EXPERIMENTAL MECHANICS

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experimental mechanics 2

Proceedings of the

Second SESA International Congress

on

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on September 28–October 1, 1965

B. E. Rossi, Editor

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foreword

Collected in this volume are papers presented at the Second SESA International Congress on Experimental Mechanics, convened in Washington, D. C., Sept. 28–Oct. 1, 1965, under the sponsorship of the Society for Experimental Stress Analysis. This Congress follows by four years the inauguration of the First SESA International Congress on Experimental Mechanics held in New York City on Nov. 1–3, 1961. Messages from the president of the United States in each case pointed up the significance of these congresses in international relations. President J. F. Kennedy hoped that "the sinew of science can be put to work for pursuits of peace." President L. B. Johnson congratulated the SESA for its efforts "to promote international understanding and good will."

To name individually the very large number of persons within the Society and without, within the United States and abroad, who assisted the chairman of the Organizing Committee in establishing personal contact with experimentalists in nearly every country of the globe, would fill several pages in these Proceedings. The gratitude of the chairman is profound for the efforts of these many unnamed individuals. The chairman views them as informal members of the Organizing Committee, for without their unstinted cooperation it would not have been possible for him to reach, and to correspond personally with, some 400 of the leading experimentalists abroad, and to develop the truly international flavor of this Second Congress.

Special recognition is given to J. T. Pindera, of Warsaw, Poland, for his invaluable assistance in facilitating contacts in all of Eastern Europe, and to S. C. Redshaw for serving as SESA's representative in Great Britain on the Congress.

C. S. Barton, chairman of the SESA Papers Committee, is singled out for special mention. The very dedicated work performed by him and his committee in reviewing both domestic and foreign papers, in maintaining high standards of acceptance, in preparing the technical program, and in editing particularly the papers from other countries has set a challenge for future papers committees.

D. R. Olberts, chairman of the SESA Applications Committee, is also singled out for special mention. Paralleling the work of the Papers Committee, Mr. Olberts and his Applications Committee have set up a program on applications vying for first honors with that on research.

The invaluable contribution made by the Local Arrangements Committee appointed by the host Washington Area Section is also acknowledged. The committee members were M. E. Lunchick (general chairman), V. C. D. Dawson (vice-chairman), J. Bachman, R. G. Barclay, R. L. Bloss, J. O. Bryson, T. J. Culhane, M. Dean, III, T. Dimoff, F. C. Falkinburg, R. E. Heise, J. H. Herd, L. Mordfin, R. Waser and Mrs. M. E. Lunchick.

Last, but not least, the Chairman of the Organizing Committee honors the very energetic executive secretary of the SESA, B. E. Rossi, who, with his loyal staff at Westport, has borne the brunt of the grassroots organization of myriads of details involved in insuring a successful Congress of such magnitude and flavor, and in editing and producing so useful and attractive a volume of the Congress Proceedings.

Walter W. Soroka
Chairman
Organizing Committee

preface

The Second SESA International Congress on Experimental Mechanics successfully fulfilled the purpose of fostering international exchange of research information. The papers contained in this volume represent the major part of the technical program; however, there were important technical contributions given orally, but not recorded in print.

Although officially convened for four days, Sept. 28–Oct. 1, 1965, important formal and informal discussions were held on September 27 among SESA national officers, members of the executive committee and distinguished scientists, engineers and educators from other countries. This early meeting, under the able direction of SESA President John C. New, promoted a spirit of friendliness among attendees of all countries which pervaded every session of the Congress. SESA pledged itself to the support and coordination of international meetings of the future.

On Monday evening, September 27, President New hosted a reception for the Congress attendees around the pool of the International Inn in Washington. Again, the traditional SESA friendliness was manifest, setting a proper frame for the technical sessions to follow. On Tuesday morning, at 9:00 a.m., President New formally declared the Congress to be in session, and thus began the most extensive series of technical meetings yet sponsored by the SESA.

The technical program generally consisted of two research paper sessions and one applications paper session each half-day. This resulted in the largest number of technical papers ever presented at an SESA-sponsored program. On Tuesday evening, additional technical sessions were held in order to free Wednesday afternoon for a tour of the new National Bureau of Standards facilities.

On Wednesday morning, the applications session presented an unusual program entitled "Management Views Experimental Mechanics" with F. G. Tatnall as moderator. On Thursday morning, an applications panel, moderated by M. M. Leven, discussed the present state-of-the-art of the moiré method. Special emphasis was given to photoelasticity at this Congress under the guidance of M. M. Frocht and R. Guernsey which resulted in a special Thursday afternoon photoelasticity symposium for all Congress attendees. An international panel chaired by Dr. Frocht of the USA and composed of A. Kuske of West Germany, M. Nisida of Japan, J. T. Pindera of Poland, H. Pohl of East Germany, S. C. Redshaw of England, and A. J. Robert of France reviewed world-wide activities in this field of research.

