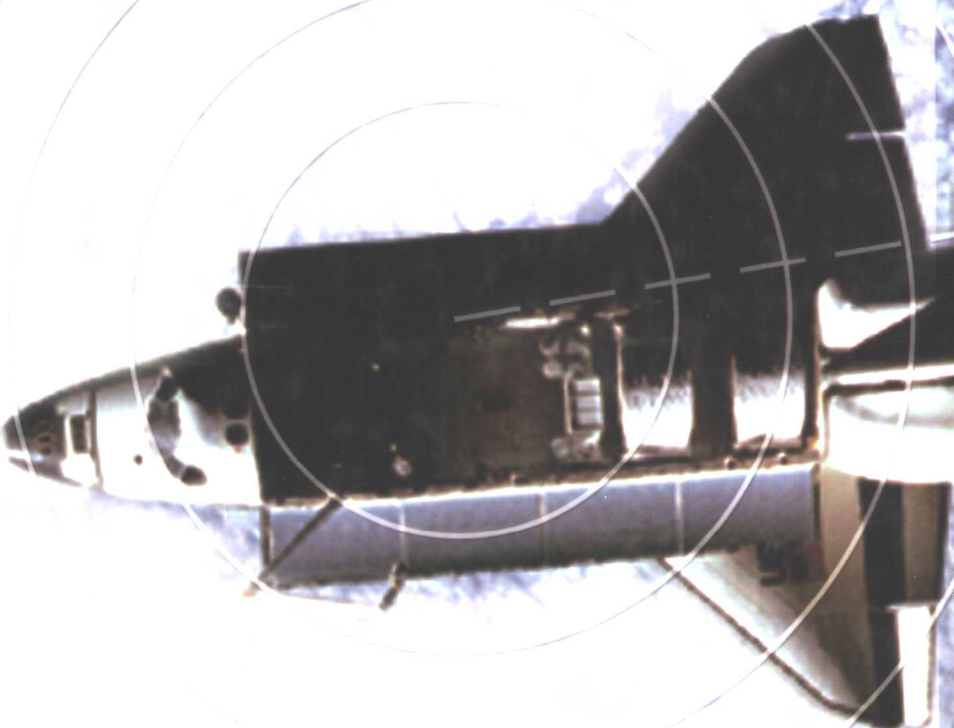
# Digital Signal Processing with Examples in MATLAB®



Samuel D. Stearns

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# Digital Signal Processing

## with Examples in MATLAB®

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# *Dedication*

*An intelligent man acquires knowledge,*
*and the ear of the wise seeks knowledge.*

*—Proverbs 18:15*

# *Foreword in Memory of Richard W. Hamming (1915-1998)*

The information age in which we are living has underlined the importance of signal processing, while the development of solid-state integrated circuits, especially in the form of general-purpose minicomputers, has made practical the many theoretical advantages of digital signal processing. It is for these reasons that a good book on digital signal processing is welcomed by a wide range of people, including engineers, scientists, computer experts, and applied mathematicians.

Although most signal processing is now done digitally, much of the data originates as continuous, analog signals, and often, the result of the digital signal processing is converted back to analog form before it is finally used. Thus, the complex and often difficult to understand relationships between the digital and analog forms of signals need to be examined carefully. These relationships form a recurring theme throughout this book.

The theory of digital signal processing involves a considerable amount of mathematics. The nonmathematically inclined reader should not be put off by the number of formulas and equations in the book, because the author has been careful to motivate and explain the physical basis of what is going on and, at the same time, avoid unnecessarily fancy mathematics and artificial abstractions. It is a pleasure, therefore, to recommend this book to the serious student of digital signal processing. It is carefully written and illustrated by many useful examples and exercises, and the material is selected to cover the relevant topics in this rapidly developing field of knowledge.

*This foreword is included in memory of Professor Richard W. Hamming, one of the world's great mathematicians and a pioneer in the development of digital signal processing. The fact that his comments on signal processing, written in 1975 for the progenitor of this text, are just as relevant now as they were when they were written, is a testimony to Dr. Hamming's foresight and genius.*

**Samuel D. Stearns**

# *Foreword by Delores M. Etter*

The combination of digital signal processing techniques with today's computing capabilities has allowed us to address many difficult problems. For example, computers can "understand" human speech, translate it to another language, and then synthesize the translated speech in a new voice. Satellites can transmit information from one part of our world to another in the blink of an eye. Medical doctors can diagnose, and surgically correct, heart valve problems in a baby before it is born. Biometric signals, such as fingerprints and iris scans, can be stored in a "smart card" to allow positive identification of the person using the card.

