or lines on either side of the lane may be considered as the input. The heading of the automobile is the output of the system. The driver controls this output by constantly measuring it with his or her eyes and brain, and correcting it with his or her hands on the steering wheel. The major components of this control system are the driver's hands, eyes and brain, and the vehicle.

1.3 OPEN-LOOP AND CLOSED-LOOP CONTROL SYSTEMS

Control systems are classified into two general categories: *open-loop* and *closed-loop* systems. The distinction is determined by the **control action**, that quantity responsible for activating the system to produce the output.

The term *control action* is classical in the control systems literature, but the word *action* in this expression does not always *directly* imply change, motion, or activity. For example, the control action in a system designed to have an object hit a target is usually the *distance* between the object and the target. Distance, as such, is not an action, but action (motion) is implied here, because the goal of such a control system is to reduce this distance to zero.

Definition 1.5: An **open-loop** control system is one in which the control action is independent of the output.

Definition 1.6: A **closed-loop** control system is one in which the control action is somehow dependent on the output.

Two outstanding features of open-loop control systems are:

1. Their ability to perform accurately is determined by their calibration. To **calibrate** means to establish or reestablish the input-output relation to obtain a desired system accuracy.
2. They are not usually troubled with problems of *instability*, a concept to be subsequently discussed in detail.

Closed-loop control systems are more commonly called *feedback* control systems, and are considered in more detail beginning in the next section.

To classify a control system as open-loop or closed-loop, we must distinguish clearly the components of the system from components that interact with but are not part of the system. For example, the driver in Example 1.5 was defined as part of that control system, but a human operator may or may not be a component of a system.

EXAMPLE 1.6. Most *automatic toasters* are open-loop systems because they are controlled by a timer. The time required to make "good toast" must be estimated by the user, who is not part of the system. Control over the quality of toast (the output) is removed once the time, which is both the input and the control action, has been set. The time is typically set by means of a calibrated dial or switch.

EXAMPLE 1.7. An *autopilot mechanism and the airplane it controls* is a closed-loop (feedback) control system. Its purpose is to maintain a specified airplane heading, despite atmospheric changes. It performs this task by continuously measuring the actual airplane heading, and automatically adjusting the airplane control surfaces (rudder, ailerons, etc.) so as to bring the actual airplane heading into correspondence with the specified heading. The human pilot or operator who presets the autopilot is not part of the control system.

1.4 FEEDBACK

Feedback is that characteristic of closed-loop control systems which distinguishes them from open-loop systems.

Definition 1.7: **Feedback** is that property of a closed-loop system which permits the output (or some other controlled variable) to be compared with the input to the system (or an input to some other internally situated component or subsystem) so that the appropriate control action may be formed as some function of the output and input.

More generally, feedback is said to exist in a system when a *closed* sequence of cause-and-effect relations exists between system variables.

EXAMPLE 1.8. The concept of feedback is clearly illustrated by the autopilot mechanism of Example 1.7. The input is the specified heading, which may be set on a dial or other instrument of the airplane control panel, and the output is the actual heading, as determined by automatic navigation instruments. A comparison device continuously monitors the input and output. When the two are in correspondence, control action is not required. When a difference exists between the input and output, the comparison device delivers a control action signal to the controller, the autopilot mechanism. The controller provides the appropriate signals to the control surfaces of the airplane to reduce the input-output difference. Feedback may be effected by mechanical or electrical connections from the navigation instruments, measuring the heading, to the comparison device. In practice, the comparison device may be integrated within the autopilot mechanism.

1.5 CHARACTERISTICS OF FEEDBACK

The presence of feedback typically imparts the following properties to a system.

