

# LASER PHYSICS

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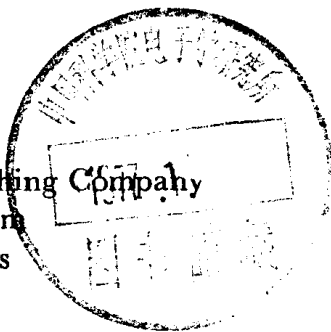
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## PREFACE

This book treats the interaction of radiation with matter, particular attention being paid to the laser. Knowledge is assumed of the usual half-year introduction to quantum mechanics found in undergraduate physics curricula. The material can be covered in two semesters, or, alternatively, the first part (Chaps. 1-13) can be used as a one-semester course in which quantum-mechanical aspects of the electromagnetic field are ignored. Each chapter is accompanied by problems that illustrate the text and give useful (occasionally new) results.

Existing laser media are intrinsically quantum mechanical and are most easily studied with the quantum theory. Understanding the laser along these lines enlivens one's understanding of quantum mechanics itself. In fact, the material constitutes a viable, applied alternative for the usual second and third semesters of quantum mechanics.

The text format lends itself quite well to reference use, particularly with regard to the fundamental concepts of laser physics. There is leeway for some detailed extensions, notably in the problems, but we have deliberately sacrificed generality for the sake of clarity. An understanding of the simpler theories enables one to work with the more general extensions available in the original literature or to treat new problems oneself.

With the exception of Chaps. 19 and 20, the laser theory discussed in this book tends to follow the approaches of the Lamb school. Parallel work of the Bell Telephone Laboratories group has been presented in the book by W. H. Louisell, *Quantum Statistical Properties of Radiation*, (John Wiley & Sons, New York, 1973), while H. Haken's contribution, "Laser Theory," in *Encyclopedia of Physics*, Vol. XXV/2c, edited by S. Flügge, Springer-Verlag, Berlin, 1970; also Chap. 23 in *Laser Handbook*, gives a very complete account of the Stuttgart work.

In keeping with the text format, no uniform attempt is made to assign credit to the original papers. Rather, we have referred the reader to review

articles and books which he might find useful for further study. Even in this capacity, we do not give all possible references; A. E. Siegman's list of laser books (*Appl. Opt.*, **18**, A38, 1971) includes 135 entries alone and is 3 years older than this book.

The first 13 chapters use the semiclassical theory of the interaction of radiation with matter. For this, the atoms are assumed to obey the laws of quantum mechanics and the field is governed by classical Maxwell equations. Chapters 14 through 20 treat both field and atoms quantum mechanically. More specifically, Chaps. 1 and 2 present basic quantum preliminaries. Chapter 3 discusses classical and quantum dipole moments and their interaction with electric fields. Chapter 4 traces historical analogs of some laser phenomena from Huygens to Van der Pol. Chapter 5 applies the theory developed to the first maser, introducing important laser concepts like cavity  $Q$ , saturated gain, and cavity tuning. Chapter 6 reviews quantum formalism especially useful for Chaps. 14 through 20 (Dirac notation, Schrödinger, interaction, and Heisenberg pictures). Chapter 7 introduces the density matrix, which is used to considerable analytical and pedagogical advantage in subsequent chapters.

Chapters 8 through 11 develop the theory of the laser with single- and multi-mode electromagnetic fields, with homogeneously and Doppler broadened media, and with two mirror and ring cavities. The atoms are approximated by two level systems, and the field is taken to be scalar. In Chap. 12 these restrictions are removed, and an applied DC magnetic field is considered. Chapter 13 treats pulse propagation, self-induced transparency, and photon echo in terms of the semiclassical theory.