The capabilities that allow solutions such as the ones mentioned above all use digital signal processing algorithms to extract information from signals collected from the environment around us. These algorithms are based on fundamental principles from mathematics, linear systems, and signal analysis. This text will guide you through the mathematics and electrical engineering theory using real-world applications. It will also use MATLAB, a software tool that allows you to easily implement the signal processing techniques using the computer, and to view the signals graphically. Digital signal processing (DSP) and MATLAB together open up a fascinating new world of possibilities.

The author of this text, Sam Stearns, brings a unique perspective from a career of solving difficult problems at one of our country's finest laboratories — Sandia National Laboratory. He also brings a lifelong passion for education. Sam has been a teacher, a guide, a mentor, and a friend to me for over twenty years. I shall always be in his debt for introducing me to the wonders of digital signal processing.

*Dr. Delores M. Etter, Professor, Electrical Engineering, United States Naval Academy at Annapolis, MD, is a former Deputy Under Secretary of Defense for Science and Technology. She is also the author of a number of engineering textbooks, including several on MATLAB.*

# *Preface*

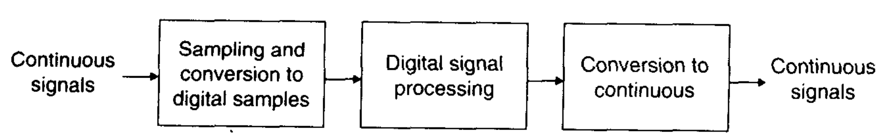
This book is intended to be a text for courses in digital signal processing. It is written primarily for senior or first-year graduate engineering students, and it is designed to teach digital signal processing and how it is applied in engineering.

A progenitor of this text, *Digital Signal Analysis,* was published in 1975. At that time, there were only a handful of texts in the signal processing area, but now there are many texts, and we must give reasons why still another is needed.

One reason is that there is always room for another text in an area like digital signal processing, which is still expanding and finding new applications, provided the text is up to date and covers the fundamental areas, yet does not leave gaps in its development that must somehow be bridged by the reader. We endeavored in this text to provide a complete development of the main subjects in modern digital signal processing. These subjects, we believe, have changed over time in nature and relative importance, and this book is meant to respond to these changes and present the main subjects, or "basics," applicable in today's technology.

In a way, it is much more challenging now than it used to be to produce a text on DSP, because the field has grown in so many directions. It is more difficult now in a single book to cover the relevant theory and applications, and yet present the whole subject in a constructive manner, that is, in a manner conducive to teaching that proceeds logically from one topic to the next without omitting derivations and proofs.

A second and perhaps more important reason for this text, in particular, lies in our understanding of current applications of digital signal processing to engineering problems and systems. In most applications, digital signals, which are simply vectors or arrays with finite numbers of discrete elements, are worth processing and analyzing only because they represent discrete samples of continuous phenomena. That is, the engineer who applies the techniques described in a text like this is normally working with at least two, and usually all three, of the following operations:




Our premise is that the reader will find the text more useful, because we do not treat the central operation above as a subject in isolation. We try always to relate digital signal processing to continuous signal processing and to treat digital signals as samples of physical phenomena, just as the engineer must do when he or she applies digital signal processing to solve real problems.

Other important features of this text include the use of MATLAB® to provide examples of signal processing, digital system design, solutions to exercises, coding and compression algorithms, etc. The MATLAB language has become a standard in signal processing, because it is so easy to understand and use. Its plotting and graphics functions make it ideally suited to signal processing applications. We provide a brief MATLAB tutorial in Chapter 1, so that even if the reader does not use MATLAB, he or she can easily read and understand the algorithms and examples in the text. All of the examples and functions presented in the text, as well as answers to exercises, are included with the software provided for the reader on the CRC PRESS LLC website (www.crcpress.com).

As indicated in the table of contents, other features include areas that are now considered basic but are not always covered in signal processing texts, including an introduction to statistical signal processing, the discrete cosine transform, an introduction to time-frequency and wavelet transforms, coding and compression of signals and other digital data, least-squares system design, and an introduction to adaptive signal processing.

The author of this text is greatly indebted to a number of students, colleagues, and friends, who have been patient and interested enough to comment on large portions of the manuscript. Several have reviewed the entire manuscript and suggested many changes and improvements, in particular, Professors Chaouki Abdallah, Scott Acton, Majid Ahmadi, Victor DeBrunner, Dr. Robert Ives, Jeff Kern, Chip Stearns, and Dr. Li Zhe Tan. The students are those who have attended classes where the manuscript was first used, at the University of New Mexico, and in short courses taught in industry. Other friends and colleagues who have encouraged the author and commented on the text's contents include Dr. Nasir Ahmed, Professors James Matthews, Wasfy Mikhael, Marios Pattichis, Balu Santhanam, Michael Soderstrand, and Dr. Stanley White.

The author is also thankful for his children, who kept asking, "How's the book coming?" (and were really interested in the answer), and to his wonderful wife, Mary, who has patiently read and helped correct the entire manuscript.