The 1965 William M. Murray Lecture was presented on Friday afternoon by August J. Durelli; his topic was "Visual Representation of the Kinematics of the Continuum."

For the record, it should be noted that more than 60 reviewers of the Papers Committee were involved in selecting 54 research papers for the Congress. Thirty of these papers were from 11 countries outside the United States and 24 were from the United States. Specifically, the following countries were represented: Canada, Czechoslovakia, England, France, Germany (East and West), Greece, Hungary, Italy, Japan, Poland, USA, and USSR. Compared to the First SESA International Congress on Experimental Mechanics held in New York City, in 1961, which hosted 9 countries and presented 24 papers, the Second Congress showed important progress toward a broadened international exchange of research and technology.

The applications sessions were unusually interesting and were drawn from many areas of important research activity.

All told, the combined applications and research presentations totaled over 80, thus providing a rich choice for the congress attendees. In addition to the papers published in this volume, the technical program included the following papers or talks for which no formal manuscripts were available at the time that book was made ready for printing:

Fatigue Characteristics of Foil Strain Gages
by H. R. Becker, The Budd Company

A Statistical Approach to the Quality Control of Strain Gages
by G. A. Hulbert, BLH Electronics

Residual Stresses—Their Nature and Significance
by W. M. Murray, Massachusetts Institute of Technology

Investigation of Residual Stresses Using Photoelastic Coatings
by W. E. Greenwald, The Budd Company

A Study of the Effect on the Induction Hardening Variables on the Residual Stresses in Gears
by R. A. Celletti and J. A. Halgren, International Harvester Company, and V. H. Pagano, U. S. Army Tank-Automotive Center

Management Wants to Be Sure the Product is Good
by N. L. Mochel (Retired), Westinghouse Electric Corporation

Management Wants to Be Sure the Product is Adequate for the Job
by A. H. Keil, David Taylor Model Basin

Management Views the Question of Product Liability
by J. G. Jackson, Patent Attorney

Educational Needs of a Space-age Technical Staff
by E. J. Ott, NASA Headquarters

Managements Wants the Product Designed, Tested and Evaluated in the Best Ways
by R. E. Peterson, Westinghouse Electric Company

Management Wants the Most Suitable Materials in Its Products
by A. A. Watts, General Electric Company

Management Needs Technical Information for Decision Making
by E. Wenk, Jr., Library of Congress

Instrumentation Philosophy from Management Viewpoint
by N. Cohn, Leeds and Northrup Company

Fundamental Explanation of Moiré Method, Techniques, Etc.
by A. J. Durelli, The Catholic University of America

Transfer Grids, Remote Master Grids, Etc.
by F. Zandman, Vishay Instruments, Inc.

Limitations of Method
by W. F. Riley, IIT Research Institute

Developments of the Moiré Method in Europe
by A. Kuske, Technische Hochschule Stuttgart

Report on Visit to Measurement Laboratories in Europe and Japan
by M. Dean, III., David Taylor Model Basin

Impulsive Tests on Zircaloy Tubes
by E. A. Davis and G. H. Eng, Westinghouse Electric Corporation

Some Properties and Applications of Thin-film Semiconductor Strain Gages
by O. Watanabe and K. Shioda, Kyowa Electronics Instrument Company, Ltd.

The complete technical program was divided into 19 sessions, plus the Murray Lecture presentation. The titles of the individual sessions, some of which have already been mentioned, were as follows:

1. Research Session—APPLIED RESEARCH IN PHOTOELASTICITY
   Chairman—C. E. Taylor, University of Illinois
   Co-Chairman—J. S. Brock, David Taylor Model Basin

2. Research Session—BEHAVIOR OF MECHANICAL COMPONENTS AND SYSTEMS
   Chairman—I. Vigness, U. S. Naval Research Laboratory
   Co-Chairman—C. E. White, Goddard Space Flight Center

3. Applications Session—STRAIN GAGES AND FATIGUE PROBLEMS
   Chairman—C. R. Smith, Convair
   Co-Chairman—D. J. DeMichele, General Electric Company

4. Research Session—NEW METHODS IN PHOTOELASTICITY
   Chairman—M. Hetényi, Stanford University
   Co-Chairman—J. O. Bryson, National Bureau of Standards

5. Research Session—BEHAVIOR OF MECHANICAL COMPONENTS AND SYSTEMS
   Chairman—J. H. Meier, IBM Corporation
   Co-Chairman—H. M. Forkois, Naval Research Laboratory

6. Applications Session—STRAIN GAGES AND HEAT PROBLEMS
   Chairman—R. J. Whitehead, General Motors Corporation
   Co-Chairman—R. M. Kinney, Kinney Technical Products, Inc.