Chapter 14 presents the quantum theory of radiation in a form suitable for treatment of the laser. The electromagnetic field is shown to consist of simple harmonic oscillators with straightforward quantization rules. The Weisskopf-Wigner theory of spontaneous emission is given as an important example. An accompanying appendix deals with the phenomenon of superradiance. Chapter 15 develops the coherent state—the state most closely approximating the classical electromagnetic field. The state provides not only a quantum-classical bridge, but also an instructive formalism for the quantum laser. Chapter 16 defines the system-reservoir problem with the use of the reduced density operator. This problem is fundamental to all physical situations involving the interaction of a system of interest with an environment having many degrees of freedom. The techniques of Chaps. 14 through 16 are used in Chap. 17 to treat the laser from a fully quantum-mechanical point of view, yielding the semiclassical theory in the appropriate limit and providing information about photon statistics, laser linewidth, and buildup from noise. Chapter 18 discusses the theory of measurement as applied to laser problems. Chapter 19 revisits the system-reservoir problem from a Brownian motion viewpoint, which is based on the Heisenberg picture of quantum mechanics. This ap-

proach is employed in Chap. 20 to treat the laser. These two chapters are patterned after the work of Lax with suitable pedagogical modifications. Chapter 21 provides an overview of laser physics, relating it to other subjects in the field loosely called "quantum optics," including Josephson radiation. In this connection, the approaches developed in the book are eminently applicable to laser problems; they are, furthermore, general and hence can often be applied elsewhere. It is hoped that the reader can obtain from the book not only an increased understanding of the laser, but also a more profound comprehension of many-system physical phenomena in general.

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## LASER THEORY NOTATION

$a$	annihilation operator (boson)	(14.11)
$a^\dagger$	creation operator (boson)	(14.12)
$a$	as subscript, refers to the upper level $a$ of a two-level atom	(2.14)
$a', a''$	magnetic quantum numbers for level $a$	(Chap. 12)
$a_0$	Bohr radius ( $0.53 \text{ \AA}$ )	(1.27)
$a_s$	annihilation operator for mode $s$ (boson)	(14.41)
$a_\mu(t)$	general Heisenberg picture operator	(19.52)
$\{a\}$	set of quantum operators $a_1, \dots, a_\mu, \dots$ (not necessarily annihilation operators)	(19.52)
$ a\rangle$	energy eigenstate for level $a$	(6.46)
$A$	Einstein $A$ coefficient	(2.38)
$\mathbf{A}$	vector potential	(2.7)
$A(t)$	slowly varying annihilation operator [ $a(t) = A(t) \exp(i\Omega t)$ ]	(19.33)
$A_\mu(t)$	slowly varying operator for $a_\mu$ [ $a_\mu(t) = A_\mu(t) \exp(-i\Omega_\mu t)$ ]	(19.58)
$\mathcal{A}$	linear gain constant in quantum theory of laser	(17.13)
$\mathcal{A}_b$	active medium linear absorption constant	(17.58)
$\mathcal{A}_c$	complex linear gain coefficient in quantum theory of laser	(20.49)
$b$	as subscript, refers to the lower level $b$ of two levels	(2.14)
$b_k(t)$	annihilation operator for mode $k$	(19.24)
$ b\rangle$	energy eigenstate of lower of two levels	(6.46)
$B$	Einstein $B$ coefficient	(2.38)
$\mathbf{B}$	magnetic field	(8.2)
$\mathcal{B}$	lowest-order saturation coefficient in quantum theory of laser	(17.14)
$\mathcal{B}_c$	complex saturation coefficient for $\mathcal{B}$	(20.50)
$\mathcal{B}$	effective magnetic field	(7.77)
$c$	speed of light ( $2.99793 \times 10^8 \text{ m/sec}$ )	
$c_a(t)$	probability amplitude for upper level $a$ (Schrödinger picture)	(6.71)
$c_b(t)$	probability amplitude for lower level $b$ (Schrödinger picture)	(6.72)

