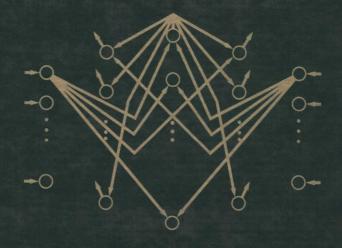
# Neurocomputing



Robert Hecht-Nielsen

## **NEUROCOMPUTING**

#### Robert Hecht-Nielsen

HNC, Inc. and University of California, San Diego



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### **Preface**

Neurocomputing is a subject that has captivated the interest of thousands of technologists, scientists, and mathematicians. The idea of training a system to carry out an information processing function (instead of programming it) has intrinsic appeal, perhaps because of our personal familiarity with training as an easy and natural way to acquire new information processing capabilities. Neurocomputing systems are often endowed with a "look and feel" vaguely reminiscent of animals. Like a pet, this is a technology that is easy to fall in love with. I did, 22 years ago.

I hope you find this book useful and enjoyable.

#### **About This Book**

The author of this book is an industrialist, an adjunct academic, and a philanthropist without financial portfolio. Activities of these sorts heighten one's awareness of the need for technology textbooks to have industrially oriented material sprinkled in with the purely academic grist. This book is designed with this idea in mind. It contains auxiliary material on subjects as diverse as business plan development, venture capital, proposal writing, and development project planning and management. Discussions about the history of neurocomputing and historical vignettes summarizing the stories of pioneering neurocomputing companies are also presented. The observations that have led to the decision to include this material are sketched below.

Until about 1960, the primary focus of technology was the near-term betterment of the life of the average human being (with the development of weaponry running a close second). However, over the past 30 years some countries (such

as the United States and the Soviet Union — but not, for example, Japan) have built up large technology research infrastructures (separate from the science and mathematics infrastructures) that are, to a distressing extent, disconnected from the practical civilian and military applications of technology (both short-term and long-term). These infrastructures have diverted large numbers of highly talented technologists from useful work. A concomitant of the rise of these infrastructures has been the emergence of a reprehensible snobbery that places all technological efforts into one of two categories: high-tech or I'd-rather-die-thando-that. The social damage caused by these aberrations has been enormous.

It is now time for technology to regain its traditional primary focus. The formula is the same one that has always worked. Namely: young technologists should work on design, project management, product development, and/or manufacturing. Those of supreme talent and/or pedagogic ability who are appropriately motivated and who have proven themselves in practical work can, later in their careers, carry out research and/or teach technology in universities. By addressing some of the issues surrounding the practice of technology in industry, as part of the presentation of a popular and potentially important new information processing technology, perhaps this book can, in some small way, serve to help bring about this renaissance.

#### Use of this Book in a Course

This text is designed to provide technologists, scientists, and mathematicians with an introduction to the field of neurocomputing. It is intended for use at the graduate level, although seniors would typically have all of the required background. The text is written to support a year-long course. It can also be used for shorter courses if some of the material is skipped or skimmed.

A year-long course can go through the whole book, in order. This would afford an opportunity to explore all three aspects of the subject (theory, implementation, and applications). Instructors can expand or contract the various topics to fit their personal preferences.

A semester-long course can probably still cover the whole book. Chapters 1, 2, 3, 4, 5, and 9 can be covered in detail, and the rest of the book can be skimmed. A quarter-long course can cover most of Chapters 1, 2, 3, 4, and 5, along with a small amount of material selected by the instructor from other chapters.

#### **Concurrent Laboratory Projects**

One of the realities of neurocomputing is that hands-on practice is as important as classroom learning. Thus, students learning neurocomputing should, if possible, be involved in laboratory projects concurrent with their classroom instruction. This activity can be done on a group basis or, preferably, on an

individual basis. To help in the planning of a laboratory section to complement a course based on this book, the following suggestions for projects are offered:

- Develop a backpropagation image compression system based on the example in Section 9.2.2. Images of the members of the class can be used for training and testing, with images of automobiles used to demonstrate the problems caused by data that are not statistically consistent with those used during training.
- Simulate a broomstick-balancing system based upon visual feedback, as discussed in Section 9.3.1.
- Write a neurosoftware description (in AXON, for example) of a famous neural network architecture, such as the Boltzmann Machine or GMDH and build a demonstration program that illustrates the architecture's capabilities.
- Solve a "toy problem" using multiple neural networks. The group can first define a suitable toy problem, and then solve it.
- Carry out experimental validation of theoretically derived results from Chapters 3 through 6.
- Develop variants of some of the simpler neural networks described in Chapters 4, 5, and 6.
- Define and solve a simple application problem from start to finish.

