Transform Coding of Images

R. J. CLARKE

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R. J. CLARKE
Department of Electronic and Electrical Engineering
University of Technology
Loughborough, England

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MICROELECTRONICS AND SIGNAL PROCESSING

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Preface

Over the past twenty years or so, the field of electrical engineering has been revolutionised by the advent and the subsequent rapidly increasing use of the digital computer. Not only are many analytical operations which were originally carried out in analogue form now performed more conveniently and speedily via the (discrete) digital computation process, but, and perhaps of more significance, procedures can now be executed which were once considered quite impossible, for example, the manipulation or processing of frames of video data on a realtime basis. Even where still images are concerned, the computer now allows complex high-speed processing operations, which typically involve large numbers of non-integer multiplications, to be applied to high-resolution data emanating, for example, from earth satellites. More generally, the development of information provision services (including the transmission of pictorial material) has been fostered by the close relationship between the digital computer and digital communication systems.

Digital communication links, although offering several advantages when compared with their analogue counterparts, do require a much higher equivalent bandwidth, and this is particularly a problem where image data is concerned, since such source material requires a large bandwidth in any case. It is not surprising, therefore, that interest in bandwidth compression techniques, originally analogue but now digital, has a long history, particularly as applied to image communication or storage, and the present volume is concerned with one appropriate procedure—transform coding, which, to risk a sweeping oversimplification, allows the reproduction of images of reasonable quality at average rates of perhaps 0.5 to 1.0 bit/element.

The main part of this book is concerned with the theoretical and practical aspects of transform coding systems intended to process still, or moving sequences of, images. However, two other topics are given more consideration than might, at first sight, seem appropriate. The successful operation of any bandwidth compression system is crucially dependent upon specific properties of the source data, and here such statistical measures are given detailed consideration in the light of their effect on coder performance. Again, there is much interest nowadays in the development of image processing systems having regard to the perception, by an observer, of degradation in the reconstructed picture. Furthermore, it is possible to use simple models of the human visual system within an image coder design in order to improve its performance, and the book, therefore, includes material on these aspects of image coding also.

An attempt has been made to exclude complex mathematical detail except where its presence is unavoidable (in particular, in parts of Chapters 2 and 9) and to demonstrate fundamental relationships via numerical examples. The book is intended for postgraduate students pursuing either course options or research in image coding and for those actively engaged in the design of coding systems, but it is suitable, in part, for final year undergraduate courses which include an image processing element. The list of references has been made as complete as practicable, and suggestions for supplementary reading have been made for those who wish to pursue specific topics in more detail.

Acknowledgements

Naturally, a volume of the present kind would not be possible without the dedicated work, over many years, of all those individuals and research groups throughout the world who have developed the subject to its present level of sophistication and have, moreover, freely published their results. It is also appropriate to acknowledge here the foresight of the electronics division of the Institution of Electrical Engineers (London), who, a few years ago, established professional group E4 (Electronic Image Processing), with whose members I have had many long and fruitful discussions on the topic. More specifically, I should like to thank the Society of Motion Picture and Television Engineers and the British Broadcasting Corporation for allowing me to employ their test-image material and to Taylor and Francis, Ltd., London, for permitting me to make use of the content of the paper "The Application of Image Covariance Models to Transform Coding", which appeared in Volume 56 of the *International Journal of Electronics* in 1984. Acknowledgement is also due to the Ministry of Defence for permitting the reproduction of Fig. 8.1 and to the IEE for allowing me to use material which forms the basis of Section 4.3.

It is worth emphasising that the book could not have been written without the financial support provided by the research laboratories of British Telecom, Martlesham, United Kingdom, which has enabled the author and his research students to pursue an active research programme in transform image coding at Loughborough University over the past six years. In turn, the enthusiastic labours of those students, in particular D. Allott (who carried out the coding simulations described in Chapter 5), W. K. Cham, K. N. Ngan and W. C. Wong, as well as of other members of the image coding research group (M. G. B. Ismail, M. N. Pauls and Badi Moussa), have, it is hoped, contributed usefully to the sum total of knowledge on the subject. Needless to say, any errors, omissions or, equally significant, points of contention concerning the material within the volume remain solely the responsibility of the author.