7. Research Session—NEW RESEARCH IN FATIGUE AND STRAIN GAGES
   Chairman—R. L. Bloss, National Bureau of Standards

8. Research Session—EXPERIMENTAL MECHANICS OF MATERIALS
   Chairman—M. E. Lunchick, Booz-Allen Applied Research, Inc.

9. Applications Session—RESIDUAL-STRESS MEASUREMENTS
   Chairman—P. M. Palermo, David Taylor Model Basin
   Co-Chairman—V. C. D. Dawson, U. S. Naval Ordnance Laboratory

10. Research Session—DYNAMIC AND STATIC LOADINGS IN PHOTOELASTICITY
    Chairman—W. F. Riley, IIT Research Institute
    Co-Chairman—C. T. Moyer, The Budd Company

11. Research Session—DYNAMIC BEHAVIOR OF MATERIALS
    Chairman—E. A. Ripperger, University of Texas
    Co-Chairman—R. H. Waser, Naval Ordnance Laboratory

12. Applications Session—MANAGEMENT VIEWS EXPERIMENTAL MECHANICS
    Moderator—F. G. Tatnall, Consultant

13. Research Session—DYNAMIC BEHAVIOR OF MATERIALS
    Chairman—D. C. Drucker, Brown University
    Co-Chairman—B. Reznek, Harry Diamond Laboratory

14. Research Session—STRUCTURAL RESEARCH—DESIGN AND ANALYSIS
Chairman—T. J. Dolan, University of Illinois
Co-Chairman—R. R. Palmisano, Harry Diamond Laboratory

15. Applications Session—PANEL DISCUSSION ON MOIRÉ METHOD—PRESENT STATE OF THE ART
Moderator—M. M. Leven, Westinghouse Electric Corporation

16. PHOTOELASTICITY SYMPOSIUM
Chairman—M. M. Frocht, Illinois Institute of Technology

17. Research Session—OPTICAL METHODS IN EXPERIMENTAL ANALYSIS
Chairman—G. Mesmer, Washington University
Co-Chairman—S. Edelman, National Bureau of Standards

18. Research Session—STRUCTURAL RESEARCH—DESIGN AND ANALYSIS
Chairman—L. Mordfin, National Bureau of Standards

19. Applications Session—APPROACHES TO TESTING AROUND THE WORLD
Chairman—D. Olberts, Deere and Company

For the convenience of the reader, all applications and research papers which appear in this volume have been grouped, as far as possible, in common areas.

As usual, in addition to the technical program, many other events occurred as part of the Congress. A special ladies' program provided a full schedule of activities for the women attendees. Authors' breakfasts, Society committee meetings, luncheons and dinners, the president's reception, tours, and the annual business meeting at which President-elect R. Guernsey was introduced along with other elected society officers for the 1965-1966 terms all were well-planned supplementary functions. The technical-equipment exposition was the largest and among the finest ever assembled by the Exhibits Committee.

The SESA cooperative spirit and willingness to share the fruits of research prevailed in an international atmosphere. With nearly 15 percent of SESA membership from countries other than the United States, this Congress provided a fine representation of this group for the benefit of all who attended and who will read this volume.

As chairmen of the applications and research papers committees, we are pleased to have been a part of this Congress. Deep appreciation and thanks are expressed to all those who helped on both national and local levels. Special mention must be given to Walter W. Soroka, chairman of the Organizing Committee, who spent many hours corresponding on the international level, and who made possible, to a very large extent, the international success of this Congress; and to Bonney Rossi, the Society's executive secretary and editor, who indeed gave "life" to all that occurred and provided the "unseen" power behind every activity.

We are indebted to the technical papers reviewers, authors and session chairmen, and all others who helped make the Second SESA International Congress on Experimental Mechanics possible. It is our sincere hope that you, the reader, may derive great benefit from this volume.

