

Experimental Robotics I

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139

V. Hayward, O. Khatib (Eds.)

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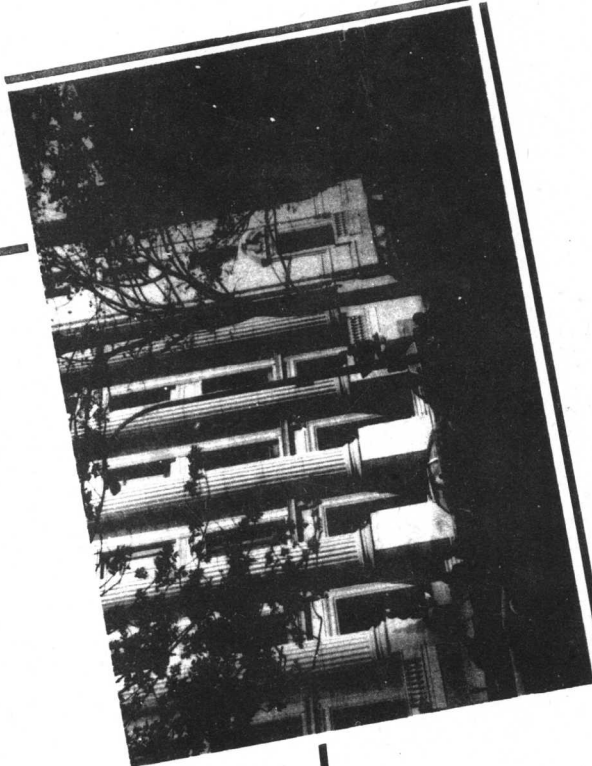
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About the Editors

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Preface

Experimental Robotics 1 — The First International Symposium is the first attempt at collecting works in Robotics from the point of view of experimental research. The meeting at which these contributions were presented took place in Montréal in June 1989. It is the first of a series to be organized in a circular fashion around North America, Europe, and Asia.

The series of events that led to this meeting can be traced back to Spring 1987 in Albany, New York. Several members of the organizing committee, who were attending a SIAM conference on Applied Geometry, began to discuss the major trends that would underly Robotics Research toward the rapidly looming end of this Century. We all agreed that experiments were called to play a larger role in the very way Robotics Research will be approached. Of course we did not mean that theoretical developments are not important, we simply felt that their importance can only be assessed through experimentation, that is, synthesis.

We proposed that the presentations be centered on theories and principles, as applied to robotics, that are validated by experiments. One of the conclusions of the meeting was thus to draw a clear distinction between "demonstrations" and "experiments". An increasing amount of researchers in the field of robotics no longer feel satisfied with hypothetical developments. They strive to distinguish what *can* be done from what *could* be done, and are willing to submit their theories to the test of physical implementation. The content of this collection of contributions reflects a cross-section of the current state of robotic research from one particular aspect: experimental work, and how it affects the theoretical basis of subsequent developments.

Of course, the selection of topics: the study of friction, calibration issues, and design of manipulators, for example, reflects this theme. Some of these problems may have been considered in the past as secondary ones, but they are now revealed as being critical, and we conjecture that progress toward their solution will have a significant impact on most aspects of robotics research. A cursory examination of the contents of the papers shows that, in Robotics, once again, practice is sometimes ahead of theory: successes and failures are rarely fully explained by the theory and investigators are left with conjectures. This is a further justification for the choice of the experimental theme.

The international program committee was composed of the following individuals: V. Hayward, *McGill University*, Canada; O. Khatib, *Stanford University*, USA; J. Angeles, *McGill University*, Canada; R. Chatila, *LAAS/CNRS*, France; J. Craig, *Silma*, USA; P. Dario, *University of Pisa*, Italy; B. Espiau, *IRISA/INRIA*, France; G. Hirzinger, *DFVLR*, FRG; K. Salisbury, *MIT*, USA; and T. Yoshikawa, *Kyoto University*, Japan. A total of 35 papers have been included in the program, representing Australia, Belgium, Canada, England, France, Germany, Italy, Japan, USA, and Yugoslavia. A keynote lecture given by Prof. Lozano-Perez from MIT on the topic "Tasks, Experiments, and Strategies" kicked off the meeting with the delivery of numerous inspiring ideas. Due to the small size of this meeting and the quality of the attendees, a large amount of interaction took place during the three days of this meeting.

The international committee has proposed that the next meeting should take place in Toulouse, France, in the Spring 1991. The committee has asked Dr. Raja Chatilla from LAAS-CNRS, Toulouse, France, and Dr. Gerd Hirzinger from DLR, German Aerospace Research Establishment, Institute for Flight Systems, to co-chair the next meeting.

