Transform Coding of Images

R. J. CLARKE

73.772 C 599

Transform Coding of Images

R. J. CLARKE

Department of Electronic and Electrical Engineering University of Technology Loughborough, England





1985

15 2

ACADEMIC PRESS

Harcourt Brace Jovanovich, Publishers
London Orlando San Diego New York
Austin Montreal Sydney Tokyo Toronto

8850152

COPYRIGHT © 1985 BY ACADEMIC PRESS INC. (LONDON) LTD. ALL RIGHTS RESERVED.

NO PART OF THIS PUBLICATION MAY BE REPRODUCED OR TRANSMITTED IN ANY FORM OR BY ANY MEANS, ELECTRONIC OR MECHANICAL, INCLUDING PHOTOCOPY, RECORDING, OR ANY INFORMATION STORAGE AND RETRIEVAL SYSTEM, WITHOUT PERMISSION IN WRITING FROM THE PUBLISHER.

ACADEMIC PRESS INC. (LONDON) LTD. 24–28 Oval Road LONDON NW1 7DX

United States Edition published by ACADEMIC PRESS, INC.
Orlando, Florida 32887

British Library Cataloguing in Publication Data

Clarke, R.J.

Transform coding of images.—(Microelectronics and signal processing)

- 1. Image processing
- I. Title II. Series

621.3819'598 TA1632

Library of Congress Cataloging in Publication Data

Clarke, R. J.

1

85-1379

Transform coding of images.

(Microelectronics and signal processing) Includes index.

1. Image transmission. 2. Bandwidth compression (Telecommunication) 3. Coding theory. 1. Title. II. Series.

TK5105.2.C57 1985 621.38'043 ISBN 0-12-175730-7 (alk. paper)

ISBN 0-12-175731-5 (paperback)

PRINTED IN THE UNITED STATES OF AMERICA

85 86 87 88 9 8 7 6 5 4 3/2 1

Transform Coding of Images

05/8/03

MICROELECTRONICS AND SIGNAL PROCESSING

Series editors: P. G. Farrell, University of Manchester, U.K. J. R. Forrest, University College London, U.K.

About this series:

The topic of microelectronics can no longer be treated in isolation from its prime application in the processing of all types of information-bearing signals. The relative importance of various processing functions will determine the future course of developments in microelectronics. Many signal processing concepts, from data manipulation to more mathematical operations such as correlation, convolution and Fourier transformation, are now readily realizable in microelectronic form. This series aims to satisfy a demand for comprehensive and immediately useful volumes linking microelectronic technology and its applications.

Key features of the series are:

- Coverage ranging from the basic semiconductor processing of microelectronic circuits to developments in microprocessor systems or VLSI architecture and the newest techniques in image and optical signal processing.
- Emphasis on technology, with a blend of theory and practice intended for a wide readership.
- Exposition of the fundamental theme of signal processing; namely, any aspect of what happens to an electronic (or acoustic or optical) signal between the basic sensor which gathers it and the final output interface to the user.
- 1. Microprocessor Systems and Their Application to Signal Processing: C. K. YUEN, K. G. BEAUCHAMF, and G. P. S. ROBINSON
- Applications of Walsh and Related Functions: With an Introduction to Sequency Theory: K. G. BEAUCHAMP
- 3. Secure Speech Communications: HENRY J. BEKER and FRED C. PIPER
- 4. Transform Coding of Images: R. J. CLARKE

Preface

Over the past twenty years or so, the field of electrical engineering has been revolutionised by the advent and the subsequent rapidly increasing use of the digital computer. Not only are many analytical operations which were originally carried out in analogue form now performed more conveniently and speedily via the (discrete) digital computation process, but, and perhaps of more significance, procedures can now be executed which were once considered quite impossible, for example, the manipulation or processing of frames of video data on a realtime basis. Even where still images are concerned, the computer now allows complex high-speed processing operations, which typically involve large numbers of non-integer multiplications, to be applied to high-resolution data emanating, for example, from earth satellites. More generally, the development of information provision services (including the transmission of pictorial material) has been fostered by the close relationship between the digital computer and digital communication systems.

Digital communication links, although offering several advantages when compared with their analogue counterparts, do require a much higher equivalent bandwidth, and this is particularly a problem where image data is concerned, since such source material requires a large bandwidth in any case. It is not surprising, therefore, that interest in bandwidth compression techniques, originally analogue but now digital, has a long history, particularly as applied to image communication or storage, and the present volume is concerned with one appropriate procedure—transform coding, which, to risk a sweeping oversimplification, allows the reproduction of images of reasonable quality at average rates of perhaps 0.5 to 1.0 bit/element.

