

MANUAL OF ACTIVE FILTER DESIGN

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MANUAL OF ACTIVE FILTER DESIGN

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PREFACE

In this book we present simplified methods for obtaining a complete, practical filter design by inspection of a graph, requiring no computations whatsoever. The book therefore is useful to all filter designers, from the novice to the expert. The filter circuit elements used are operational amplifiers and standard values of resistances and capacitances.

The type of filters which one may construct using the graphs are the following:

1. Low-pass (Butterworth or Chebyshev of second or fourth orders)
2. High-pass (Butterworth or Chebyshev of second or fourth orders)
3. Band-pass (second and higher orders)
4. Band-reject (notch)
5. Phase-shift or delay (all-pass or Bessel)

Each of the filter types is discussed in a separate chapter. At the end of each chapter the design procedure is summarized and the appropriate graphs are presented. Practical design suggestions are given for each circuit considered.

Examples are given of every type of filter considered and actual photographs of the results are included. A detailed example is presented in Sec. 2.3, which may be used as a design guideline. However, it is not necessary to read the chapters in order to use the handbook, since all the necessary information is presented on the summary pages of each chapter.

*John L. Hilburn
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INTRODUCTION



CHAPTER

1.1 Active Filters

A filter is a device which passes signals of certain frequencies and rejects or attenuates those of other frequencies. Passive filters are constructed with inductors, capacitors, and resistors, but for certain frequency ranges inductors, because of their size and practical performance limitations, are undesirable. Consequently there has been, for some years, a trend toward replacing inductors by active devices which simulate the effect of inductors. This trend has accelerated with the advances in miniaturization which have made the active devices available at prices competitive with, and in many cases cheaper than, those of inductors.

In this handbook we present simplified methods for constructing a variety of active filters having specified characteristics with standard element values. The active device we use is the integrated circuit (IC) operational amplifier, which is briefly described in the remainder of this chapter. Graphs are presented for each type of filter, and, depending on the specifications, the designer may simply choose the appropriate graph and read off the circuit element values. For the designer interested in the theoretical details, there is a chapter of background material with numerous references provided for each filter type. However, to use the handbook, one needs only to refer to the summary sheet preceding the graphs at the end of each chapter.

1.2 The Operational Amplifier

The basic element we shall use in the construction of an active filter is the operational amplifier (op-amp), the symbol for which is shown in Fig. 1.1. Only three terminals are shown in the figure, the inverting input terminal ($-$), the noninverting input terminal ($+$), and the output terminal. However, a practical op-amp is actually a multi-terminal device. The purposes of the other terminals are specified by the manufacturer and include, in general, power supply connections, frequency compensation terminals, and offset null terminals.

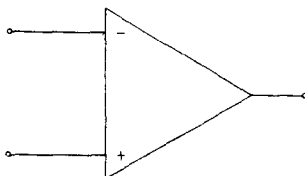


Fig. 1.1. A differential op-amp.

The equations we have derived in the following chapters are obtained assuming zero voltage between the two input terminals of the op-amp and zero current into the two input terminals. This is true of the ideal op-amp, and is closely approximated by practical op-amps, if used according to the manufacturer's specifications.

Numerous publications are available describing in detail the characteristics and uses of commercially available operational amplifiers. (See for example, [1]–[12].)* In addition, most manufacturers publish detailed catalogs containing information on their specific op-amps. An extensive list of manufacturers is given in [1]. Some well-known manufacturers include Burr-Brown Research Corp., Fairchild Semiconductor, Motorola, National Semiconductor, RCA, Signetics Corp., and Texas Instruments.

The op-amp of Fig. 1.1 is a differential-input amplifier, which is a commonly manufactured type. In general, for stable operation, IC op-amps require frequency compensation. Some, such as the 741, are internally compensated. The μ A741, AD741, MC1741, LM741, RC741, SN72741, CA3056A, etc. are all type 741 op-amps. The different representations are used to identify the manufacturer.

* References thus cited are listed in the Bibliography.

Other types of compensated op-amps include the 536, 107, 5556, 740, and 747 (dual 741). Other op-amps require external compensation, specified by the manufacturer, but generally are useful for much higher frequencies and gains. Some examples of these are the types 709, 748, 101, and 531.

For best results in the circuit configurations given in the following chapters, the designer should use op-amps which perform adequately for the gains and frequency ranges of interest. For example, the open-loop gains as specified by the manufacturer should be at least 50 times the filter gain [4]. Other suggestions will be made on the summary sheets at the end of each chapter.