The McGill Research Center for Intelligent Machines (McRCIM) hosted this meeting on the McGill University campus. All the persons involved in this center must be thanked for helping to make this event possible. In particular, Prof. Martin Levine, director of McRCIM, gave a warm welcome address which created an informal, yet productive atmosphere. Dean Bélanger, forefather of McRCIM, then discussed the important connection between theory and experiments in Robotics. He must be thanked for his contribution and support. Prof. Lozano-Perez's keynote lecture alone made the effort of putting this meeting together worthwhile. The members of the program committee must be gratefully acknowledged for accepting to shoulder the difficult task of selecting the papers.

This meeting could not have taken place without the backing of many individuals, institutions and companies: Norman Kaplan from the National Science Foundation, Washington D.C.; Christine Quérido from "Les Fonds pour la Formation des Chercheurs et l'Aide à la Recherche (FCAR), Sainte Foy, Québec"; Pierre Girard and René Blais from "L'Institut de Recherche d'Hydro-Québec (IREQ), Varennes, Québec"; Fred Christie and Pierre Maltais from The Canadian Space Agency, Ottawa, Ontario; Len Allen and Roy Hoffman from CAE Electronics Limited, Saint Laurent, Québec; Samad Hayati from the Jet Propulsion Laboratory, Pasadena, California; Ian Rowe and Ravi Ravindran from SPAR Aerospace Limited, Weston, Ontario.

In the end, the credits must go to the authors for the quality of their contributions and their availability during the conference. The final kudos go to Margaret Dalziel, Manager of McRCIM, who skillfully engineered the organization of this meeting and whose talent was very much appreciated.

Finally, we hope that the video document which accompanies this collection of contributions will prove to be a useful illustration of the reported research.

Montréal, November 1989

Vincent Hayward (McGill University)
Oussama Khatib (Stanford University)

Introduction

The classification adopted in this collection of contributions proceeds along the lines of a nearly traditional division of topics in robotics. The reader will notice numerous cross-correlations between sections. As with most classifications, it is perhaps artificial and somewhat unsatisfactory.

Section 1 covers the control of flexible limbs, intermittent tasks, the control of cooperating robots, force control, and adaptive control. The second section deals with design issues: control of friction at low velocities, actuators, joints, and manipulators. Section 3 investigates questions in perception in terms of model construction, task guidance, visual servoing, and tactile feedback. Section 4 tackles kinematic problems such as inversion and calibration. Finally, section 5 deals with problems in motion planning.

Most of the papers share three overriding concerns that we might wish to use as guidelines for future work in Experimental Robotics.

Dealing with uncertainty, is one of these concerns. It is certainly not a new issue in engineering, but it is treated in various and particular ways by the robotics approach. Experimental robotics research suggests that there might be dual perspectives to reducing uncertainty. Conventional wisdom tells us that we need to "see to act" (as in control), whereas the dialectic reversal of this proposition requires us to "act to see" (as in perception). Dealing with uncertainty is an attribute of intelligence and autonomy. Whether it lies in the effectors, the sensors or elsewhere is still an open question; what is sure is that robots need a lot of it.

The second general theme which stems from these contributions is the *extension of the task repertoire*. This certainly indicates a major trend in robotics research. Clearly, no systematic methodology is proposed; however, biological systems seem to provide the largest source of inspiration (i.e. running, juggling, etc...). It is of course an exercise in synthesis, for which the tools are difficult to find, whereas analytical tools are already abundantly available.

The third general direction suggested by the reading of the papers is *redundancy and cooperation*. Advanced robots will undoubtedly be endowed with large amount of redundancy from the viewpoints of action and perception. This redundancy must be orchestrated to achieve cooperation. This is the running theme of a large amount of current research. The necessity of multi-sensory perception systems is generally agreed upon. Similarly, multi-actuator action systems are also coming into focus. Cooperative action, for lack of a better word, should become the center of focus for understanding advanced robotic systems.

Each of these three themes encompasses the others and it can be observed that most of the papers incorporate elements of all three.

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Theory and Experiments in Selecting Mode Shapes for Two-Link Flexible Manipulators

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Abstract

The design of a control system is typically based on a model of the actual plant. The achievable performance is thus intimately related to the modelling accuracy. A popular method for modelling flexible manipulators is the assumed-modes method. However, in order to generate an accurate model with a minimal number of modes, appropriate component mode shapes must be selected. This paper explores the selection of component mode shapes to be used in models for two-link flexible manipulators. The theoretical natural frequencies and system mode shapes predicted by these models are compared to those of the experimental Stanford Multi-Link Flexible Manipulator. Strobe photographs taken to capture the experimental system mode shapes are included.

