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Optical Data Processing

Applications

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D. Casasent and H.J. Caulfield Basic Concepts

B.J. Thompson Optical Transforms and Coherent
Processing Systems – With Insights From
Crystallography

P. S. Considine and R. A. Gonsalves Optical Image
Enhancement and Image Restoration

E. N. Leith Synthetic Aperture Radar

N. Balasubramanian Optical Processing in
Photogrammetry

N. Abramson Nondestructive Testing and Metrology

H.J. Caulfield Biomedical Applications of Coherent
Optics

D. Casasent Optical Signal Processing



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With Contributions by

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With 170 Figures

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Preface

This book was written to satisfy a variety of needs. First, it is intended to provide an updated summary of the present status of optical data processing: what it has accomplished, what it can accomplish, and how it accomplishes these things. We feel that this presentation of an updated summary is best achieved by treating different applications of optical data processing. A commonality of fundamental tools and operations is used in all cases. These cases are adequately treated in many textbooks and highlighted here in Chapter 1. Since the objective of optical data processing is to apply fundamental techniques to the solution of specific problems, a discussion of specific applications seems appropriate.

This discussion enables the potential user to see the steps required in deciding: if one should use optical processing for a solution to a specific problem; what application areas have been considered, and why; how one uses the fundamental concepts to configure a candidate system solution; how one must modify these basic concepts, refine them and develop new ones to solve specific features of one problem area.

However, this text is also intended for those researchers presently engaged in various aspects of optical processing. By providing an updated review of where various aspects of our field presently stand, we can best direct our future work in the proper direction. By pausing to organize diverse papers and reports and to seek fresh perspectives, we discover what gaps exist in our present understanding, and we can exercise wise choices in deciding which areas to pursue next. Without such a perspective, there is a great danger of overspecialization. By exposing researchers in one area of optical processing to work in other application areas, we often find that techniques and solutions appropriate for one area can be applied to problems in other areas. Quite often, solutions that have been considered for use in one area and found not to be appropriate are quite applicable for use in solving problems in other areas.

For the sake of completeness we include an introductory chapter in which these fundamental concepts and the common concepts of many of the following chapters are highlighted. The specific application areas we select are: crystallography, image enhancement and restoration, synthetic aperture radar, photogrammetry, holographic interferometry and non-destructive testing, biomedical applications and signal processing. A more detailed discussion and review of each of these chapters can be found at the end of Chapter 1.

We thus hope that this text will be of use to those engineers engaged in any form of data processing, to those contemplating the use of optical processing,

and to those researchers who are presently involved in one aspect of optical processing. The material is presented in a form which should make it useful for reference, for individual study, and for short courses. If need be, it and Chapter 1 (as well as the rest of the material) can be supplemented by a basic text or by a companion Topics Appl. Phys. volume (Optical Data Processing, Fundamentals, ed. by S.Lee) to provide more detail on the fundamental principles and thus, with the two books, a text for a graduate level course can be assembled.

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November, 1977

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1. Basic Concepts

D. Casasent and H. J. Caulfield

With 6 Figures

1.1 Introduction

Coherent optics, optical spatial filtering, optical Fourier transformations, lasers, and holography are among the key items which are spawning a revolution in our concepts of optics. Once the lonely domain of lens designers and astronomers, optics has become a vital field affecting all areas of basic research (from quantum theory to medicine) and all areas of applied technology (from nuclear fusion to telephones). This book is a progress report on the underlying field of coherent optical signal and image processing. In this chapter we review the key concepts just mentioned as an aid to understanding subsequent chapters. With this review and the more specialized chapters which follow, the reader will have a self-contained survey of the entire field. Readers already familiar with these basic concepts can proceed immediately to the specialized chapters without fear of missing new concepts.

This brief survey of basic concepts proceeds from the most basic (coherent light) to the most technical (holography and correlation). Numerous books treat one or more of these areas in detail. An annotated bibliography of some of these is appended to this introduction.