C. S. Barton,
Chairman
Papers Committee

D. R .Olberts,
Chairman
Applications Committee

contents

Visual Representation of the Kinematics of the Continuum

Lecture illustrates physically the principles, considerations and conclusions of the theories of continua

by August J. Durelli

ABSTRACT—The purpose of this paper is to present some experimentally determined properties of the continuum. The development starts with the illustration of deformed shapes, or metamorphoses, and the introduction of the concept of displacement. From there on, the development follows the classic mechanics-of-continuum presentation, spatial and time derivatives of displacement, strains, stresses and rigid rotations. To unify the presentation, all considerations are applied to the case of a circular ring subjected to diametral compression. Particular advantage is taken of grids, moiré, brittle-coatings and photoelasticity methods to obtain the physical representations of the properties. The following loci are considered: isokinetics, isothetics, isogonics, isoclinics of displacement, displacement trajectories, loci of partial derivatives, isogyros, isotenics, isochromatics, isoclinics of strain and stress, isostatics, isopachics, isobars, isoetrophics and isoentatics. The understanding of the properties of these fields helps in the visualization of the mechanics of the continuum, in general, and may prove to be particularly useful in the fields of plasticity and finite strain.

Preface

The text of this paper has been prepared to be delivered as a lecture, in honor of Professor William M. Murray, at the Second SESA International Congress on Experimental Mechanics. The opportunity seems to be appropriate to make some comments about the author's relationship with Dr. Murray and about the development of the research work that is reported in the lecture.

August J. Durelli is Professor, Mechanics Division, The Catholic University of America, Washington, D. C.

Lecture was presented at Second SESA International Congress on Experimental Mechanics held in Washington, D. C., on Sept. 28 – Oct. 1, 1965.

When I first came to the United States twenty-five years ago, I was sent to the Massachusetts Institute of Technology to take Dr. Murray's course on photoelasticity and learn his experimental techniques.

It is impossible to forget the understanding shown by Dr. Murray to that unknown foreigner coming to his laboratory, totally unable to communicate in English. It is impossible to forget his kindness and patience, and his unselfish and humble attitude.

learned a great deal during those years at MIT, not only scientifically and technically, but also about the way of life of the scientific community in this country. I was very fortunate to have had Dr. Murray as a guide at that time. It was with great pride that I saw my name associated with his in a couple of papers we published then, and I had been hoping to see once again my name associated with Dr. Murray's. This lecture, named after him, is to me more than an honor of the Society—it is an opportunity to reappraise those days of twenty-five years ago and to express my very deep feeling of gratitude to Dr. Murray.

In these twenty-five years, I have had the opportunity to publish a very large number of papers and reports. Anyone familiar with experimental work knows that a production of that size is possible only by a group of people working as a team. The subject matter of this lecture has been of great interest to several of us for many years and is, to a large extent, the result of a cooperative effort. I

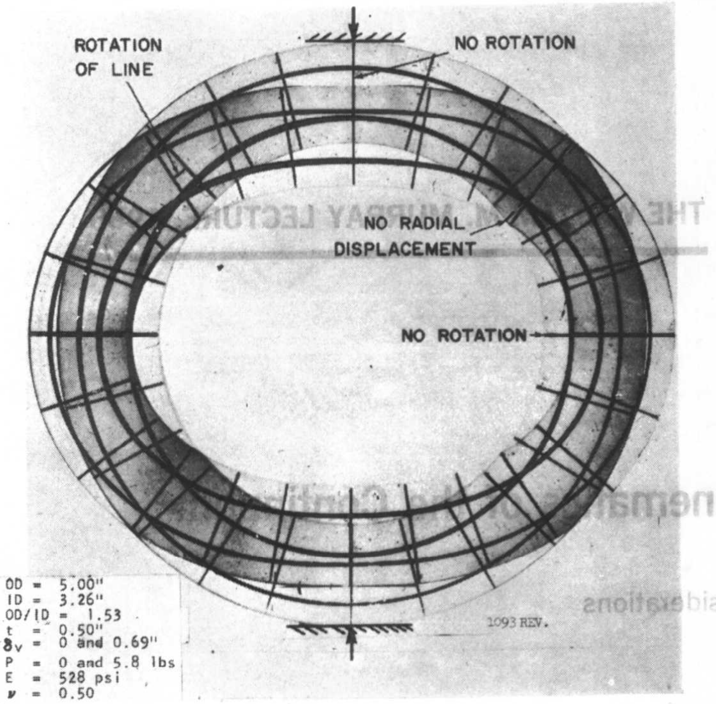

Fig 1.—Grid network on a deformed and undeformed ring subjected to diametral compression