$c_n(t)$	probability amplitude for every level $n$ (Schrödinger picture)	(6.67)
$C_a(t),$	as above, but in interaction picture [ $C_x =$	
$C_b(t), C_n(t)$	$c_x \exp[(i\omega_x t)], x = a, b, n$	(2.14), (1.13)
$C_{an}(t)$	probability amplitude for atom in upper level, field with $n$ photons (interaction picture)	(14.68)
$c_{n_1, n_2, \dots, n_r, \dots} \equiv c(n_r)$	probability amplitude for multimode field with $n_1$ photons in mode 1, $n_r$ photons in mode $r$ .	(14.48)
$C_n(t)$	in Sec. 10-3 and Prob. 8-1 only, cosine (in-phase) component of polarization	(8.61)
$\mathcal{C}_{an}(t)$	probability amplitude for atom in upper state, an $n$ -photon laser field and vacuum otherwise (interaction picture)	(I.18)
c.c.	abbreviation for "complex conjugate"	
$d$	constant term in mode-locking equation	(9.58)
$\mathbf{D}$	displacement vector in Maxwell's equations	(8.2)
$D_{vv}$	diffusion coefficient for Brownian motion	(19.6)
$D(\varphi)$	phase diffusion coefficient in quantum theory laser	(20.74)
$D(a)$	displacement operator for coherent states	(15.61)
$D(z, v, t)$	population difference for two-level systems	(10.58)
$D_\mu$	drift coefficient for operator $A_\mu$	(19.60)
$\langle D_{\mu\nu} \rangle$	diffusion coefficient for operators $A_\mu$ and $A_\nu$	(19.63)
$\mathcal{D}_x(\Delta\omega)$	complex denominator $= 1/(\gamma_x + i \Delta\omega)$	(9.6)
$\mathcal{D}_n$	sums of complex denominators	(E. 12)
$\mathfrak{D}(\omega)$	density of free-field radiation states	(14.101)
$e$	electron charge $= -1.9 \times 10^{-19}\text{C}$	(1.24)
$e^n$	$(e)^n, n$ an integer	
$\hat{e}_1, \hat{e}_2, \hat{e}_3$	unit vectors in abstract Cartesian coordinate system, used for the pictorial representation of the density matrix	(7.70)
$E_0$	scalar electric field amplitude	(2.13)
$E(z, t)$	plane wave electric field	(8.8)
$\mathbf{E}(\mathbf{R}, t)$	electric field vector	(2.6)
$E_n(t)$	slowly varying Fourier amplitude for $E(z, t)$	(8.8)
$\mathcal{E}(z, t)$	complex electric field envelope in pulse propa- gation	(13.1)
$\mathcal{E}(t)$	Positive frequency part of electric field $E(t)$	(16.36)
$\mathcal{E}$	electric field "per photon"	(14.17)
$\bar{E}$	dimensionless electric field amplitude $= \sqrt{I}$	(E. 11)