1.2 Coherence

"Coherence" is a property of two or more beams of light, and thus is more properly called "mutual coherence". Mutually coherent overlapping beams form a spatial pattern of light which differs from the sum of the patterns of each individual beam. This spatial pattern is called an "interference pattern". When a beam of coherent light encounters a physical aperture, the interference pattern formed by the rays of light emerging from various parts of the aperture is called a "diffraction pattern". In the usual interferometer (any device used to generate an interference pattern) a beam of light from a single source is divided into two parts, an optical path length difference is introduced between the two beams, and the two beams are recombined to form an interference pattern. Analysis of this system is easy. If the two paths from the source point merge at a given point, the interference is constructive and this region of the output plane is bright. When the path difference is a half wavelength plus or minus an integral (including zero)

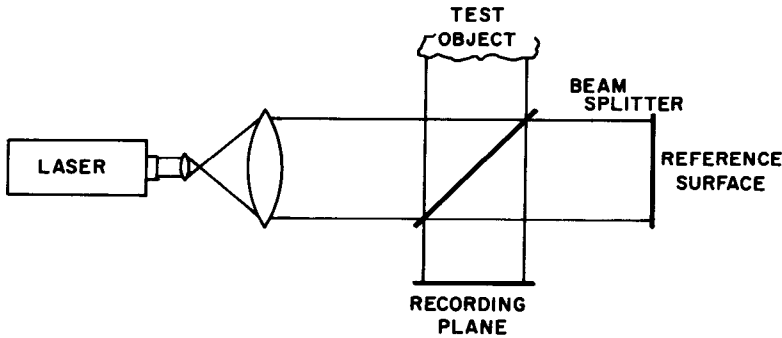


Fig. 1.1. Schematic diagrams of an interferometer

number of wavelengths, the interference is destructive and the region of the output plane is dark.

Unfortunately, there are two inevitable errors in such a simple analysis. Because fully monochromatic light is impossible for many reasons, a path difference cannot be precisely one wavelength (or any other multiple of one wavelength other than zero) for all wavelengths present. The need for partially monochromatic light is called the "temporal coherence" requirement for interference. Indeed, the path difference can become so large that constructive interference occurs for one wavelength at the same point where destructive interference occurs for another wavelength. This allowable path length mismatch is called the "temporal coherence length". Beams mismatched by this length or greater lengths produce very low contrast interference patterns.

Because mathematical point sources are also excludable in principle, rays reach any point in the interference pattern from all points within the source "point". As a result, perfect path matching is impossible. The requirement for point-like sources is often called the "spatial coherence" requirement for interference. If rays travel directly from the source to the interference plane by the shortest route (called the "optical axis") the spatial coherence problem is not troublesome. If the angle between rays leaving the point-like source becomes large enough, the paths cannot be balanced at any point. This maximum angle subtended at the physical aperture of the system determines a distance called the "spatial coherence length". Thus to achieve good coherence (temporal and spatial), a source must be as small and as monochromatic as possible.

1.3 Lasers

Before the advent of the laser, we could achieve good coherence only by spectral filtering (to make the usual "thermal" sources as monochromatic as possible) and spatial filtering (to make the source as small as possible). Unfortunately

these filtering operations result in considerable light loss. Thus a strong, coherent light source was a contradiction in terms. That situation changed with the availability of lasers which automatically produced highly monochromatic light capable of being focused to a very small point. Thus the laser is the key source for coherent optics because it can be very bright and very “coherent” simultaneously.

“Laser” is an acronym for *Light Amplification by Stimulated Emission of Radiation*. For our purposes, the key feature is that the laser radiation is an amplified version of a very few stimulating photons which initiated the process by being present in the laser and traveling in the right direction with the right wavelength by naturally occurring spontaneous emission. The laser can only amplify photons if their wavelengths and directions of travel fit within certain narrow bands. Thus the amplified photons are precisely those we would like to select out by spectral and spatial filtering to achieve good coherence. Therefore we can say that for ordinary thermal light sources coherence filtering must be external to the source and hence very lossy, whereas for a laser the filter is built into the source and causes no loss.

1.4 The Optical Fourier Transform

The use of lenses with coherent light leads quite naturally to the concept of the optical Fourier transform. It is well known that the light amplitude distribution in the back focal plane of a spherical lens is the two-dimensional complex Fourier transform of the amplitude transmittance in the lenses front focal plane. This property is the hallmark of a coherent optical system and is used constantly in all applications throughout this volume. This operation of a coherent optical processor is considered in more detail in Chapter 2 in which other conditions and alternate placements of the input and transform lens are discussed.