It is suggested that projects be organized via the writing of a brief (one or two page) development plan in accordance with the planning approach presented in the Appendix. Projects can last from 1 week to 4 weeks and can be concluded with a detailed report describing the development plan, the work carried out, the results, the software and neurosoftware developed (with listings and permanent archival diskette provided), and recommendations for future work.

#### Acknowledgments

Writing a textbook involves a major commitment of time and energy and a commensurate loss of availability of that time and energy for other uses. Robert L. North and Gerald I. Farmer (President and Executive Vice President of HNC, Inc.) have been extremely generous in supporting the development of this book during the past 3 years. Thanks are also due to the 100+ students who have patiently sat through an entire year of ECE-270 A/B/C (Neurocomputing) in the Electrical and Computer Engineering Department of the University of California, San Diego in 1986/1987, 1987/1988, and 1988/1989. They have been both the guinea pigs for the material of the text and the most helpful assistants in making improvements. The class of 1989/1990 has been helpful in doing final proofreading. Special thanks are also due Lawrence Milstein and Manuel Rotenberg, the successive Chairs of the UCSD ECE Department over the past 4 years, for their unwavering support. Also, the lively interdisciplinary interaction in neural networks at UCSD (across 14 academic departments and the Salk Institute) has been of great value in this project.

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## **Contents**

11111	oduction: What Is Neurocomputing?	1
1.1		
	1.1.1 Overview of Neurocomputing	1
1.2	Neurocomputing as a Subject	
1.3	The Relationship Between Neurocomputing and Neuroscience	12
1.4		
1.5		
		21
2.1	Neural Networks	21
	2.1.1 Definition of a Neural Network	
2.2	2.1.1 Definition of a Neural Network	22
2.2		22 26
2.2	Connections	22 26 27
2.2	Connections	22 26 27 28
	Connections	22 26 27 28
	Connections  2.2.1 Connection Signal Data Types  2.2.2 Input Classes  2.2.3 Connection Geometries  Processing Elements	22 26 27 28 29
	Connections  2.2.1 Connection Signal Data Types  2.2.2 Input Classes  2.2.3 Connection Geometries  Processing Elements	22 26 27 28 29 37 37
	Connections  2.2.1 Connection Signal Data Types  2.2.2 Input Classes  2.2.3 Connection Geometries  Processing Elements  2.3.1 Transfer Functions and Local Memories  2.3.2 Slabs	22 26 27 28 29 37 37
	1.2 1.3 1.4	1.1.1 Overview of Neurocomputing  1.2 Neurocomputing as a Subject  1.3 The Relationship Between Neurocomputing and Neuroscience 1.3.1 Neurocomputing and Neuroscience 1.3.2 Hype  1.4 History of Neurocomputing 1.4.1 The Beginning 1.4.2 First Successes 1.4.3 The Quiet Years 1.4.4 Neurocomputing Takes Off  1.5 Guide to This Book  Neural Network Concepts, Definitions, and Building Blocks

#### x Contents

		2.4.2	Spheres and Cubes		•	•	42
3	Lea		aws: Self-Adaptation Equations				46
	3.1	Defini	itions				46
		3.1.1	Information Environments				46
		3.1.2	Weight Space				47
		3.1.3	Varieties of Training				49
	3.2	Coinc	idence Learning				51
		3.2.1	Hebb's Biological Learning Law				51
		3.2.2	The Linear Associator				52
		3.2.3	Hebb's Learning Law				53
		3.2.4	The Pseudoinverse Formula				55
	3.3	Perfor	rmance Learning				57
		3.3.1	The ADALINE				58
		3.3.2	The Least Mean Squared Error Goal				59
		3.3.3	The Widrow Learning Law				60
	3.4		etitive Learning	•	•	•	64
	•••	3.4.1	Kohonen's Layer				66
		3.4.2	The Kohonen Learning Law		•	•	67
		3.4.3	Estimation of the Probability Density Function				68
	3.5		Learning				72
	5.5	3.5.1	The Flywheel Equation				72
		3.5.2	The Instar				74
		3.5.3	Grossberg's Learning Law				74
	3.6		otemporal Learning				76
	3.0	3.6.1	Temporal Sequences				76
		3.6.2	The Kosko/Klopf Learning Law				77
		3.0.2	The Rosko/Riopi Learning Law		٠	٠	//
4			Networks: Data Transformation Structures				79
	4.1	Basic	Definitions		٠	•	79
	4.2	Linear	r Associator Network			•	81
	4.3	The L	earnmatrix Network		•	•	87
			Definition of the Learnmatrix Network				88
		4.3.2	Learnmatrix Optical Analysis				89
		4.3.3	Learnmatrix Capacity				91
	4.4	Recur	rent Associative Networks				95
		4.4.1					
		4.4.2					100
		4.4.3	Associative Network Theorems				101
	4.5	Associ	iation Fascicles				107
5	Mar	oping N	letworks: Multi-Layer Data Transformation Structures				110
_	5.1		Sapping Implementation Problem				
		5.1.1	Mapping Neural Networks				
		5.1.2	Measuring Function Approximation Accuracy				
		J. I. L	measuring i unction approximation accuracy		٠	٠	111