$f(x)$	arbitrary function of $x$	
$f(t)$	rapidly varying noise operator for annihilation operator $a(t)$	(19.32)
$F(t)$	slowly varying noise operator for $A(t)$ [ $F(t) = f(t) \exp(i\Omega t)$ ]	(19.34)
$F_{\mu}(t)$	slowly varying noise operator for $A_{\mu}(t)$	(19.60)
$F_v(t)$	random classical force on particle in Brownian motion	(19.2)
$F_1$	first-order factor in laser coefficients	(Tables 8-1, 10-1)
$F_3$	third-order factor in laser coefficients	(Tables 8-1, 10-1)
$\mathcal{F}\{f(x)\}$	Fourier transform of $f(x)$	(15.48)
$\mathfrak{F}$	real part of continued fraction in strong signal theory	(E. 32)
$g$	coupling constant in quantum theory of radiation	(14.60)
$g(\omega)$	coupling constant for frequency $\omega$	(14.101)
$g_s$	coupling constant for sth mode of field	(14.88)
$g_a$	Landé $g$ factor for upper level of two-level atom	(12.18)
$g_b$	Landé $g$ factor for lower level of two-level atom	(12.18)
$g_e$	electron $g$ factor	(1.37)
$g_{ij}$	( $i, j = \pm$ ) conductivity matrix elements	(12.7)
$G$	conductivity matrix (in ring and Zeeman laser theory)	(12.7)
$G(x, x_0, t)$	Green's function	(H. 17)
$G^{(n)}(x_1, \dots, x_{2n})$	$n$ -th order correlation function	(15.59)
$h$	Planck's constant = $6.6256 \times 10^{-34}$ joule-sec	
$\hbar$	Planck's constant/ $2\pi$ = $1.027 \times 10^{-34}$ joule-sec	(1.6)
$H$	magnetic field	(1.34)
$H$	magnitude of magnetic field $\mathbf{H}$	(1.34)
$H_y$	$y$ component of magnetic field	(14.7)
$H_n(\xi)$	Hermite polynomial	(1.22)
$\mathcal{H}$	total Hamiltonian	(2.1)
$\mathcal{H}_0$	unperturbed Hamiltonian	(2.1)
$\mathcal{H}_c$	classical energy	(1.24)
$i$	$\sqrt{-1}$	
$I_n$	dimensionless intensity for $n$ th mode = $\frac{1}{2}(\mathcal{E} E_n / \hbar)^2 (\gamma_a \gamma_b)^{-1}$	(8.45)
$\mathcal{I}$	identity operator for matrix	(6.13)

$\text{Im}(\mathcal{I})$	imaginary part of $\mathcal{I}$	
$I(z, t)$	intensity envelope in pulse propagation	(13.28)
$\mathcal{I}(z, t)$	partial energy integral	(13.32)
$\mathbf{J}$	current density in Maxwell's equations	(8.2)
$J_n(\Gamma)$	$n$ th-order Bessel function	(9.101)
$J_a, J_b$	angular momenta for upper and lower levels	(Fig.12-2)
$k_B$	Boltzmann's constant = $1.38054 \times 10^{-23}$ J/°K	
$K$	wave number $\approx K_n$ (when difference between $K_n$ 's does not matter)	(3.38)
$K_n$	wave number for $n$ th mode of electric field	(8.5)
$Ku$	= $K$ times $u$ : Doppler broadening constant	(10.2)
$l, l_c, l_s$	locking parameters	(9.59), (9.54)
$l_x, l_y, l_z$	lengths of $x, y$ , and $z$ dimensions in maser cavity	(5.23)
$L$	mirror separation in laser	(Fig. 8-2a)
$\mathbf{L}$	orbital angular momentum vector	(after (6.41))
$L_z$	$z$ component of orbital angular momentum $\mathbf{L}$	(after (6.41))
$\mathcal{L}_x(\omega - \nu)$	dimensionless Lorentzian = $\gamma_x^2/[\gamma_x^2 + (\omega - \nu)^2]$ ( $x$ can be blank)	(8.36)
$m$	particle mass	(1.8)
$M$	mass constant for radiation field oscillator	(14.6)
$M(z, \nu, t)$	population sum in strong-signal laser theory	(10.59)
$M_n$	$n$ th-order moment in Langevin process	(19.8)
$ n\rangle$	energy eigenstate with eigenvalue $\hbar\omega_n$ (usually photon number state)	(6.28)
$\langle n(t) \rangle$	average number of photons	(16.31)
$n_a$	number of atoms in upper level (Einstein theory)	(2.38)
$n_b$	number of atoms in lower level (Einstein theory)	(2.38)
$\bar{n}_{ss}$	steady-state average number of photons in laser field	(17.33)
$ n_1 n_2 \dots n_r\rangle \equiv  \{n_s\}\rangle$	multimode field eigenstate	(14.46)
$N$	number of modes in multimode laser operation [or $N(z, t)$ , see below]	(sec. 9-4)
$N$	number of atoms in Langevin theory	(20.10)
$\bar{N}$	average population inversion density	(8.42)
$N_{2l}$	$2l$ th component of the population inversion density	(9.16)
$N(z, t)$	population inversion density	(8.39)
$N_a, N_b$	number of atoms in upper and lower states	(20.39)