1 Introduction

There are many robotic manipulators that possess significant structural link flexibility. These manipulators, used in both space and industrial environments, are utilized for dangerous situations, handling large payloads, and executing precise maneuvers. The speed and accuracy with which these tasks are performed depends intimately on the control system implemented. Since the design of the control system is based on a model of the actual manipulator, the achievable performance is limited by the modelling accuracy. For example, if the model is simplified by ignoring link flexibility, the control bandwidth must be significantly lower than the first natural frequency of the manipulator [1].

If, however, link flexibility is suitably modelled and advanced control strategies are formulated from these models, the performance of existing flexible manipulators can be improved and certain characteristics of link flexibility can be advantageously used in future designs. For instance, flexible manipulators offer high payload-to-mass ratios in addition to light-weight and power-efficient configurations.

Single-link flexible manipulators can often be adequately described by a linear model [2]. That is, poles and zeros of the transfer function of a single-link manipulator can be accurately identified experimentally, and a control system can be designed. The more useful configuration of a two-link manipulator, however, must be described by a nonlinear model. In this case, the physical manipulator parameters must be identified and a model suitably developed so that the control system can be designed using accurate equations of motion.

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[†]Charles Lee Powell Professor and Chairman, Department of Aeronautics and Astronautics.

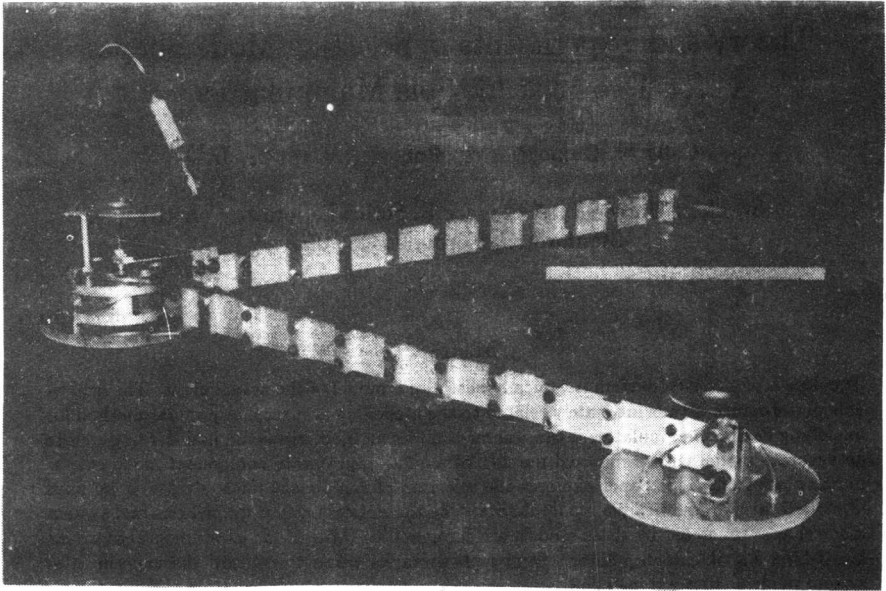


Figure 1: Photograph of the Stanford Multi-Link Flexible Manipulator

The motivation to pursue more appropriate mode shapes arose when a rather large number of cantilever modes was required to sufficiently match the model of an experimental manipulator to a sine-sweep identification [5]. (The experimental two-link manipulator was configured with a rigid upper arm and a very flexible forearm.) Proper mode shape selection was even more critical for controller design. As a result of limited computing power with which to control the experimental manipulator, a model consisting of a small number of cantilever modes was first employed to design a high-performance optimal end-point controller. The resulting controller worked fine in simulation, but when implemented on the experimental manipulator, was easily be driven unstable, indicating that refinement of the theoretical model (or mode shapes) was required [6].

We consequently set out to explore both theoretically and experimentally the use of different component mode shapes for an assumed-modes model of a two-link manipulator with flexibility in both the links. (The word "component" is used in the sense that these mode shapes are "building-blocks" for predicting the true system mode shapes.) Of particular importance is the number of states (or order) necessary to accurately model the physical manipulator, as this number will influence the complexity and implementability of the control system. This paper presents some of our results and identifies how revisions to the model were guided by the experimental facility.