		5.1.3	Training and Overtraining	. 115
		5.1.4		
	5.2	Kolm	ogorov's Theorem	. 122
		5.2.1	Implications for Neurocomputing	123
	5.3	The B	Backpropagation Neural Network	. 124
		5.3.1	Architecture of the Backpropagation Network	. 125
		5.3.2	Backpropagation Error Surfaces	128
		5.3.3	Function Approximation with Backpropagation	131
		5.3.4	Backpropagation Learning Laws	133
	5.4		Organizing Map	138
		5.4.1	The second of th	138
		5.4.2	Examples of the Operation of the Self-Organizing Map	
	5.5		terpropagation Network	147
		5.5.1	Architecture of the Counterpropagation Neural Network	147
		5.5.2	Variants of the Counterpropagation Network	152
	5.6	Group	Method of Data Handling	155
		5.6.1	The GMDH Neural Network	156
		5.6.2	GMDH Lessons	162
	C	49 - 4		
6			oral, Stochastic, and Hierarchical Networks: Frontiers of Neu	
	6.1	mputing		164
	0.1	6.1.1	stemporal Networks	164
		6.1.2	Spatiotemporal Pattern Recognizer Neural Network	166
	6.2		Recurrent Backpropagation Neural Network	182
	0.2	6.2.1	astic Networks	192
		6.2.2	The Boltzmann Machine Network	192
	6.3		rchical Networks	100
	0.5	6.3.1	Neocognitron Network	198
		6.3.2	Combinatorial Hypercompression	198
		6.3.3	Attention Mechanisms: Segmentation and Object Isolation	210
		0.5.5	Attention Mechanisms. Segmentation and Object Isolation	214
7	Neu	rosoftwa	are: Descriptions of Neural Network Structure	220
	7.1		software: Coded Descriptions of Neural Network Structure	
	7.2	Softwa	are Interfaces Between Computers and Neurocomputers	222
	7.3		V Language	
			AXON Structure	
			Parameter Definition Block	
		7.3.3	Processing Element and Slab Definition Block	
		7.3.4	Network Creation and Connection Definition Block	
		7.3.5	Scheduling Function Block	
		7.3.6	Function Definition Block	239
	7.4	AXON	V Examples	
		7.4.1	Backpropagation	
		7.4.2	Counterpropagation	

8	Neu	rocomputers: Machines for Implementing Neural Networks	259
•	8.1	Neurocomputer Fundamentals	260
		8.1.1 Neurocomputers as Computer Coprocessors	260
		8.1.2 Performance Measures	263
		8.1.3 Taxonomy	266
	8.2	Analog and Hybrid Neurocomputer Design Fundamentals	272
		8.2.1 Overview	272
		8.2.2 Primacy of Input Processing and Weight Modification.	276
		8.2.3 Transfer Function Implementation	284
	8.3	Analog and Hybrid Neurocomputer Design Examples	287
		8.3.1 Electro-Optic Neurocomputer	287
		8.3.2 Optical Neurocomputer	289
		8.3.3 MNOS/CCD Electronic Neurocomputer Chip	292
	8.4	Digital Neurocomputer Design Fundamentals	297
		8.4.1 Overview	297
		8.4.2 Fully Implemented Design Approaches	298
		8.4.3 Virtual Design Approaches	301
	8.5	Digital Neurocomputer Design Examples	305
		8.5.1 Mark III Neurocomputer	305
		8.5.2 Mark IV Neurocomputer	307
_		a training Designation of D	
9		rocomputing Applications: Sensor Processing, Control, and D	rata 317
	Ana	lysis  Neurocomputing Applications Engineering	
	9.1	9.1.1 Solving Problems with Neurocomputing	318
		9.1.2 Functional Specification Development	320
	9.2		323
	9.2	9.2.1 Character Recognizer	323
			325
		9.2.2 Cottrell/Munro/Zipser Technique	337
	9.3	Control	
	9.3	9.3.1 Vision-Based Broomstick Balancer	342
		9.3.2 Automobile Autopilot	
	9.4	Data Analysis	349
	7.4	9.4.1 Loan Application Scoring	349
		9.4.2 NETtalk	351
		9.4.3 The Instant Physician	354
		7.4.5 The instant I hysician	
A	Neu	procomputing Projects: Developing New Capabilities that Succeed	
	the	Marketplace	358
	<b>A.</b> 1	Business Plan Development	359
		A.1.1 Project Definition	361
		A.1.2 Defining Goals	362
		A.1.3 Technical Feasibility	
		A.1.4 Market Analysis	363