1.3. Resistors and Capacitors

There are three types of resistors in common use. The carbon composition resistor is the most widely used and is acceptable in most noncritical filter applications. This is particularly true if the filter is used at room temperature. In all our examples the filters were constructed with 5% tolerance carbon composition resistors. These were used because they are the most economical and commonly available. For high-performance applications, or in instances where temperature is important, one should use either metal-film or wire-wound resistors.

In the case of capacitors, the ceramic disk capacitor is a very common and economical type. However, these should be used in the most noncritical applications. A more acceptable common type is the Mylar capacitor, which is the type we used in most of our examples. For critical applications and high performance, polystyrene and Teflon capacitors are good choices in most cases.

For a good discussion of resistors and capacitors, the reader is referred to [2], pp. 317–319.

LOW-PASS FILTERS

CHAPTER

2

2.1. General Circuit and Equations

A low-pass filter is a device which passes signals of low frequencies and suppresses or attenuates those of high frequencies. Its performance may be illustrated by its amplitude response, which is a plot of the amplitude $|H(j\omega)|$ of its transfer function $H(s)$ versus frequency ω (radians/sec) or f (Hz), where $\omega = 2\pi f$. In all cases we shall take $H(s) = V_2(s)/V_1(s)$, where V_2 is the output voltage and V_1 is the input voltage. An example is shown in Fig. 2.1, where the response represented by the broken line is the ideal case and the response represented by the solid line is a realizable approximation to the ideal. The value ω_c (or in Hz, $f_c = \omega_c/2\pi$) is the cutoff frequency defined as the point at which $|H(j\omega)|$ is $1/\sqrt{2} = 0.707$

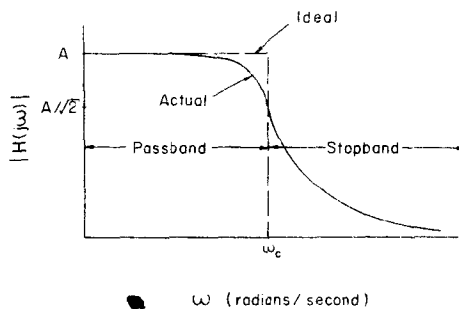


Fig. 2.1. Low-pass amplitude response.

times its maximum value, shown here as A . The passband is the range $0 \leq \omega < \omega_c$ and the stopband is the range $\omega > \omega_c$.

Alternately the amplitude response may be plotted as amplitude in decibels (dB), which we denote by α , versus frequency ω (or f), or versus $\log \omega$ (or $\log f$). An example is shown in Fig. 2.2, where it may be seen that cutoff corresponds to $\alpha = -3$ dB.

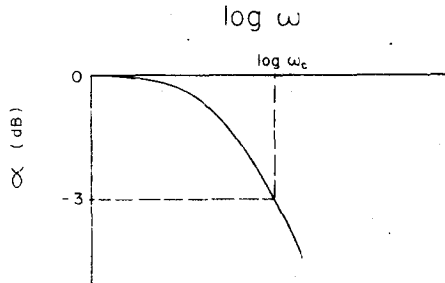


Fig. 2.2. Amplitude response in dB.

A second-order approximation to an ideal low-pass filter is achieved by the transfer function

$$\frac{V_2(s)}{V_1(s)} = \frac{K}{s^2 + as + b} \quad (2.1)$$

where a and b are properly chosen constants and K is a constant [13]. The term “second-order” refers to the degree of the denominator polynomial. Higher order transfer functions are like Eq. (2.1) except that the denominator is of higher degree. The gain of the low-pass filter is the value of its transfer function at $s = 0$, and is given in the case of Eq. (2.1) by $\text{gain} = K/b$.

There are any number of ways of obtaining low-pass filters using active devices in lieu of inductors. (See, for example [2], [6], [13], [14].) The method we use is that of Sallen and Key [15], in which the active device is an operational amplifier (op-amp), described in Chapter 1. A Sallen and Key second-order low-pass filter is shown in Fig. 2.3, where the resistors and capacitors are properly chosen to realize given values of a and b in Eq. (2.1). The op-amp, together with the resistors R_3 and R_4 , constitutes a voltage-controlled voltage source (VCVS), and hence the Sallen and Key network is of the VCVS type.