1. Increased accuracy. For example, the ability to faithfully reproduce the input. This property is illustrated throughout the text.
2. Tendency toward oscillation or instability. This all-important characteristic is considered in detail in Chapters 5 and 9 through 19.
3. Reduced sensitivity of the ratio of output to input to variations in system parameters and other characteristics (Chapter 9).
4. Reduced effects of nonlinearities (Chapters 3 and 19).
5. Reduced effects of external disturbances or noise (Chapters 7, 9, and 10).
6. Increased bandwidth. The **bandwidth** of a system is a frequency response measure of how well the system responds to (or filters) variations (or frequencies) in the input signal (Chapters 6, 10, 12, and 15 through 18).

1.6 ANALOG AND DIGITAL CONTROL SYSTEMS

The signals in a control system, for example, the input and the output waveforms, are typically functions of some independent variable, usually time, denoted t.

Definition 1.8: A signal dependent on a continuum of values of the independent variable t is called a **continuous-time** signal or, more generally, a **continuous-data** signal or (less frequently) an **analog** signal.

Definition 1.9: A signal defined at, or of interest at, only discrete (distinct) instants of the independent variable t (upon which it depends) is called a **discrete-time**, a **discrete-data**, a **sampled-data**, or a **digital** signal.

We remark that *digital* is a somewhat more specialized term, particularly in other contexts. We use it as a synonym here because it is the convention in the control systems literature.

EXAMPLE 1.9. The continuous, sinusoidally varying voltage $v(t)$ or alternating current $i(t)$ available from an ordinary household electrical receptable is a continuous-time (analog) signal, because it is defined at *each and every instant* of time t electrical power is available from that outlet.

EXAMPLE 1.10. If a lamp is connected to the receptacle in Example 1.9, and it is switched on and then immediately off every minute, the light from the lamp is a discrete-time signal, on only for an instant every minute.

EXAMPLE 1.11. The mean temperature T in a room at precisely 8 A.M. (08 hours) each day is a discrete-time signal. This signal may be denoted in several ways, depending on the application; for example $T(8)$ for the temperature at 8 o'clock—rather than another time; $T(1), T(2), \ldots$ for the temperature at 8 o'clock on day 1, day 2, etc., or, equivalently, using a subscript notation, T_1, T_2, etc. Note that these discrete-time signals are *sampled* values of a continuous-time signal, the mean temperature of the room at all times, denoted $T(t)$.

EXAMPLE 1.12. The signals inside digital computers and microprocessors are inherently discrete-time, or discrete-data, or digital (or digitally coded) signals. At their most basic level, they are typically in the form of sequences of voltages, currents, light intensities, or other physical variables, at either of two constant levels, for example, ± 15 V; light-on, light-off; etc. These *binary signals* are usually represented in alphanumeric form (numbers, letters, or other characters) at the inputs and outputs of such digital devices. On the other hand, the signals of analog computers and other analog devices are continuous-time.

Control systems can be classified according to the types of signals they process: continuous-time (analog), discrete-time (digital), or a combination of both (hybrid).

Definition 1.10: **Continuous-time control systems**, also called **continuous-data control systems**, or **analog control systems**, contain or process only continuous-time (analog) signals and components.

Definition 1.11: **Discrete-time control systems**, also called **discrete-data control systems**, or **sampled-data control systems**, have discrete-time signals or components at one or more points in the system.

We note that discrete-time control systems can have continuous-time as well as discrete-time signals; that is, they can be hybrid. The distinguishing factor is that a discrete-time or digital control system *must* include at least one discrete-data signal. Also, digital control systems, particularly of sampled-data type, often have both open-loop and closed-loop modes of operation.

EXAMPLE 1.13. A target tracking and following system, such as the one described in Example 1.3 (tracking and pointing at an object with a finger), is usually considered an analog or continuous-time control system, because the distance between the "tracker" (finger) and the target is a continuous function of time, and the objective of such a control system is to *continuously* follow the target. The system consisting of a person driving an automobile (Example 1.5) falls in the same category. Strictly speaking, however, tracking systems, both natural and manufactured, can have digital signals or components. For example, control signals from the brain are often treated as "pulsed" or discrete-time data in more detailed models which include the brain, and digital computers or microprocessors have replaced many analog components in vehicle control systems and tracking mechanisms.