$\mathcal{X}$	relative excitation	(8.54)
$\mathcal{N}_{nm}, \mathcal{N}_{nm}'$	number factors in quantum theory of the laser	(17.15)
$\mathcal{N}$	population inversion operator	(20.45)
$\mathcal{N}$	normalization constant	(8.22)
$\mathfrak{N}$	set of index values whose corresponding field amplitudes are nonzero	(9.72)
$\mathcal{O}$	arbitrary quantum-mechanical operator	(1.4)
$P_n$	probability of $n$ photons	(15.13)
$\mathbf{p}$	particle momentum	(1.7)
$\mathbf{P}$	polarization vector in Maxwell's equations	(8.2)
$P(z, t)$	scalar polarization of medium	(8.9)
$P(a), P(a, t)$	diagonal coherent state representation of density operator	(15.2)
$P(x, t)$	probability density for a particle at $x$ (time $t$ )	(16.66)
$P(x, y), P(r, \theta)$	two-dimensional probability densities	(16.119), (16.86)
$P_a$	transition probability to state $ a\rangle$	(2.31)
$P_s$	probability of stimulated emission	(2.50)
$P_\psi$	probability of the state vector $ \psi\rangle$ in a mixture	(7.17)
$\mathcal{P}_n(t)$	slowly varying complex polarization for mode $n$	(8.9)
$\mathcal{P}(z, t)$	slowly varying complex polarization in pulse propagation	(13.2)
$\wp$	electric-dipole matrix element between levels $a$ and $b$ (taken real)	(2.16)
$\wp_{a'b'}$	electric-dipole matrix element between levels $a$ and $b'$	(12.23)
$q$	position coordinate	(14.6)
$q_n$	Fourier coefficient of population difference $D(z, \nu, t)$ and in quadrature coefficients $S(z, \nu, t)$	(10.60)
$Q$	cavity quality factor	(5.19)
$Q_n$	cavity quality factor for $n$ th mode	(8.10)
$Q_x, Q_y$	$Q$ 's for $x$ and $y$ polarizations of electric field	(12.16)
$r$	radial coordinate in polar coordinate system	(Fig. 1-4)
$\mathbf{r}$	position vector in polar coordinates	(Fig. 1-4)
$r_a, r_b$	excitation rates to states $ a\rangle$ and $ b\rangle$	(16.1)
$r_n$	complex ratios of atomic Fourier coefficients $(q_n/q_{n-1})$	(E.14)
$r_0$	classical electron radius $\approx 2.8 \times 10^{-15}$ m	(3.22)