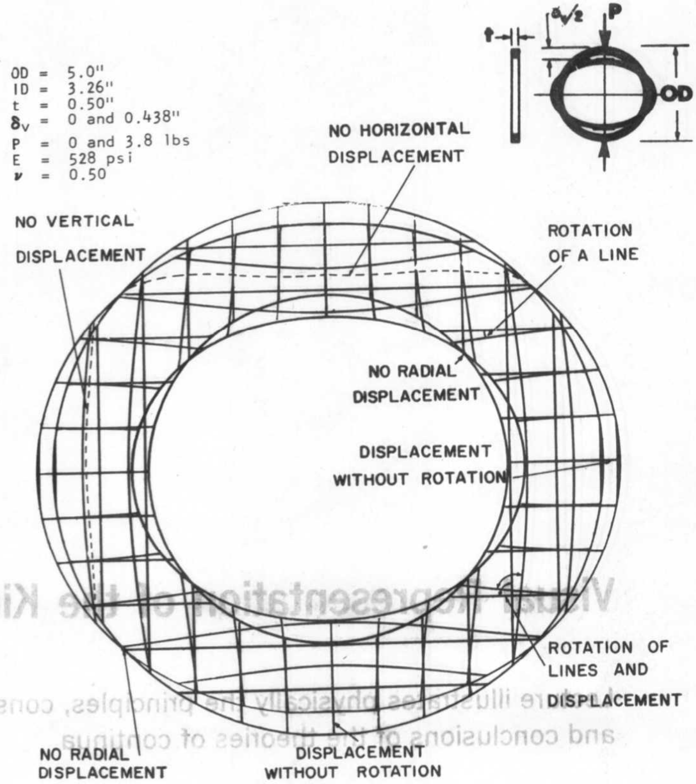

Fig. 2—Illustration of displacement of points and rotation of lines in the metamorphosis of a circular ring subjected to diametral compression

should like to acknowledge, briefly, the contributions of some of my associates.

About twenty years ago, in the basement of Chapin Hall at the old Armour Research Foundation, Seichi Okubo and I tried to make some sense out of a then-popular brittle coating. We managed to obtain some very clear and attractive isostatic patterns, and we learned the hard way that there is nothing wrong with the laws of nature, but that they do not always present themselves in an easy-to-advertise manner. At about the same time, on a shoestring and against all the rules of the book, we built our first polariscope. Since then, activities developed for us at an accelerated rate, and we enjoyed our work tremendously. Personally, I have been fascinated by field representations ever since.

Rex Lake and I started the first applications of photoelasticity to the design of solid-propellant grains. We published then some of those striking star-shaped isochromatic patterns. The association with Earl Phillips and Ching Tsao was particularly pleasant and fruitful. Besides several papers, we coauthored a book in which field representation was emphasized. Shortly after, James Dally, William Riley, and later, Isaac Daniel joined the group, and I became interested in dynamic photoelasticity. There was a particular enjoyment in the contemplation of the family of stress waves traveling at high speed through a body and in the development of the ingenuity necessary to record them. The association and close friendship with Riley continued after I left the Illinois Institute of Technology. The result of our cooperation was expressed in the publication of dozens of papers and in a recently published book. In the meantime, Albert Kobayashi and Kenneth Hofer became interested in brittle coatings, electrical strain gages and plasticity problems. The neatness and inherent beauty of some of the isostatic patterns in elasticity and Lueder's lines in plasticity obtained by Kobayashi have never since been reproduced.

The fascination of whole-field patterns was not limited to photoelasticity and brittle coatings, however, and with Cesar Sciammarella, S. Morse, and later with Vincent Parks and Margaret DeMarco, I became interested in moiré techniques. In the development of this paper, some of the powerful potentialities of the moiré effect will be shown. It suffices to say here that every progress in the new field has been an exciting experience.

I should add that I have been most fortunate in having Vincent Parks as an associate for almost ten years. The influence of his critical mind on our work is deeper than what could be gathered from the acknowledgments in papers and reports. None of the work in the last five years could have been possible without his contribution. His originality and the rigor of his thinking have influenced all our work on moiré and, in particular, the development of this paper. Important contributions to the paper have also been made by Margaret DeMarco, Harold Miller, Henry Feng, J.

Phillips and H. Vanderveldt. Several of the drawings were made by G. Gaunaurd, and the typing, editing and reproduction were the responsibility of Mrs. J. Maio.

The financial support necessary for the research reported in this paper was provided by the Office of Naval Research, the Army Research Office (Durham), and the National Science Foundation. The interest and understanding shown by H. Liebowitz, J. Crowley, J. Murray, and M. P. Gaus are very gratefully acknowledged.