Y Preface

The main part of this book is concerned with the theoretical and practical aspects of transform coding systems intended to process still, or moving sequences of, images. However, two other topics are given more consideration than might, at first sight, seem appropriate. The successful operation of any bandwidth compression system is crucially dependent upon specific properties of the source data, and here such statistical measures are given detailed consideration in the light of their effect on coder performance. Again, there is much interest nowadays in the development of image processing systems having regard to the perception, by an observer, of degradation in the reconstructed picture. Furthermore, it is possible to use simple models of the human visual system within an image coder design in order to improve its performance, and the book, therefore, includes material on these aspects of image coding also.

An attempt has been made to exclude complex mathematical detail except where its presence is unavoidable (in particular, in parts of Chapters 2 and 9) and to demonstrate fundamental relationships via numerical examples. The book is intended for postgraduate students pursuing either course options or research in image coding and for those actively engaged in the design of coding systems, but it is suitable, in part, for final year undergraduate courses which include an image processing element. The list of references has been made as complete as practicable, and suggestions for supplementary reading have been made for those who wish to pursue specific topics in more detail.

Acknowledgements

Naturally, a volume of the present kind would not be possible without the dedicated work, over many years, of all those individuals and research groups throughout the world who have developed the subject to its present level of sophistication and have, moreover, freely published their results. It is also appropriate to acknowledge here the foresight of the electronics division of the Institution of Electrical Engineers (London), who, a few years ago, established professional group E4 (Electronic Image Processing), with whose members I have had many long and fruitful discussions on the topic. More specifically, I should like to thank the Society of Motion Picture and Television Engineers and the British Broadcasting Corporation for allowing me to employ their test-image material and to Taylor and Francis. Ltd., London. for permitting me to make use of the content of the paper "The Application of Image Covariance Models to Transform Coding", which appeared in Volume 56 of the International Journal of Electronics in 1984. Acknowledgement is also due to the Ministry of Defence for permitting the reproduction of Fig. 8.1 and to the IEE for allowing me to use material which forms the basis of Section 4.3.

It is worth emphasising that the book could not have been written without the financial support provided by the research laboratories of British Telecom, Martlesham, United Kingdom, which has enabled the author and his research students to pursue an active research programme in transform image coding at Loughborough University over the past six years. In turn, the enthusiastic labours of those students, in particular D. Allott (who carried out the coding simulations described in Chapter 5), W. K. Cham, K. N. Ngan and W. C. Wong, as well as of other members of the image coding

research group (M. G. B. Ismail, M. N. Pauls and Badi Moussa), have, it is hoped, contributed usefully to the sum total of knowledge on the subject. Needless to say, any errors, omissions or, equally significant, points of contention concerning the material within the volume remain solely the responsibility of the author.

Finally, it is a pleasure to acknowledge the wholehearted support of the editorial staff of Academic Press and of the editors of the series, Professors J. R. Forrest and P. G. Farrell, in this project.

Symbols

The main symbols used are defined below. All notation employed is defined upon its first appearance, and, of necessity, some symbols are used for more than one quantity.

a_i	Predictor weighting coefficient
A(f)	HVS spatial frequency response
$A_{\mathbf{R}}$.	Row transform
$b_{(0,0)}$	Bit allocation for the dc coefficient
b_i	Bit allocation for the <i>i</i> th coefficient
\boldsymbol{B}	Binary wordlength
\boldsymbol{B}	Colour component
$B_{\mathbf{C}}$	Column transform
$[B_N]$	Matrix in DCT decomposition
c_j	jth transform coefficient
c _j ĉ _j C	Estimated value of c_i
C	Transform coefficient vector
\boldsymbol{C}	Channel capacity
C(p,q)	Transform coefficient array
$COR(\cdot)$	Correlation matrix of (·)
ČOV(∙)	Covariance matrix of (·)
D_{\parallel}	Distortion parameter
$DCT(\cdot)$	Discrete cosine transform basis matrix of order (·)
DFT(I)	Imaginary part of the discrete Fourier transform basis matrix
DFT(R)	Real part of the discrete Fourier transform basis matrix
$DLB(\cdot)$	Discrete linear basis matrix of order (·)