$R$	rate constant	(8.35)
$\mathbf{R}$	vector in pictorial representation of density matrix	(7.70)
$R_1, R_2, R_3$	components of $\mathbf{R}$ in abstract Cartesian space	(7.66), (7.67), (7.69)
$R_{nl}(r)$	associated Laguerre polynomials	(1.26)
$R_s$	saturation parameter $= 1/(\gamma_a^{-1} + \gamma_b^{-1})$	(8.38)
$\mathcal{R}_a, \mathcal{R}_b$	single-mode rate coefficients in quantum theory of reservoirs	(16.25) (16.22)
$R(a^*, \beta)$	coherent state representation of density operator	(15.36)
$\text{Re}\{\mathcal{P}\}$	real part of complex quantity $\mathcal{P}$	
$s_n$	constants in third-order complex polarization integrals $T_{lw}$	(D.6)
$S$	similarity transformation (Zeeman theory)	(12.14)
$S(z, v, t)$	in-quadrature component of polarization (strong-signal theory)	(10.56)
$S_n(t)$	in-quadrature component of $n$ th-mode polarization	(8.60)
$\mathcal{S}$	stabilization factor (laser theory)	(8.57)
$t, t', t'', t''', t_1, t_2, t_0, t_n$	various times	
$T$	temperature in degrees Kelvin	(16.1)
$T_1$	level lifetime of atomic system $[= \frac{1}{2}(\gamma_a^{-1} + \gamma_b^{-1})]$	(13.26)
$T_2$	dephasing time of atomic dipoles ( $= 1/\gamma$ )	(13.26)
$T_2^*$	inhomogeneous dephasing time of atomic dipoles	(13.56)
$T_{lw}$	third-order complex polarization integrals	(D.7)
$\mathcal{T}(z)$	total energy crossing $x$ - $y$ plane at position $z$ (pulse propagation)	(13.35)
$u$	average atomic speed in a gas	(10.2)
$u_a(\mathbf{r}), u_b(\mathbf{r})$	energy eigenfunctions for upper and lower levels	(2.14)
$u_n(x), u_n(\mathbf{r})$	energy eigenfunctions for eigenvalue $\hbar\omega_n$	(1.10)
$U_n(z)$	$n$ th-mode function of a cavity (has wave number $K_n$ )	(8.8)
$\mathcal{U}(\omega)$	energy distribution, e.g., for blackbody radiation	(2.31)
$v$	$z$ component of velocity in gas laser (and for Brownian motion)	(10.2)
$V$	volume of a cavity	(5.20)
$V(r)$	atomic potential energy	(2.7)

$\mathcal{V}(t)$	atom-field interaction energy (usually electric dipole)	(2.1)
$W(v)$	dimensionless velocity distribution (usually Maxwellian)	(10.2)
$W(\omega)$	dimensionless frequency distribution for inhomogeneously broadened line	(10.2)
$\tilde{W}(T)$	Fourier transform of $W(\omega)$	(13.25)
$x, y, z$	$x, y$ , and $z$ Cartesian coordinates	(Fig. 1-4)
$\hat{x}, \hat{y}, \hat{z}$	unit vectors for $x, y$ , and $z$ Cartesian axes	(Fig. 1-4)
$X(t)$	slowly varying complex displacement [ $x(t) = \text{Re } X(t) \exp(i\omega t)$ ]	(3.29)
$Y_{lm}(\theta\phi)$	spherical harmonics	(1.26)
$Z(v)$	plasma dispersion function	(10.29)
$\mathfrak{J}$	set of index values whose corresponding field amplitudes vanish	(9.72)
$a$	complex dimensionless field amplitude of coherent state $ a\rangle$ (eigenvalue of annihilation operator $\{a a\rangle = a a\rangle\}$ )	(15.12)
$a$	energy gain coefficient in pulse propagation	(13.30)
$a$	as subscript, means $a$ or $b$ for energy levels of two-level atom	(8.20)
$a$	net gain coefficient in classical sustained oscillator	(4.1)
$a'$	gain parameter	(13.18)
$ a\rangle$	coherent state	(15.12)
$a_n$	net gain coefficient for $n$ th laser mode	(8.50)
$\beta$	as subscript, means $a$ or $b$ for energy levels of two-level atom	(14.90)
$\beta$	self-saturation coefficient in classical sustained oscillator	(4.1)
$ \beta\rangle$	auxiliary coherent state	(15.35)
$\beta_n$	self-saturation coefficient for the $n$ th laser mode	(8.50)
$\gamma$	atomic dipole decay constant ( $=1/T_2 = \gamma_{ab} + \gamma_{ph}$ )	(7.48)
$\gamma_a, \gamma_b$	upper and lower-level decay constants	(2.46) (2.47)