EXAMPLE 1.14. A closer look at the thermostatically controlled heating system of Example 1.2 indicates that it is actually a sampled-data control system, with both digital and analog components and signals. If the desired room temperature is, say, 68°F (22°C) on the thermostat and the room temperature falls below, say, 66°F, the thermostat switching system closes the circuit to the furnace (an analog device), turning it on until the temperature of the room reaches, say, 70°F. Then the switching system automatically turns the furnace off until the room temperature again falls below 66°F. This control system is actually operating open-loop between switching instants, when the thermostat turns the furnace on or off, but overall operation is considered closed-loop. The thermostat receives a continuous-time signal at its input, the actual room temperature, and it delivers a discrete-time (binary) switching signal at its output, turning the furnace on or off. Actual room temperature thus varies continuously between 66° and 70°F, and *mean* temperature is controlled at about 68°F, the *setpoint* of the thermostat.

The terms discrete-time and discrete-data, sampled-data, and continuous-time and continuous-data are often abbreviated as *discrete*, *sampled*, and *continuous* in the remainder of the book, wherever the meaning is unambiguous. *Digital* or *analog* is also used in place of discrete (sampled) or continuous where appropriate and when the meaning is clear from the context.

1.7 THE CONTROL SYSTEMS ENGINEERING PROBLEM

Control systems engineering consists of *analysis* and *design* of control systems configurations.

Analysis is the investigation of the properties of an existing system. The **design** problem is the choice and arrangement of system components to perform a specific task.

Two methods exist for design:

1. Design by analysis
2. Design by synthesis

Design by analysis is accomplished by modifying the characteristics of an existing or standard system configuration, and **design by synthesis**, by defining the form of the system directly from its specifications.

1.8 CONTROL SYSTEM MODELS OR REPRESENTATIONS

To solve a control systems problem, we must put the specifications or description of the system configuration and its components into a form amenable to analysis or design.

Three basic representations (models) of components and systems are used extensively in the study of control systems:

1. Mathematical models, in the form of differential equations, difference equations, and/or other mathematical relations, for example, Laplace- and z-transforms
2. Block diagrams
3. Signal flow graphs

Mathematical models of control systems are developed in Chapters 3 and 4. Block diagrams and signal flow graphs are shorthand, graphical representations of either the schematic diagram of a system, or the set of mathematical equations characterizing its parts. Block diagrams are considered in detail in Chapters 2 and 7, and signal flow graphs in Chapter 8.

Mathematical models are needed when quantitative relationships are required, for example, to represent the detailed behavior of the output of a feedback system to a given input. Development of mathematical models is usually based on principles from the physical, biological, social, or information sciences, depending on the control system application area, and the complexity of such models varies widely. One class of models, commonly called *linear systems*, has found very broad application in control system science. Techniques for solving linear system models are well established and documented in the literature of applied mathematics and engineering, and the major focus of this book is linear feedback control systems, their analysis and their design. Continuous-time (continuous, analog) systems are emphasized, but discrete-time (discrete, digital) systems techniques are also developed throughout the text, in a unifying but not exhaustive manner. Techniques for analysis and design of *nonlinear* control systems are the subject of Chapter 19, by way of introduction to this more complex subject.

In order to communicate with as many readers as possible, the material in this book is developed from basic principles in the sciences and applied mathematics, and specific applications in various engineering and other disciplines are presented in the examples and in the solved problems at the end of each chapter.

Solved Problems

INPUT AND OUTPUT

1.1. Identify the input and output for the pivoted, adjustable mirror of Fig. 1-2.

The input is the angle of inclination of the mirror θ, varied by turning the screw. The output is the angular position of the reflected beam $\theta + \alpha$ from the reference surface.