	Contents					
A.1.5	Development Plan		. 368			
	Marketing and Sales Plan					
A.1.7	Production Plan		. 374			
A.1.8	Organization and Personnel		. 376			
A.1.9	Schedule		. 377			
	Budget					
A.1.11	Financing and Ownership	<b></b> .	. 383			
A.2 Writing	g a Proposal		. 388			
A.2.1	RFPs and RFQs		. 389			
A.2.2	Proposal Organization	• • • • • • ·	. 389			
A.2.3	Proposal Writing	<b></b> .	. 392			
	ng and Managing Development					
A.3.1	The Development Planning Process	<b></b> .	. 394			
A.3.2	Project Management		. 402			
INDEX			422			

## Introduction: What Is Neurocomputing?

Neurocomputing is the technological discipline concerned with information processing systems (for example, neural networks) that autonomously develop operational capabilities in adaptive response to an information environment. Neurocomputing is a fundamentally new and different approach to information processing. It is the first alternative to programmed computing, which has dominated information processing for the last 45 years. This book provides a graduate-level introduction to neurocomputing, including theory, implementation, and applications.

This chapter begins with an overview of neurocomputing. The structure of the field of neurocomputing is then discussed, followed by a discussion of the relationship between neurocomputing and neuroscience. The history of the subject is then surveyed, and finally, a brief overview of the structure of the rest of the book is presented.

#### 1.1 Introduction

#### 1.1.1 Overview of Neurocomputing

From the advent of the first useful electronic digital computer (ENIAC) in 1946 [23] until the late 1980s, essentially all information processing applications used a single basic approach: programmed computing. Solving a problem using programmed computing involves devising an algorithm and/or a set of rules for solving the problem and then correctly coding these in software (and making necessary revisions and improvements).

Clearly, programmed computing can be used in only those cases where the processing to be accomplished can be described in terms of a known procedure

or a known set of rules. If the required algorithmic procedure and/or set of rules are not known, then they must be developed — an undertaking that, in general, has been found to be costly and time consuming. In fact, if the algorithm required is not simple (which is frequently the case with the most desirable capabilities), the development process may have to await a flash of insight (or several flashes of insight). Obviously, such an innovation process cannot be accurately planned or controlled. Even when the required algorithm or rule set can be devised, the problem of software development still must be faced.

Because current computers operate on a totally logical basis, software must be virtually perfect if it is to work. The exhaustive design, testing, and iterative improvement that software development demands makes it a lengthy and expensive process.

A new approach to information processing that does not require algorithm or rule development and that often significantly reduces the quantity of software that must be developed has recently become available. This approach, called *neurocomputing*, allows, for some types of problems (typically in areas such as sensor processing, pattern recognition, data analysis, and control), the development of information processing capabilities for which the algorithms or rules are not known (or where they might be known, but where the software to implement them would be too expensive, time consuming, or inconvenient to develop). For those information processing operations amenable to neurocomputing implementation, the software that must be developed is typically for relatively straightforward operations such as data file input and output, peripheral device interface, preprocessing, and postprocessing. The Computer Aided Software Engineering (CASE) tools often used with neurocomputing systems can frequently be utilized to build these routine software modules in a few hours. These properties make neurocomputing an interesting alternative to programmed computing, at least in those areas where it is applicable.

Formally, neurocomputing is the technological discipline concerned with parallel, distributed, adaptive information processing systems that develop information processing capabilities in response to exposure to an information environment. The primary information processing structures of interest in neurocomputing are neural networks (although other classes of adaptive information processing structures are sometimes also considered, such as learning automata, genetic learning systems, data-adaptive content addressable memories, simulated annealing systems, associative memories, and fuzzy learning systems). The formal definition of a neural network follows.

■ DEFINITION 1.1.1 A neural network is a parallel, distributed information processing structure consisting of processing elements (which can possess a local memory and can carry out localized information processing operations) interconnected via unidirectional signal channels called connections. Each processing element has a single output connection that branches ("fans out") into as many collateral connections as desired; each carries the same signal

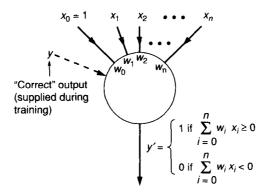
— the processing element output signal. The processing element output signal can be of any mathematical type desired. The information processing that goes on within each processing element can be defined arbitrarily with the restriction that it must be completely local; that is, it must depend only on the current values of the input signals arriving at the processing element via impinging connections and on values stored in the processing element's local memory.