Since the mathematical concept of the Fourier transformation is the basis for many sciences and data processing applications, we discuss its highlights briefly in this introduction. The supporting theory can require considerable mathematical sophistication; however, the basic concepts are quite simple. Let us consider the one-dimensional case for simplicity. If $f(x_1)$ is a suitable input function (a concept we will not expound on because we will seldom if ever encounter an unsuitable function), its Fourier transform is:

$$F(u) = \int_{-\infty}^{\infty} f(x_1) \exp(-j2\pi x_1 u) dx_1 \quad (1.1)$$

where $j = \sqrt{-1}$, $x_1 u$ is dimensionless, and u has units of reciprocal length. We refer to the coordinate u of the Fourier transform plane as a “spatial frequency” with typical units of line pairs/mm = lp/mm. The spatial frequency variable u can be related to the actual distance coordinate x_2 of the transform plane by

Table 1.1. Several simple Fourier transform pairs and their physical significance

Input functions $f(x)$	Fourier transform $F(u)$	Comment
$\delta(x)$	1	A point in x goes to a constant in u
1	$\delta(u)$	Converse of above
$ag(x)$	$aG(u)$	Multiplication by a constant
$g(ax)$	$\frac{1}{ a } G(u/a)$	Scaling law
$g(x-a)$	$e^{-2\pi j au} G(u)$	Shift law
$\text{rect}\left(\frac{x}{a}\right)$	$\text{sinc}(au) = \frac{\sin \pi au}{\pi au}$	$\text{rect}\left(\frac{x}{a}\right) = \begin{cases} 1 & \text{for } x/a \leq 1/2 \\ 0 & \text{otherwise} \end{cases}$
$xf(x)$	$\partial F(u)/\partial u$	Differentiation
$f(x)g(x)$	$F(u) * G(u) \equiv \int_{-\infty}^{\infty} F(\eta)G(\eta-u)d\eta$	Convolution

$x_2 = f_1 \lambda u$ where f_1 is the focal length of the transform lens and λ is the wavelength of the laser light used.

The inverse Fourier transform operation is also easily proven to produce:

$$f(x) = \int_{-\infty}^{\infty} F(u) \exp(2\pi j xu) du. \quad (1.2)$$

We follow conventional notation in representing spatial functions by lower case variables and their Fourier transforms by the corresponding upper case variable. If we consider these basic equations more closely, we note that each point u in the Fourier transform plane receives a contribution from every point x in the input plane and vice versa. Several typical Fourier transform pairs are listed in Table 1.1 and are easily obtained by substitution into (1.1). The physical meaning of these simple transform pairs sheds considerable light on the process involved. The first transform pair states that a point x in the input plane produces a uniform response in the transform plane or that a point in x transforms into a plane wave in Fourier space. The last transform pair states that multiplication in input space is equivalent to convolution in transform space and vice versa. We will revisit this last transform pair in more detail in discussing optical pattern recognition.

For now let us attempt to gain further insight into this key Fourier transform operation performed by an optical processor. Refer to Fig. 1.2. We denote the input plane as P_1 and the transform plane as P_2 with coordinates (x_1, y_1) and (x_2, y_2) respectively. We place an input transparency with amplitude transmittance $f(x_1, y_1)$ in the front focal plane of the Fourier transform lens L_1 and

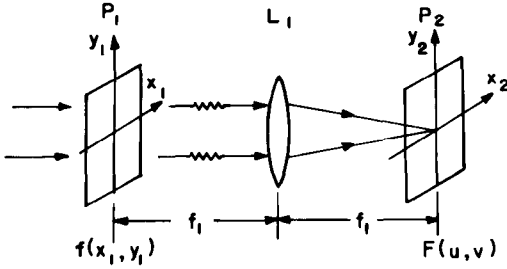


Fig. 1.2. Schematic representation of an optical Fourier transform system

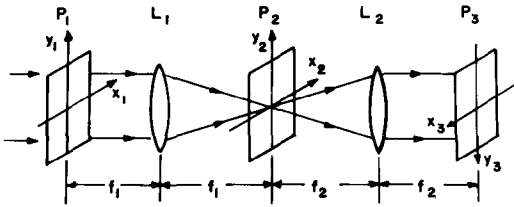


Fig. 1.3. Schematic representation of an optical image processing and spatial filtering system