1.2. Identify a possible input and a possible output for a rotational generator of electricity.

The input may be the rotational speed of the prime mover (e.g., a steam turbine), in revolutions per minute. Assuming the generator has no load attached to its output terminals, the output may be the induced voltage at the output terminals.

Alternatively, the input can be expressed as angular momentum of the prime mover shaft, and the output in units of electrical power (watts) with a load attached to the generator.

1.3. Identify the input and output for an automatic washing machine.

Many washing machines operate in the following manner. After the clothes have been put into the machine, the soap or detergent, bleach, and water are entered in the proper amounts. The wash and spin cycle-time is then set on a timer and the washer is energized. When the cycle is completed, the machine shuts itself off.

If the proper amounts of detergent, bleach, and water, and the appropriate temperature of the water are predetermined or specified by the machine manufacturer, or automatically entered by the machine itself, then the input is the time (in minutes) for the wash and spin cycle. The timer is usually set by a human operator.

The output of a washing machine is more difficult to identify. Let us define *clean* as the absence of foreign substances from the items to be washed. Then we can identify the output as the percentage of cleanliness. At the start of a cycle the output is less than 100%, and at the end of a cycle the output is ideally equal to 100% (*clean* clothes are not always obtained).

For most coin-operated machines the cycle-time is preset, and the machine begins operating when the coin is entered. In this case, the percentage of cleanliness can be controlled by adjusting the amounts of detergent, bleach, water, and the temperature of the water. We may consider all of these quantities as inputs.

Other combinations of inputs and outputs are also possible.

1.4. Identify the organ-system components, and the input and output, and describe the operation of the biological control system consisting of a human being reaching for an object.

The basic components of this intentionally oversimplified control system description are the brain, arm and hand, and eyes.

The brain sends the required nervous system signal to the arm and hand to reach for the object. This signal is amplified in the muscles of the arm and hand, which serve as power actuators for the system. The eyes are employed as a sensing device, continuously "feeding back" the position of the hand to the brain.

Hand position is the output for the system. The input is object position.

The objective of the control system is to reduce the distance between hand position and object position to zero. Figure 1-3 is a schematic diagram. The dashed lines and arrows represent the direction of information flow.

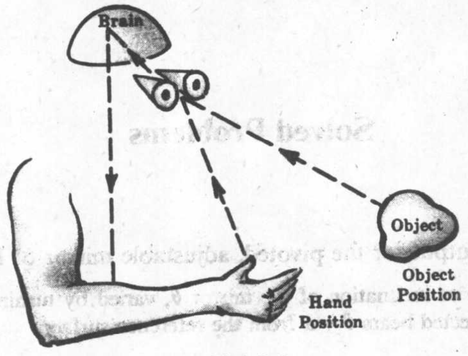

Fig. 1-3

OPEN-LOOP AND CLOSED-LOOP SYSTEMS

1.5. Explain how a closed-loop automatic washing machine might operate.

Assume all quantities described as possible inputs in Problem 1.3, namely cycle-time, water volume, water temperature, amount of detergent, and amount of bleach, can be adjusted by devices such as valves and heaters.

A closed-loop automatic washer might continuously or periodically measure the percentage of cleanliness (output) of the items being washing, adjust the input quantities accordingly, and turn itself off when 100% cleanliness has been achieved.

1.6. How are the following open-loop systems calibrated: (*a*) automatic washing machine, (*b*) automatic toaster, (*c*) voltmeter?

(*a*) Automatic washing machines are calibrated by estimating any combination of the following input quantities: (1) amount of detergent, (2) amount of bleach or other additives, (3) amount of water, (4) temperature of the water, (5) cycle-time.

On some washing machines one or more of these inputs is (are) predetermined. The remaining quantities must be estimated by the user and depend upon factors such as degree of hardness of the water, type of detergent, and type or strength of the bleach or other additives. Once this calibration has been determined for a specific type of wash (e.g., all white clothes, very dirty clothes), it does not normally have to be redetermined during the lifetime of the machine. If the machine breaks down and replacement parts are installed, recalibration may be necessary.