$\gamma_{ab}$	$\frac{1}{2}(\gamma_a + \gamma_b)$ , spontaneous emission and in-elastic collision contribution to decay of atomic dipole	(7.37)
$\gamma_{ph}$	elastic collision contribution to dipole decay	(7.43)
$\Gamma$	modulation depth	(9.104)
$\Gamma$	damping constant in Brownian motion	(19.2)
$\delta(x - x')$	one-dimensional Dirac delta function	
$\delta(a - a_0)$	two-dimensional Dirac delta function	
$\delta(\mathbf{r} - \mathbf{r}')$	three-dimensional Dirac delta function	
$\delta_{ij}$	Kronecker delta function = $\begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$	
$\delta\omega$	small frequency shift	(7.40)
$\delta t$	small time interval	
$\Delta$	intermode beat frequency ( $= \nu_n - \nu_{n-1}$ )	(9.79)
$\Delta\nu$	low-frequency beat note	(9.61)
$\Delta t$	time interval	(19.8)
$\epsilon$	permittivity of medium	(13.3)
$\epsilon_n$	small displacement from stationary intensity $I_n$	(9.73)
$\epsilon_0$	permittivity of vacuum $= (4\pi c^2)^{-1} 10^7 \simeq 8.85 \times 10^{-12}$ F/m	(8.2)
$\hat{\epsilon}_+, \hat{\epsilon}_-$	complex unit vectors for circular polarization of electric field	(12.1)
$ \zeta\rangle,  \xi\rangle$	arbitrary vectors in quantum-mechanical space	(6.16)
$\eta$	index of refraction	(8.18)
$\theta$	polar angle in polar coordinates	(Fig. 1-4)
$\theta_{nm}$	real, cross-saturation coefficients (by mode $E_m$ on $E_n$ )	(9.71)
$\theta(z)$	area under pulse envelop at position $z$	(13.40)
$\vartheta(z, t)$	partial area under pulse envelop (up to time $t$ )	after (13.53)
$\vartheta_{n, \rho\sigma}$	complex, third-order general saturation coefficient	(9.18)
$\Theta$	Multimode stability matrix	(9.41)
$\kappa$	loss coefficient in amplifier theory	(13.3)
$\lambda$	wavelength of light	
$\lambda_n$	wavelength of mode $n$ ( $= 2\pi/K_n$ )	
$\lambda_a, \lambda_b$	numbers of atoms per second-per volume excited to upper and lower levels	(8.25)

$A_a, A_b$	excitation operators for upper and lower states	(20.7)
$\mu$	power-broadened frequency in Rabi flopping	(2.61)
$\mu$	as subscript, indexes field amplitude and phase: $E_\mu(t), \phi_\mu(t)$	(9.18)
$\mu$	magnetic-dipole moment	(1.37)
$\mu_B$	Bohr magneton = $9.27 \times 10^{-24}$ J/°K	(1.37)
$\mu_n$	eigenvalue in Rabi flopping	(2.58)
$\mu_0$	permeability of vacuum = $4\pi \times 10^{-7}$ H/m	(8.2)
$\nu$	laser (optical) oscillation frequency in radians/sec not Hertz (circular frequency)	(8.10)
$\nu_n$	laser frequency for mode $n$	(8.8)
$\nu_M$	modulation frequency	(9.84)
$\nu_0$	average frequency = $\frac{1}{2}(\nu_+ + \nu_-)$	(Table 12-1)
$\nu_\pm$	frequency of $\pm$ circular polarizations in Zeeman laser or right- and left-traveling waves in ring laser	(11.2), (12.1)
$\xi$	dimensionless coordinate for simple harmonic oscillator = $(m\omega/\hbar)^{1/2}x$	(1.22)
$\pi$	3.1415926535897. . . .	
$\rho, \rho(t)$	density matrix or operator	(7.5), (7.17)
$\rho$	mode subscript in laser theory: for example, $E_\rho(t)$	(9.18)
$\rho(z, t)$	population matrix for laser medium	(8.23)
$\rho(z, v, t)$	population matrix for ensemble moving with $z$ component of velocity, $v$	(10.6)
$\rho_{aa}(z, t)$	$a = a, b$ , number of atoms in the $a$ th level	(8.23)
$\rho_{ab}(z, t)$	population matrix element proportional to the complex polarization	(8.29)
$\rho(a, z_0, t_0, v, t)$	pure case density matrix for atom excited to level $a = a$ or $b$ , at time $t_0$ , with $z$ component of velocity $v$ , located at $z$ at time $t$	(10.4)
$\rho_{nm}(t)$	number representation of the density operator $\rho(t)$	(7.19)
$\rho_A(t), \rho_B(t)$	reduced density operators for the system and reservoir	(16.87), (16.88)
$\rho_c(t)$	correlation part of system-reservoir density operator	(16.91)