Vocabulary

Isobar	Locus of points exhibiting the same value of normal stress (equal stress)
Isochromatic	Locus of points exhibiting the same value of maximum shear stress, or maximum shear strain (equal shear stress, or strain)
Isoclinic	Locus of points at which the principal stress, or strain, exhibits the same inclination (equal inclination)
Isoclinic of displacement	Locus of points at which the displacement exhibits the same inclination
Isoentatic	Locus of cracked ends in a brittle-coating test
Isogonic	Locus of points exhibiting the same value of angular displacement (equal angle)
Isogyro	Locus of points exhibiting the same value of rotation of a line that goes through the point (equal line rotation)
Isokinetic	Locus of points exhibiting the same value of displacement (equal displacement)
Isopachic	Locus of points exhibiting the same value of transverse strain in a plate (equal thickness)
Isostatic	Line, the tangent to which coincides with the direction of the principal stress or strain
Isostrophic	Locus of points exhibiting the same value of rigid rotation (equal rigid rotation)
Isotachic	Locus of points exhibiting the same value of velocity
Isotenic	Locus of points exhibiting the same value of normal strain (equal strain)
Isothetic	Locus of points exhibiting the same value of displacement component (equal displacement component)
Metamorphosis	A deformed shape
Trajectory of Displacement	Line, the tangent to which coincides with the direction of the displacement

Introduction

The usual development of a theory of strain and, in general, of theories of mechanics of continua, begins with considerations of the motion to which a point in a body is subjected when the body is deformed, or when it moves rigidly. These considerations are reasonable. They are based on geometry, or mathematics (Taylor series expansion). They are not, however, associated with any physical phenomenon that may substantiate the reasoning or illustrate it. One of the objectives of this paper is to illustrate physically the principles, considerations and conclusions of the theories of continua.

To simplify the presentation, the plane-stress case of the circular ring ($OD/ID = 1.53$) subjected to diametral compression will be used for all the illustrations. Since the small-deformation theories, in general, attract the greatest interest, the ring will be subjected to a level of deformation sufficiently small for the phenomenon to be considered linear. However, it is believed that in the future a great deal of interest will be given to the large-deformation phenomena, and that the greatest usefulness of the approaches presented here will be found in the large-deformation field. In some instances, some aspects of the large deformation of the ring will, therefore, also be shown to give the reader a glimpse of the potentialities of the method presented. It would be easy to show that, contrary to what happens when mathematical tools are used, the experimental solution of nonlinear problems is easier than the solution of linear problems.

One type of nonlinearity considered here is the geometric nonlinearity. The material used for these illustrations is a polyurethane rubber exhibiting a practically linear stress–strain relationship up to about 20-percent strain.

A second type of nonlinearity is illustrated by a structural aluminum loaded beyond its yield point. Several of the methods presented will apply equally well to any type of nonlinear problem.

Fundamental Concepts

A polar system of lines has been drawn on the face of a circular ring (Fig. 1). The ring is subjected to a diametral compression load P, associated with a vertical displacement δ_v of the point of application of the load. A superposed photograph of the deformed and undeformed shape of the ring shows the genesis of a final configuration by the metamorphosis through which the ring goes in the process of deformation. The word "metamorphosis" indicates the process of transformation, but more particularly will be used to specify a deformed shape.

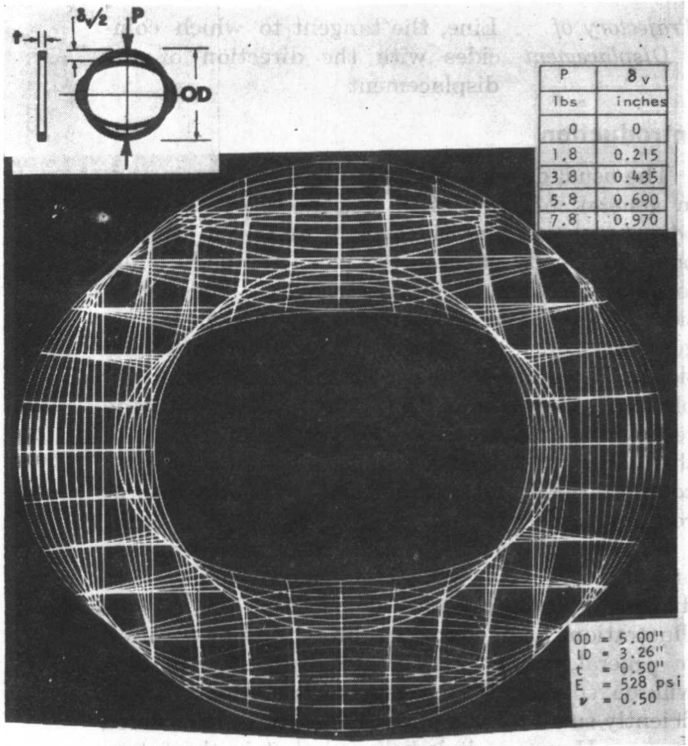

Fig. 3—Metamorphosis of a circular ring subjected to diametral compression (Cartesian grid)