Finally, it is a pleasure to acknowledge the wholehearted support of the editorial staff of Academic Press and of the editors of the series, Professors J. R. Forrest and P. G. Farrell, in this project.

Symbols

The main symbols used are defined below. All notation employed is defined upon its first appearance, and, of necessity, some symbols are used for more than one quantity.

a_i	Predictor weighting coefficient
$A(f)$	HVS spatial frequency response
$A_{\mathbf{R}}$	Row transform
$b_{(0,0)}$	Bit allocation for the dc coefficient
b_i	Bit allocation for the ith coefficient
B	Binary wordlength
B	Colour component
B_{C}	Column transform
$[B_N]$	Matrix in DCT decomposition
c_j	jth transform coefficient
$\hat{c}_j$	Estimated value of c_j
$\mathbf{C}$	Transform coefficient vector
C	Channel capacity
$C(p, q)$	Transform coefficient array
COR$(\cdot)$	Correlation matrix of $(\cdot)$
COV$(\cdot)$	Covariance matrix of $(\cdot)$
D	Distortion parameter
DCT$(\cdot)$	Discrete cosine transform basis matrix of order $(\cdot)$
DFT(I)	Imaginary part of the discrete Fourier transform basis matrix
DFT(R)	Real part of the discrete Fourier transform basis matrix
DLB$(\cdot)$	Discrete linear basis matrix of order $(\cdot)$

DST(·)	Discrete sine transform basis matrix of order (·)
e	Error term (suitably subscripted and/or indexed)
e	Quantisation distortion
$E(\cdot)$	Expectation operator
f	Spatial frequency (cycles/deg)
$\mathbf{f}$	Data vector
$f(k, l)$	Input array of picture elements
$[F]$	Two-dimensional data array
G	Colour component
$G(\omega)$	Visual contrast sensitivity
$h(\cdot)$	Differential entropy of (·)
$h(\theta)$	Line spread function
$[H]$	Horizontal process correlation matrix
$H(\cdot)$	Walsh–Hadamard transform basis matrix of order (·)
H_N	Entropy of N level quantiser
$H(x)$	Per-element entropy of data sequence
$H(x, y)$	Joint entropy of two data sequences
$H(y/x)$	Conditional entropy of two data sequences
$H(\omega)$	Spatial frequency response
HA(·)	Haar transform basis matrix of order (·)
HCT(·)	High-correlation transform basis matrix of order (·)
HVS	Human visual system
$i(x)$	Self information of occurrence of event $x = X$
$i(x; y)$	Mutual information between events $x = X$ and $y = Y$
$I(x; y)$	Average mutual information between element sequences
I	Chrominance signal in NTSC system
I_N	Identity matrix of order N
$[J]$	Matrix in the development of the sinusoidal transform family
k	Data or coefficient index
KLT(·)	Karhunen–Loève transform basis matrix for the first-order Markov process of interelement correlation coefficient (·)
$K_{1,2,3}$	Colour conversion components
L	Total number of allowable luminance levels ($L = 2^B$)
L	Luminance (suitably subscripted)
m	Optical contrast
m.s.e	Mean square error
M	Image or sub-block dimension
n_l	Number of image elements taking on luminance level l
N	Transform, image or sub-block dimension
N_B	Number of bits allocated
N_L	Number of quantisation levels
n	Transform coefficient order
p_i	Probability of occupation of the ith quantiser interval