illuminate it with a uniform amplitude and uniphase beam of coherent light. We assume an input beam of unit amplitude and describe the complex transmittance of plane P_1 by

$$f(x_1, y_1) = A(x_1, y_1) \exp[-j\phi(x_1, y_1)] \quad (1.3)$$

where $A(x_1, y_1)$ is a positive definite quantity representing the amplitude and $\phi(x_1, y_1)$ represents the phase of the input function. (For physical definiteness, we can identify A with the electric or magnetic vector of the electromagnetic wave describing the traveling light beam.) The phase is a mutual quantity like the coherence it is designed to represent. This means that no physical change would occur if an arbitrary fixed phase were added to all the phases in our calculations.

In two dimensions, the pattern in plane P_2 of Fig. 1.3 is

$$F(u, v) = F(x_2/\lambda f_1, y_2/\lambda f_1) = \iint_{-\infty}^{\infty} f(x_1, y_1) \exp[-j2\pi(x_1 u + y_1 v)] dx_1 dy_1 \quad (1.4)$$

by analog with (1.1), where

- u, v = spatial frequency coordinates of plane P_2 ,
- $x_2 = u\lambda f_1, y_2 = v\lambda f_1$ = physical distance coordinates of plane P_2 ,
- λ = wavelength of the light used, and
- f_1 = focal length of the transform lens L_1 .

To visualize this relationship, recall from Table 1.1 and basic geometrical optics that a lens focuses or converts an input plane wave to a point at plane P_2 . For an input point source at $(x_1, y_1) = (a, b)$, (1.4) becomes

$$\delta(x_1 - a, y_1 - b) \leftrightarrow \exp[-j2\pi(au + bv)] = \exp[(-j2\pi/\lambda)(ax_2/f_1 + by_2/f_1)], \quad (1.5)$$

where $\leftrightarrow$ denotes a Fourier transform pair. From this case, we see that the direction cosines a/f_1 and b/f_1 of the wave that results from an input point source at $(x_1, y_1) = (a, b)$ are proportional to the location of the point source in plane P_1 . We will return to this point in Section 1.8 in describing optical pattern recognition.

1.5 Image Processing and Spatial Filtering

If we place a second transform lens L_2 behind the transform plane P_2 as in Fig. 1.3, the pattern in the output plane P_3 is the double Fourier transform of the input $f(x_1, y_1)$ pattern or $f(x_3, y_3)$, where the coordinates of P_3 and P_1 are related by $x_3 - (f_2/f_1)x_1$ and $y_3 - (f_2/f_1)y_1$. As shown in Fig. 1.3, the coordinate axes of the output plane P_3 are reversed. The output image is thus a left-right and up-down inverted image of the input with the magnification factor being the ratio of the focal lengths of lenses L_2 and L_1 . This is exactly the same process that occurs in a slide projector camera.

We are now on the verge of optical image processing. Let us recall that the input light is coherent and that the pattern formed at P_2 is the Fourier transform $F(u, v)$ of $f(x_1, y_1)$. If at P_2 we place an opaque spot on axis, we block the "dc" portion of the input pattern, so only light from the higher spatial frequency components of the input image is transmitted. This produces a high-pass filtered version of $f(x_1, y_1)$ at P_3 . With a linear graded filter at P_2 whose transmittance increases with radial distance from the center, we produce the differentiated version of $f(x_1, y_1)$ at P_3 (see Table 1.1). By placing opaque spots at specific locations in P_2 , we can remove various frequency components present in the input pattern. These are some of the simpler spatial filtering methods used in image enhancement and image processing.

Far more sophisticated versions of image processing exist (see Chap. 3). If the transmittance of P_2 in Fig. 1.3 is $M(u, v)$ (where M is the Fourier transform of m), the light distribution leaving plane P_2 is $F(u, v)M(u, v)$ and at plane P_2 we find the Fourier transform of this product of two transforms or the convolution $f(x, y) * m(x, y)$ rather than simply $f(x, y)$ (see Table 1.1). By proper choice of $M(u, v)$, we can remove various distortions such as a blur or linear smear present in the input function $f(x_1, y_1)$ thus performing image enhancement or restoration. The proper choice of the filter function $M(u, v)$ for most interesting cases usually requires a complex function M .