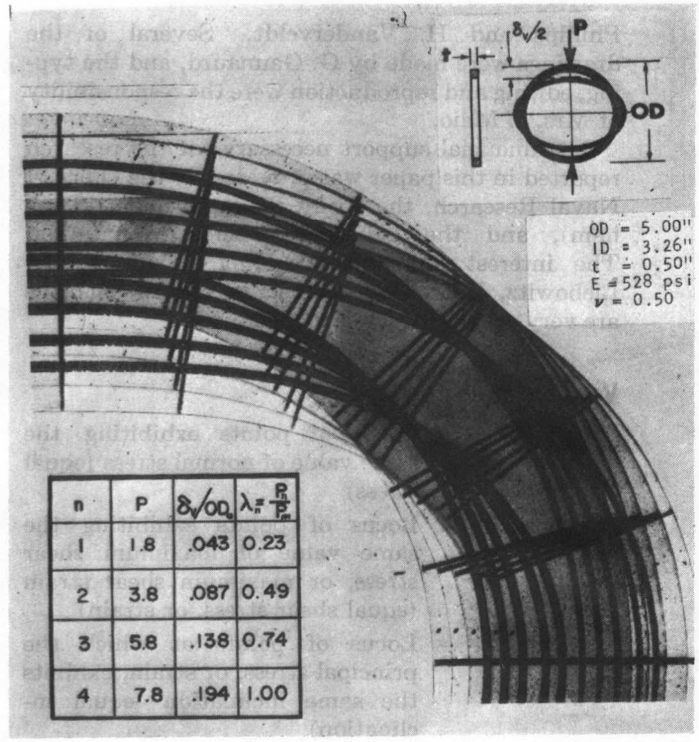

Fig. 4—Deformed shape of a ring subjected to four levels of diametral compression (metamorphosis of polar grid)

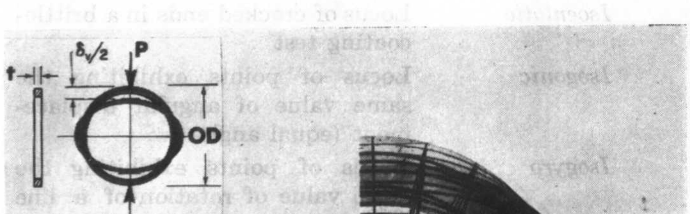

Fig. 5—Displacement velocity as a function of load

It is obvious in Fig. 1 that the axes of the ring have not rotated. All the other radial lines of the grid system have undergone an angular displacement. It is also obvious that the intersections of the original position of the circular boundaries with their metamorphosis are points at which no radial displacement occurs. This is also the case for the intersection of the circular lines of the grid with their deformed shape. Connecting the points of intersection of lines, which are originally circular, with their metamorphosis, lines are obtained which are loci of points that have not moved radially.

Similar reasoning can be applied to the ring on the surface of which a Cartesian system of lines has been drawn (Fig. 2). The metamorphosis is illustrated by the superposed photographs. The intersection of lines, originally vertical with their metamorphosis, gives points of no horizontal displacements. Connecting these points, a locus of points exhibiting no horizontal displacement is obtained. The same reasoning applied to horizontal lines gives a locus of points exhibiting no vertical displacement. Lines which originally are at right angles to each other and which, in the metamorphosis, change the angle and the position of the vertex are also shown in Fig. 2. This illustrates the general case of a pair of lines that rotate and translate.

Metamorphoses at four levels of loads are shown in Figs. 3 and 4. They have been obtained from a