Finally, in writing this text, the author has tried to adhere to the descriptions and ideals implied in the two Forewords: First, we hope this project will honor the memory of Richard W. Hamming, one of the world's great mathematicians, who made so many contributions, who inspired so many through his teaching, and who laid so much of the foundation on which signal processing is built today. Second, in this book, we have strived to meet the high standards of clarity and logic in teaching and research set by Professor Delores M. Etter, formerly Deputy Director of Defense Research and Engineering and now Office of Naval Research Distinguished Chair in Science and Technology at the United States Naval Academy.

For MATLAB product information, please contact:

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Web: www.mathworks.com

# *Biography*

**Samuel D. Stearns** has been a Distinguished Member of the Technical Staff at Sandia National Laboratories, Albuquerque, New Mexico, and is now a consultant. His principal technical areas are digital signal processing and adaptive signal processing. His most recent work has been in teaching these subjects at the University of New Mexico and in industrial short courses, as well as in consulting.

Dr. Stearns has taught and advised dissertation research at several universities throughout the United States, including the Universities of Central Florida, Colorado, and New Mexico, and Kansas State and Stanford Universities. He is a Fellow of the IEEE. His Fellow citation reads, "For contributions to education in digital and adaptive signal processing systems and algorithms." He has served in various IEEE activities and has published a number of papers in signal processing, adaptive signal processing, and related areas. He is author or coauthor of the following texts:

*Digital Signal Processing with Examples in MATLAB* (2003)
*Signal Processing Algorithms in MATLAB* (1996)
*Signal Processing Algorithms in Fortran and C* (1993)
*Digital Signal Analysis, 2nd Ed.* (1990)
*Signal Processing Algorithms* (1987)
*Adaptive Signal Processing* (1985)
*Digital Signal Analysis* (1975)

# Contents

# 1

# *Introduction*

## 1.1 Digital Signal Processing

Digital signal processing (DSP) has become an established area of electrical and computer engineering in relatively recent times. In fact, because all types of signals, when they are processed, are now most often processed in digital form, scientists and engineers in all disciplines have come to at least a nodding acquaintance with the subject in order to understand what their instruments and displays are trying to tell them.

Compared with other areas of electrical engineering that share their analytic tools with DSP, like fields and waves or communication theory or circuits or control theory, DSP has a relatively short history. The author of this text has many colleagues and friends who have built the foundations of DSP from the ground up, and most of them are still active contributors at the time of this writing. The list is too long to print here, but some of them can be found in the lists of "early" references at the end of this chapter[1–15] and subsequent chapters.

This book is the result of an attempt to organize the major contributions of these pioneers and describe subjects that are basic to DSP as it exists currently, that is, subjects that one would find essential to or at least useful in the understanding of most current applications of DSP. There are now many special topics within DSP, such as image processing, digital communications, signal coding and compression, and so on, so that no one is any longer an expert in all areas. But, hopefully, in this text, the reader will find subjects that are basic to (and useful in) all these different areas.

In other words, there is so much diversity in types of signals and types of processing and types of processors that we really only have two choices. One choice is to provide a complete text on a single subject, such as one of those just mentioned or wavelet transforms or digital filters, for example. The second choice is to provide a connected knowledge base consisting of subjects basic to all kinds of digital signal processing, so that the reader may acquire at least the fundamentals but not all of the details.

It is this second option that we have chosen here. We hope that this text will introduce the reader to the basic areas of signal processing and also provide a reference for the reader who wishes to review the fundamentals of DSP.

## 1.2 How to Read this Text

There are two main ways to read this text. One way is to read it more or less in the order in which it is written and work some of the exercises as you go. Each chapter, especially in the first part of the book, depends on subjects discussed in previous chapters. If the subjects are read and learned in order, the reader should end with a good foundation for work in most areas of signal processing.

The second way is to look in the text for a particular subject, such as filters, coding, spectral estimation, etc. The best way to do this is to look in the index. If your subject is not listed there, look in the table of contents for a related area. If you have at least some familiarity with DSP as well as the basic mathematics needed for analysis, there is no need to read all the text. The treatment of each individual subject is meant to be self-contained, although the text may refer to previous, more basic subjects.

With these two approaches in mind, the reader may now decide whether to read the remainder of this chapter or skip to a subject of interest. The rest of this chapter consists mostly of reviews of some basic mathematics and formulas useful in signal and system analysis, as well as the MATLAB language, which has become a standard in signal processing analysis and system design. If you are familiar with the mathematics and are already a MATLAB user, these reviews may be skipped and treated as reference material.