(*b*) Although the timer dial for most automatic toasters is calibrated by the manufacturer (e.g., light-medium-dark), the amount of heat produced by the heating element may vary over a wide range. In addition, the efficiency of the heating element normally deteriorates with age. Hence the amount of time required for "good toast" must be estimated by the user, and this setting usually must be periodically readjusted. At first, the toast is usually too light or too dark. After several successively different estimates, the required toasting time for a desired quality of toast is obtained.

(*c*) In general, a voltmeter is calibrated by comparing it with a known-voltage standard source, and appropriately marking the reading scale at specified intervals.

1.7. Identify the control action in the systems of Problems 1.1, 1.2, and 1.4.

For the mirror system of Problem 1.1 the control action is equal to the input, that is, the angle of inclination of the mirror θ. For the generator of Problem 1.2 the control action is equal to the input, the rotational speed or angular momentum of the prime mover shaft. The control action of the human reaching system of Problem 1.4 is equal to the distance between hand and object position.

1.8. Which of the control systems in Problems 1.1, 1.2, and 1.4 are open-loop? Closed-loop?

Since the control action is equal to the input for the systems of Problems 1.1 and 1.2, no feedback exists and the systems are open-loop. The human reaching system of Problem 1.4 is closed-loop because the control action is dependent upon the output, hand position.

1.9. Identify the control action in Examples 1.1 through 1.5.

The control action for the electric switch of Example 1.1 is equal to the input, the on or off command. The control action for the heating system of Example 1.2 is equal to the difference between the reference and actual room temperatures. For the finger pointing system of Example 1.3, the control action is equal to the difference between the actual and pointed direction of the object. The perspiration system of Example 1.4 has its control action equal to the difference between the "normal" and actual skin surface temperature. The difference between the direction of the road and the heading of the automobile is the control action for the human driver and automobile system of Example 1.5.

1.10. Which of the control systems in Examples 1.1 through 1.5 are open-loop? Closed-loop?

The electric switch of Example 1.1 is open-loop because the control action is equal to the input, and therefore independent of the output. For the remaining Examples 1.2 through 1.5 the control action is clearly a function of the output. Hence they are closed-loop systems.

FEEDBACK

1.11. Consider the voltage divider network of Fig. 1-4. The output is v_2 and the input is v_1.

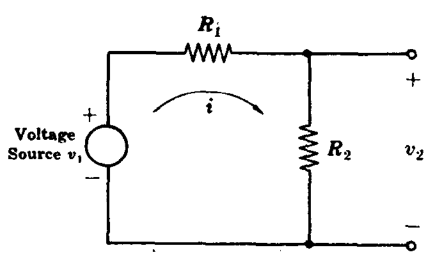

Fig. 1-4

(*a*) Write an equation for v_2 as a function of v_1, R_1, and R_2. That is, write an equation for v_2 which yields an open-loop system.

(*b*) Write an equation for v_2 in closed-loop form, that is, v_2 as a function of v_1, v_2, R_1, and R_2.

This problem illustrates how a passive network can be characterized as either an open-loop or a closed-loop system.

(*a*) From Ohm's law and Kirchhoff's voltage and current laws we have

$$v_2 = R_2 i \qquad i = \frac{v_1}{R_1 + R_2}$$

Therefore

$$v_2 = \left(\frac{R_2}{R_1 + R_2}\right) v_1 = f(v_1, R_1, R_2)$$

(*b*) Writing the current i in a slightly different form, we have $i = (v_1 - v_2)/R_1$. Hence

$$v_2 = R_2\left(\frac{v_1 - v_2}{R_1}\right) = \left(\frac{R_2}{R_1}\right) v_1 - \left(\frac{R_2}{R_1}\right) v_2 = f(v_1, v_2, R_1, R_2)$$