2 The Experimental Manipulator

The experimental Stanford Multi-Link Flexible Manipulator has been constructed to serve as a test bed for validating new theories and to suggest new directions for basic research. A photograph of the manipulator is shown in Figure 1. The manipulator operates in the horizontal

plane on top of a flat 4 ft by 8 ft granite table. The shoulder motor is mounted on the side of the table, while both the elbow motor and tip pad are supported by air cushions on the table. Located on each of the motor shafts are rotary variable differential transformers (RVDT's) to measure joint angles. Angular rate signals are generated by passing the RVDT signals through analog band-pass filters. End-point sensing is achieved using a technique developed in the ARL by Schneider [7]. A CCD television camera tracks a special variable reflectivity target located at the manipulator end-point as seen in the photograph of Figure 1.

The experimental apparatus is designed to permit a variety of configurations. Rigid or flexible links of different cross-sections, lengths, or mass distributions can be interchanged to study the effects of structural flexibility in either or both links. The configuration studied in this paper, as seen in the photograph, consists of a flexible upper arm and a flexible forearm. "Mass Intensifiers" [6], which are discrete masses evenly spaced along the length of the flexible links, have been added to the flexible links in order to increase the link inertia without changing the link flexibility. A complete set of the geometric and mass properties of the experimental two-link flexible manipulator is given in [8].

3 Modelling

We are interested in developing models of the two-link flexible manipulator that contain the significant dynamics of the system and will be suitable for control system design and implementation. Other models are also developed that are useful for verifying the models for control.

In the models that follow, each link of the manipulator has a hub and a tip, each with accountable mass, inertia, and mechanical offsets. The notation is easily extensible to configurations with more than two links.

3.1 Assumed-Modes Model

For the purpose of control system design, we have used the assumed-modes method to derive the equations of motion of a two-link flexible manipulator. Using this modelling technique we are able to obtain a finite set of ordinary differential equations (useful for controller design). We also may retain the nonlinear terms in the equations of motion so that the model will be accurate when the manipulator undergoes rapid, large-angle slews. The assumed-modes method has the potential to properly model the manipulator in a relatively small number of states. However, the choice of appropriate mode shapes is crucial.

The manipulator schematic diagram used to develop the assumed-modes model is given in Figure 2. The $\theta_{(.)}$'s, $w_{(.)}$'s, $x_{(.)}$'s, and $s_{(.)}$'s are the rigid-body rotations, deflections of the flexible links, distances along the undeflected axes of the links, and arc lengths along the links, respectively. The subscript 1 designates the upper arm and the 2 the forearm. T_1 and T_2 are the shoulder and elbow torques.

A proper analytical description of the motion of the manipulator can be obtained from the shoulder and elbow angles θ_1 and θ_2 and from the lateral displacements of the flexible links $w_1(x_1, t)$ and $w_2(x_2, t)$ and their first and second time derivatives. The assumed-modes method is then used to develop ordinary differential equations by making the approximation that the deflections of the flexible links may be expressed as a weighted superposition of a finite set of spatial mode shapes ($\phi_{(.)}(x_{(.)})$'s) that satisfy the geometric boundary conditions of the link.

The lateral displacements of the flexible links are then written as

$$w_1(x_1, t) = \sum_{j=1}^{n_1} \phi_{1j_1}(x_1) q_{1j_1}(t) \quad (1)$$

$$w_2(x_2, t) = \sum_{j=1}^{v_2} \phi_{2j}(x_2) q_{2j}(t) \quad (2)$$

where $v_{(.)}$ is the number of modes, or degrees of freedom, used to describe the deflection of a link and the $q_{(.)}(t)$'s are the time dependent modal amplitudes or weights.