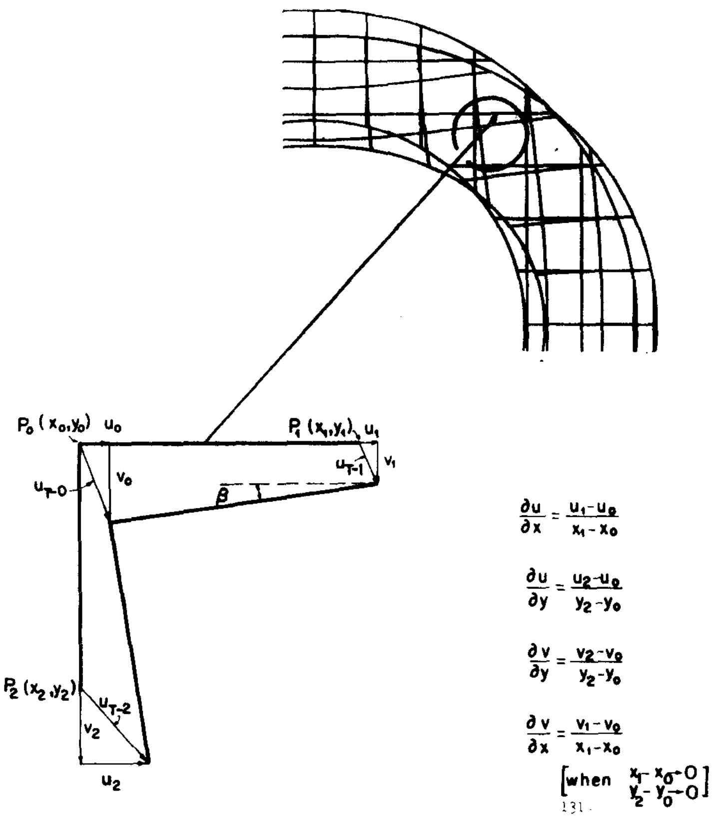

Fig. 6—Spatial derivatives of displacement

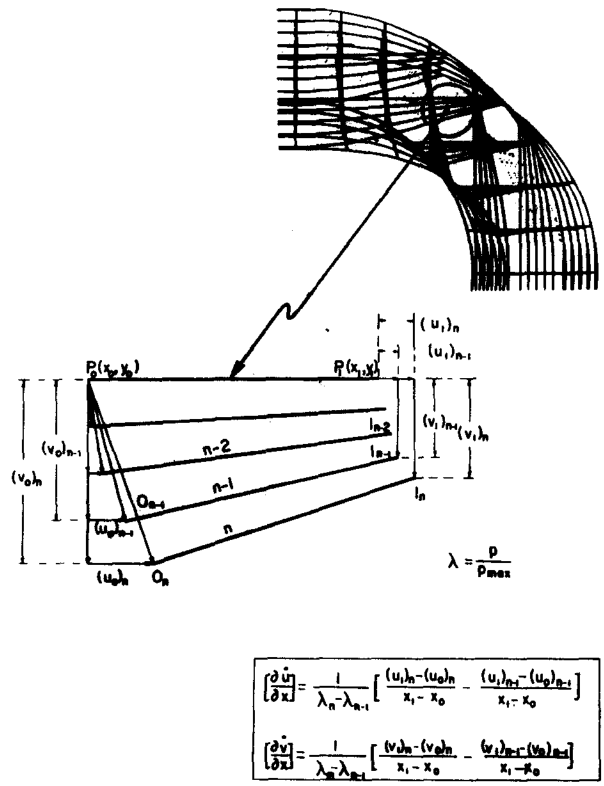

Fig. 7—Velocities of spatial derivatives of displacement

polyurethane-rubber specimen exhibiting a linear stress–strain behavior up to about 20-percent strain. These metamorphoses represent accurately the physical phenomenon of large strain and, at the same time, show in an exaggerated qualitative manner the phenomenon in small strain. (Attention should be called to the fact that these figures are not drawings, but photographs of the actual ring.)

The enlarged detail of the metamorphosis shown in Fig. 5 illustrates the case of nonlinear motion of a point and its projection on the axes of coordinates. The derivatives of the components of this motion with respect to the loading, or to some parameter related to the loading, are sometimes called the velocities of the displacement.

Looking at a single metamorphosis—for instance, the one shown in Fig. 6—it is obvious that the value and direction of the displacements change from point to point. This change in space (or as a function of the coordinates of the point) is expressed by the partial derivatives of the displacement components. It is also easy to conceive that, as there is a change in the total displacement of the point as the deformation proceeds (displacement velocity), there is also a change in the spatial derivatives as the deformation proceeds (partial-derivatives velocities). These are illustrated in Fig. 7.

The following concepts have therefore been introduced directly from the physical evidence:

(1) The displacement or motion of a point. It will be symbolized by U_T. The displacement is a vector and can be identified by its direction and its value.

(2) The components of the displacement along a system of reference. They will be called u and v in the Cartesian system, and u_θ and u_r in the polar system.

(3) The change in the value of the components of the displacement as the deformation proceeds. They will be called "increments of displacement" or "displacement velocities" and will be symbolized by $\dot{u}$ and $\dot{v}$. Since time is required for the change in deformation, they will also be called "time derivatives of the displacements."

(4) Since, at any instant, the displacements and their components change from one point of the body to the next, their change will be defined by the space derivatives

$$\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x} \quad \text{and} \quad \frac{\partial v}{\partial y}$$

(5) The change in the values of these partial derivatives as the deformation proceeds. They will be called "velocities of the derivatives of displacements."

(6) The rotation of a line as the deformation proceeds.

(7) The change in angle between two lines as the deformation proceeds.