$p(x)$	Probability of element x taking on value X
$p(x, y)$	Joint probability of x and y
$p(y/x)$	Conditional probability of y given x
$P(\omega)$	One-dimensional power spectral density
$P(\omega_h, \omega_v)$	Two-dimensional power spectral density
q	Quantisation error
Q	Chrominance signal in NTSC system
Q	Selectivity factor of human spatial frequency response
Q	Entropy power
R	Colour component
$R(\cdot)$	Correlation function (suitably indexed and subscripted)
$R(\mathrm{D})$	Rate-distortion function
$R_\mathrm{L}(\mathrm{D})$	Lower bound on $R(\mathrm{D})$
$R_\mathrm{U}(\mathrm{D})$	Upper bound on $R(\mathrm{D})$
$[R_N]$	Matrix in DCT decomposition
$\mathrm{S}(\cdot)$	Slant transform basis matrix of order $(\cdot)$
$\mathbf{t}$	Transform basis vector (appropriately subscripted)
$T(\cdot)$	General transform operator
$[T]$	General transform basis matrix (subscripts C = complex, R = real)
$T_{1,2,3}$	Colour conversion components
u	Colour conversion component
$u(i)$	Scaling constant in symmetric cosine transform
U	Colour conversion component
U_d	PAL system colour difference signal
v	Colour conversion component
V	Colour conversion component
V_d	PAL system colour difference signal
$\mathbf{V}$	Correlation separation vector
$[V]$	Vertical process covariance matrix
$\mathrm{VAR}(X)$	Variance matrix of $[X]$
W	Colour conversion component
W	$e^{-2\pi j/N}$
W_k	kth element of window function
$W(x)$	Error weighting function
$W(\omega)$	One-dimensional spectral weighting function
$W(\omega_h, \omega_v)$	Two-dimensional spectral weighting function
x	Arbitrary image element
$\hat{x}$	Predicted value of x
x_D	Data dynamic range
x_i	Quantiser decision level
X	Particular luminance value assumed by x
$\mathbf{X}$	Data column vector

$[X]$	Two-dimensional data array
$\mathbf{X}_C$	$[X]$ in column stacked form
$\mathbf{X}_R$	$[X]$ in row stacked form
y	Arbitrary image element
y_i	Quantiser reconstruction level
Y	Particular value assumed by y
Y	Luminance signal
$\mathbf{Y}$	Transform coefficient vector
α	Parameter in exponential form of the first-order Markov correlation relation
α	One-step prediction coefficient
β	Parameter in sinusoidal transform development
γ	System (non)linearity parameter
$\Gamma(\cdot)$	Gamma function
Δ	Reconstruction error
$\Delta_{h,v}$	Mean square differences between adjacent horizontal or vertical elements
Δ_L	Luminance increment
$\Delta_{x,y,z}$	Inter-element spacing in axial directions
η_C	Transform decorrelation efficiency
η_E	Transform energy packing efficiency
θ	Quantisation/bit allocation parameter
θ	Subtended angle
θ	Distortion parameter
λ	Optimisation multiplier
λ_i	ith eigenvector
$[\Lambda]$	Diagonal matrix of eigenvalues
$[\Lambda]$	DCT matrix
ρ	Inter-element correlation coefficient
ρ_d	One-step diagonal correlation coefficient
ρ_h	One-step horizontal correlation coefficient
$\rho_H(x)$	Horizontal correlation coefficient as a function of x
ρ_k	k-step correlation coefficient
ρ_n	Single letter fidelity criterion
ρ_V	One-step vertical correlation coefficient
$\rho_V(y)$	Vertical correlation coefficient as a function of y
$\rho(x, y)$	Two-dimensional correlation function
σ^2	Variance (appropriately subscripted)
τ	Element delay
ω	Angular frequency
ω_h	Horizontal angular frequency (two-dimensional case)
ω_v	Vertical angular frequency (two-dimensional case)
ω_r	Radial angular frequency

Contents

Chapter 1

Image Data Compression: An Introduction

1.1 Introduction

It is a truism nowadays to say that the advent of the digital computer has revolutionised almost every technological discipline. Not least amongst those affected has been electrical engineering (the very field which itself has been responsible for the myriad advances in computer technology), and there are many who, though working as electrical engineers, now hardly come into the close vicinity of moving electrons in a professional capacity from one year to the next. Within that sphere of interest, arguably one of the most radical changes has taken place in communications, in which the accent is now heavily placed upon representation of the input information in discrete (sampled and quantised) form, leading to a much greater flexibility in the scope and nature of the operations which can subsequently be carried out on the data. In more general terms, the information 'revolution' appears to have created a demand for a wide range of new services, and it is just such a diversity of requirements that can be catered for by the now rapidly evolving networks of digital communications channels. Many of the new services envisaged (or already in operation) require the transmission and/or storage of image detail, and the days when the only pictures to be seen 'at a distance' were those of broadcast television are long past.

Given the more or less continual demand for increased bandwidth (or channel capacity) to enable more users to benefit from communication systems, and also the great demands made upon such systems by the desire to