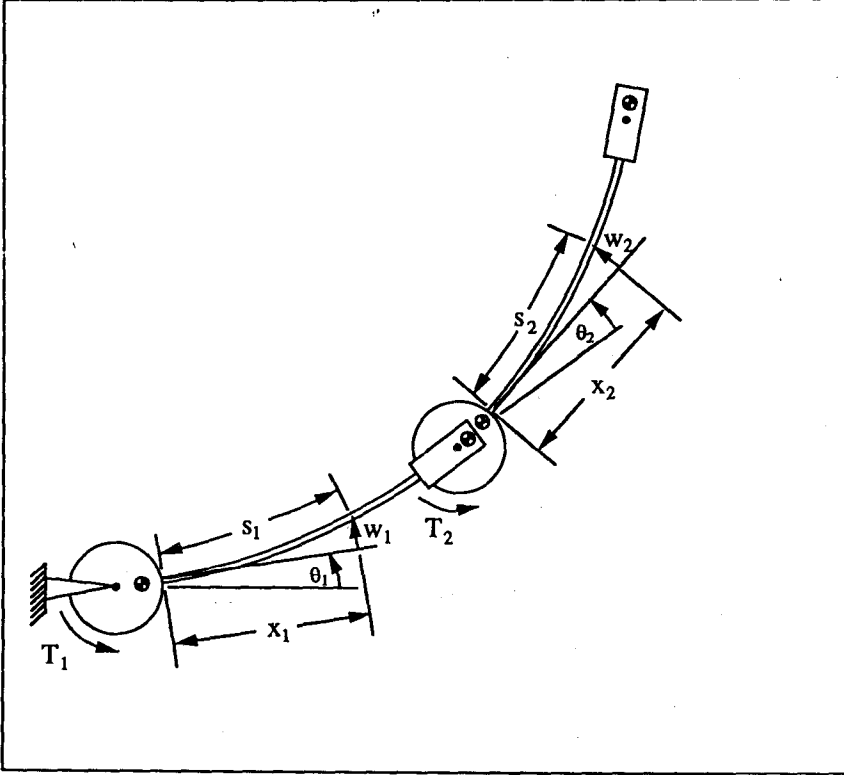


Figure 2: Assumed-Modes Manipulator Model

Link Foreshortening When using an assumed-modes model, link foreshortening may introduce a significant kinematical effect. In particular, if we excite the experimental manipulator with a sufficiently large amplitude sinusoidal disturbance (to observe a natural shape of the manipulator), link foreshortening will have a significant effect and should be included in the model.

Accounting for the link foreshortening can be simplified if we assume that the mode shapes depend on the arc length of the deflected beam [9], rather than on the distance along the undeflected axis. In this case, the projection of the flexible link on the undeformed axis can be calculated from

$$x_i = \int_0^{s_i} \sqrt{1 - \left(\frac{\delta w_i(\sigma, t)}{\delta \sigma} \right)^2} d\sigma. \quad (3)$$

An example of how this effects the spatial representation of a "generic" mode shape is presented in Figure 3. The figure shows how a *plot* of the mode shape at various amplitudes is affected by

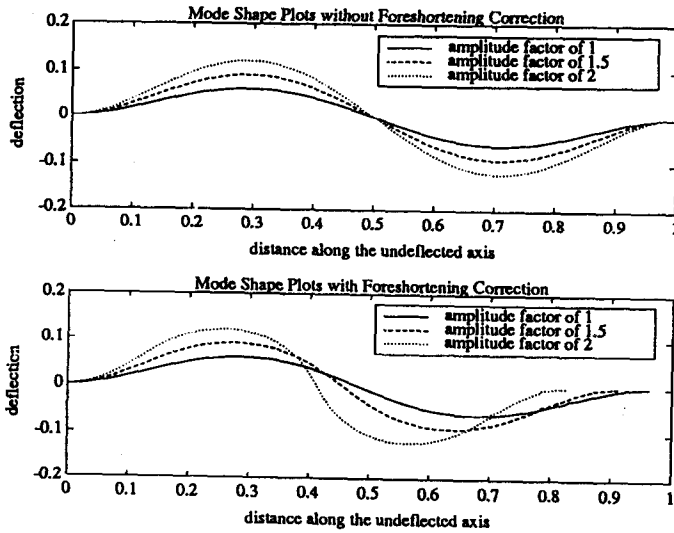


Figure 3: How the Plot of a Generic Modeshape at Various Amplitudes Changes with and without Link Foreshortening Correction

the absence and presence of the foreshortening correction.

The link foreshortening correction is used in the remainder of this paper.

3.2 Lumped Spring-Mass Model

For the purpose of verifying and further developing the assumed-modes model, we've formulated an additional model that treats each of the flexible links as a series of rigid link segments interconnected by torsional springs. This model is of particular value since the equations of motion, although high in order for a large number of segments (difficult for control), are independent of the mode shapes that must be chosen for the assumed-modes method. Note, however, that the torsional spring stiffnesses must be appropriately selected.

The manipulator schematic diagram used to develop the lumped spring-mass model is given in Figure 4. The hub and tip inertias are the same as in the assumed-modes model, and the segment geometric and mass parameters are chosen to match those of the flexible link in the assumed-modes model. The spring stiffnesses, $k_{(j)}$, are assumed to be constant for each link. n_1 is the number of segments in the upper arm, and n_2 is the number of segments in the forearm.

Selecting the Torsional Spring Stiffness The torsional spring stiffnesses are found by matching the end-point deflection of a lumped spring-mass model of a cantilevered beam, shown in Figure 5, to that of a cantilevered continuous flexible beam. A force F is applied to the end of the spring-mass model and the spring stiffness k is selected so that the deflection y at the end-point is the same as that of a continuously flexible beam. Assuming small deflections (i.e. neglecting higher than first order terms in the $q_{(j)}$'s) the appropriate spring stiffness can be