## 1.3 Introduction to MATLAB

MATLAB, a product of The MathWorks, Inc., is currently in use by a large fraction of the DSP community. Three principal reasons for this are (1) the syntax allows the user to do most DSP operations with very simple lines of code, (2) with MATLAB's graphics support, one can produce publishable plots with minimal effort, and (3) most importantly, you do not have to be an expert in MATLAB to use MATLAB. The syntax is easy to read and learn and use with almost no prior groundwork.

Most of the DSP examples in this text use the MATLAB language. This does not mean that the reader needs to own or operate MATLAB software, although having access to a computer running MATLAB gives a definite advantage. But the language is useful as a standard for describing DSP operations and algorithms. For readers unfamiliar with MATLAB notation, we begin here with some basics. If you do not own or operate MATLAB, you may wish to view the use of MATLAB in this text as a convenient and easy-to-read standard system for describing signal processing procedures. For more depth in MATLAB, the reference by Etter[19] at the end of this chapter not only describes MATLAB but also addresses several of the topics in this text, with applications.

MATLAB uses single expressions called *commands*, which may be assembled into sets of commands called *m-files* (because the file extension is "m") or *functions*, which may be called by other m-files or functions. In this text, when you see lines inside a text box, these will usually be MATLAB expressions. For example,

```
» x=4;
» y=[2,3,4];
» z=[1 2 3; 4 5 6];
```
(1.1)

Each of these lines is an individual expression (command) as indicated by the *command prompt* at the beginning of the line. Each line ends with a semicolon; if not, then MATLAB would echo the results of the line as in (1.2) below. *Row elements* of an array may be separated by commas (as in $y$) or by spaces (as in $z$). *Rows* are separated by semicolons (as in $z$). The results of (1.1) are seen in (1.2) when the MATLAB expression "*x,y,z*" is entered without a semicolon at the end.

```
» x,y,z
x=
    4
y=
    2   3   4
z=
    1   2   3
    4   5   6
```
(1.2)

From basic expressions like these, we can proceed rapidly and simply in the following sections to expressions that accomplish complicated DSP operations.

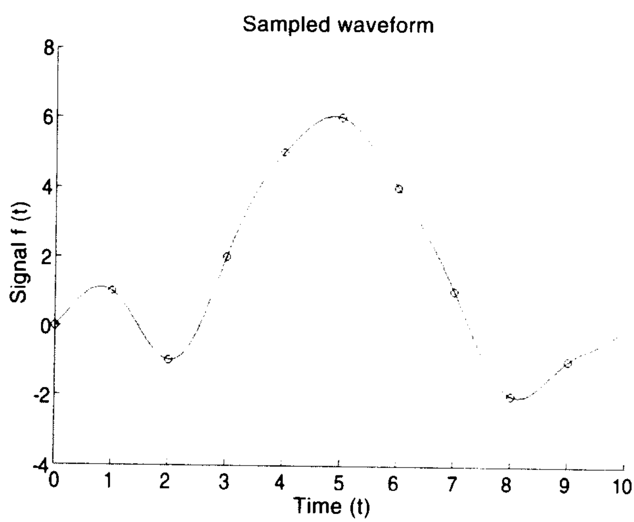
## 1.4 Signals, Vectors, and Arrays

High-level computer languages such as MATLAB are designed to process ordered sequences of elements, that is, *variables*, *vectors*, and *matrices*. As used in this text, these three terms form a hierarchy in the sense that:

- A variable is a single element (integer, real, or complex) [like $x$ in (1.1)].
- A vector is an ordered sequence of variables [like $y$ in (1.1)].
- A matrix is an ordered sequence of vectors [like $z$ in (1.1)].

We usually use *array* as an inclusive term to designate a vector or a matrix. Sometimes our use of *vector* here is confusing at first, because we are used to vectors in electromagnetics or mechanics—in three-dimensional space—with exactly three components. But these are really just ordered sequences of three variables and are thus vectors in the sense used here, and we must now allow more dimensions in order to have vectors that represent signals in a "signal space," which we define as follows.

Figure 1.1 shows a sampled waveform and its corresponding signal vector. When we say a waveform is *sampled*, we mean that its amplitude is measured and recorded at different times. Usually, these different times are equally spaced in the time domain, but not always. We assume that the samples are equally spaced. The interval between samples is called the *sampling interval* or *time step*. In Figure 1.1, the time step is $\Delta t \equiv T = 1$. In this example, there are ten sample values, and the *signal vector* consists of the ordered sequence of the ten integer samples. Because each sample is a variable, we may say that the *signal space* of the sampled waveform in Figure 1.1 has ten dimensions, one for each sample. Thus, in this sense, the signal space of a sampled waveform has as many dimensions, or *degrees of freedom*, as there are samples in the waveform.




Sample vector

[0 1 -1 2 5 6 4 1 -2 -1]

**FIGURE 1.1**
Sampled waveform and sample vector with time step $T = 1$.