One might wonder why this particular type of Multiple Instruction Multiple Data (MIMD) parallel processing architecture should be worthy of such concentrated attention (beyond the obvious fact that biological neuron networks seem to be neural networks in the above sense). In particular, why not simply study some of the more general MIMD architectures (such as dataflow architectures [57]) which contain neural networks as a subclass? Surprisingly, no completely satisfactory answer to this question is yet known. However, it seems very likely to me that the neural network definition will someday be shown to be a particularly good compromise that allows substantial information processing capability while at the same time providing sufficient structure to allow the development of efficient general-purpose implementations (methods for efficiently implementing arbitrary general MIMD architectures are not known and may not exist). All we can say for sure now is that the neural network definition does produce a class of powerful and potentially useful information processing structures that lend themselves to efficient implementation by general-purpose neurocomputers. It is these consequences of the definition that we will discuss in this book.

To illustrate the nature of neural networks we shall describe briefly a classical neural network architecture known as the *perceptron*. Because it has been largely superceded by more powerful neural networks (for example, some of those discussed in Chapters 3, 4, 5, and 6), the perceptron is primarily of historical interest, although it is still occasionally used.

The perceptron is a neural network that consists of one or more of the processing elements shown in Figure 1.1 (which are themselves also referred to individually as perceptrons). For simplicity, we shall concentrate on the operation of a single perceptron processing element.

The goal of the perceptron is illustrated in Figure 1.2. Here we see two classes of patterns (class 0 and class 1). A pattern is simply a point in n-dimensional space (the coordinates of the point represent attributes or features of the object to be classified, such as weight, height, density, or frequency). In the case illustrated in Figure 1.2 (which is the situation of interest relative to the perceptron), the two classes can be separated from each other by a simple linear hyperplane (in 2-dimensional space a hyperplane is a line, in 3-dimensional space it is an ordinary plane, and in n-dimensional space it is an (n-1)-dimensional flat surface). Classes that have this property are termed linearly separable. The goal is to find a set of weights or adaptive coefficients  $w_0, w_1, \ldots, w_n$  (which, it turns out, determine a unique hyperplane — as will



**Fig. 1.1.** • The perceptron. The perceptron has an input consisting of an (n + 1)-dimensional vector  $\mathbf{x} = (x_0, x_1, x_2, \dots, x_n)$ , where  $x_0$  is permanently set to 1 (this is called a *bias input*). The output of the perceptron is 1 if the weighted input sum  $x_0w_0 + x_1w_1 + \dots + x_nw_n$  is greater than or equal to zero; the output is 0 if this weighted input sum is less than zero.

be discussed in Chapter 3) such that the output of the perceptron is 1 if the input pattern vector  $(x_1, x_2, ..., x_n)$  belongs to class 1, and 0 if the pattern vector belongs to class 0.

The weights are stored within the processing element and are automatically modified by the processing element itself in accordance with the perceptron learning law. This learning law operates during a training process where the perceptron is shown a sequence of randomly selected x pattern vectors (one at a time). Each time an x example is presented to the perceptron (as part of a training trial), the system is also told to which class (0 or 1) the example belongs. On each training trial, the learning law modifies the weight vector w in accordance with the equation

$$\mathbf{w}^{\text{new}} = \mathbf{w}^{\text{old}} + (y - y') \mathbf{x}, \tag{1.1}$$

where y is the correct class number of the input pattern x (which is supplied, along with x, on each training trial), and y' is the output of the perceptron. The idea of this learning law is that, if the perceptron makes an error (y-y') in its output, this error indicates a need to reorient the x hyperplane so that the perceptron will tend not make an error on this particular x vector (or any other vector near it) again. Note that the output error (y-y') will be 0 if the output of the perceptron is correct. In this situation the weight will not change. If the output is wrong, then (y-y') will be either +1 or -1, and x will be modified appropriately (so that the perceptron will do better in the future).

The perceptron was invented in 1957 by Frank Rosenblatt [199] (who also wrote *Principles of Neurodynamics*, one of the two early books on neurocomputing [198] — the other being *Automat und Mensch* by Karl Steinbuch [216]). Following his invention of the perceptron, Rosenblatt proved that, given linearly separable classes, a perceptron will, in a *finite number* of training trials, develop