$\sigma$	spin-flip operator = $\frac{1}{2} (\sigma_x - i\sigma_y)$	(1.41)
$\sigma$	as subscript, indexes field amplitudes and phases, for example, $E_\sigma(t)$	(9.18)
$\vec{\sigma}$	conductivity tensor in Maxwell's equations	(12.3)
$\sigma$	scalar conductivity in Maxwell's equations	(8.2)
$\sigma$	Pauli spin vector	(1.38)
$\sigma_x, \sigma_y, \sigma_z$	components of Pauli spin vector	(1.38)
$\sigma_a(t), \sigma_b(t)$	projection operators for upper and lower states	(20.11)
$\sigma_a^i(t)$	projection operator for $i$ th atom	(20.1)
$\sigma_{k,k}$	density matrix in Josephson radiation theory	(21.24)
$\sigma_n$	linear mode pulling for laser mode $E_n(t)$	(8.52)
$\Sigma(t)$	slowly varying spin-flip operator in Langevin theory	(20.12)
$\tau, \tau', \tau'', \tau'''$	time intervals	(10.47)
$\tau_{nm}$	mode cross pushing coefficients in laser theory	(9.22)
$\tau_c$	correlation time of random functions	after (19.5)
$\tau_p$	pulse duration	(13.26)
$\tau_s$	hyperbolic secant time parameter	(13.46)
$\nu$	complex frequency, for example, $= \gamma + i(\omega - \nu)$	(10.36)
$\nu_{l\omega}$	complex frequency	(Table D-2)
$\phi$	azimuthal angle in polar coordinates	(Fig. 1-4)
$\phi_n(x)$	Hermite-Gaussian functions, eigenfunctions of simple harmonic oscillator	(1.22)
$\phi_n(t)$	slowly varying phase of the $n$ th laser mode	(8.8)
$\chi_n$	complex susceptibility for $n$ th mode	(8.13)
$\chi(z, T, t)$	complex susceptibility in pulse propagation	(13.21)
$\psi(\mathbf{r}, t)$	Schrödinger wave function	(1.1), (6.41)
$ \psi(t)\rangle$	state vector	(6.26)
$\Psi_{n,\mu\rho\sigma}$	third-order relative phase angle	(9.15)
$\Psi$	relative phase angle	(9.44)
$\Psi_1, \Psi_2$	stationary-state values of $\psi$	(9.62), (9.63)
$\Psi_0$	a relative phase angle value	(9.60)
$\omega$	atomic line center frequency in laser media $= \omega_a - \omega_b$	(2.19)
$\omega_0$	atomic line center frequency	(13.56)
$\omega_n$	eigenfrequency of an unperturbed Hamiltonian	(1.9)

$\omega_a, \omega_b$	eigenfrequencies of upper and lower levels ( $a$ and $b$ )	(2.14)
$\omega_{nm}$	frequency difference = $\omega_n - \omega_m$	
$\omega_{a'b'}$	= $\omega_{a'} - \omega_{b'}$	(Fig. 12-2)
$\Omega$	frequency of single-mode radiation	(14.6)
$\Omega_n$	frequency of $n$ th mode (passive cavity or free space)	(8.5)