xiv Symbols

$DST(\cdot)$	Discrete sine transform basis matrix of order (•)
e	Error term (suitably subscripted and/or indexed)
e	Quantisation distortion
$E(\cdot)$	Expectation operator
f	Spatial frequency (cycles/deg)
\mathbf{f}	Data vector
f(k, l)	Input array of picture elements
[F]	Two-dimensional data array
G	Colour component
$G(\omega)$	Visual contrast sensitivity
$h(\cdot)$	Differential entropy of (·)
$h(\theta)$	Line spread function
[H]	Horizontal process correlation matrix
$H(\cdot)$	Walsh-Hadamard transform basis matrix of order (·)
H_N	Entropy of N level quantiser
H(x)	Per-element entropy of data sequence
H(x, y)	Joint entropy of two data sequences
H(y/x)	Conditional entropy of two data sequences
$H(\omega)$	Spatial frequency response
$HA(\cdot)$	Haar transform basis matrix of order (·)
$HCT(\cdot)$	High-correlation transform basis matrix of order (·)
HVS	Human visual system
i(x)	Self information of occurrence of event $x = X$
i(x; y)	Mutual information between events $x = X$ and $y = Y$
I(x; y)	Average mutual information between element sequences
I	Chrominance signal in NTSC system
I_N	Identity matrix of order N
[J]	Matrix in the development of the sinusoidal transform family
\boldsymbol{k}	Data or coefficient index
$KLT(\cdot)$	Karhunen-Loève transform basis matrix for the first-order
	Markov process of interelement correlation coefficient (·)
$K_{1,2,3}$	Colour conversion components
L	Total number of allowable luminance levels $(L = 2^B)$
L	Luminance (suitably subscripted)
· m	Optical contrast
m.s.e	Mean square error
M	Image or sub-block dimension
n_1	Number of image elements taking on luminance level l
\dot{N}	Transform, image or sub-block dimension
$^{'}N_{\mathtt{B}}$	Number of bits allocated
$N_{\rm L}$	Number of quantisation levels.
n	Transform coefficient order
p_i	Probability of occupation of the <i>i</i> th quantiser interval
- •	

Symbols xv

p(x)	Probability of element x taking on value X
p(x, y)	Joint probability of x and y
p(y/x)	Conditional probability of y given x
$P(\omega)$	One-dimensional power spectral density
$P(\omega_{\rm h}, \omega_{\rm v})$	Two-dimensional power spectral density
q	Quantisation error
	Chrominance signal in NTSC system
Õ	Selectivity factor of human spatial frequency response
Q Q Q R	Entropy power
R R	Colour component
$R(\cdot)$	Correlation function (suitably indexed and subscripted)
R(D)	Rate-distortion function
$R_{\rm L}({\rm D})$	Lower bound on $R(D)$
$R_{\rm U}({\rm D})$	Upper bound on $R(D)$
$[R_N]$	Matrix in DCT decomposition
$S(\cdot)$	Slant transform basis matrix of order (·)
t .	Transform basis vector (appropriately subscripted)
$T(\cdot)$	General transform operator
[T]	General transform basis matrix (subscripts $C = complex$,
į z J	R = real
au	Colour conversion components
$T_{1,2,3}$	Colour conversion component
u u(i)	
u(i) U	Scaling constant in symmetric cosine transform
	Colour conversion component
U_{d}	PAL system colour difference signal
$\stackrel{v}{V}$	Colour conversion component
	Colour conversion component
$egin{array}{c} V_{ extbf{d}} \ V \end{array}$	PAL system colour difference signal
	Correlation separation vector
[V]	Vertical process covariance matrix
VAR(X)	Variance matrix of [X]
W	Colour conversion component $\rho^{-2\pi j/N}$
W_k	kth element of window function
W(x)	Error weighting function
$W(\omega)$	One-dimensional spectral weighting function
$W(\omega_{\rm h},\omega_{\rm v})$	Two-dimensional spectral weighting function
x	Arbitrary image element
$\hat{\mathcal{X}}$	Predicted value of x
x_{D}	Data dynamic range
X_i	Quantiser decision level
X	Particular luminance value assumed by x
X	Data column vector

Symbols

X _C	[X]	Two-dimensional data array
X _R [X] in row stacked form y Arbitrary image element y _i Quantiser reconstruction level Y Particular value assumed by y Y Luminance signal Y Transform coefficient vector α Parameter in exponential form of the first-order Markov correlation relation α One-step prediction coefficient β Parameter in sinusoidal transform development γ System (non)linearity parameter Γ(·) Gamma function Δ Reconstruction error Δ _L Luminance increment Δ _L Transform decorrelation efficient η _L Transform decorrelation efficiency η _L Transform decorrelation efficiency		▼
y Arbitrary image element y Quantiser reconstruction level Y Particular value assumed by y Y Luminance signal Y Transform coefficient vector α Parameter in exponential form of the first-order Markov correlation relation α One-step prediction coefficient β Parameter in sinusoidal transform development γ System (non)linearity parameter Γ(·) Gamma function Δ Reconstruction error Δh,v Mean square differences between adjacent horizontal or vertical elements Δ _L Luminance increment Δ _L Transform ecorrelation efficient β Quantisation/bit allocation parameter β Distortion parameter <th>-</th> <th></th>	-	
y _i Quantiser reconstruction level Y Particular value assumed by y Y Luminance signal Y Transform coefficient vector α Parameter in exponential form of the first-order Markov correlation relation α One-step prediction coefficient β Parameter in sinusoidal transform development γ System (non)linearity parameter Γ(·) Gamma function Δ Reconstruction error Δ _{h,v} Mean square differences between adjacent horizontal or vertical elements Δ _L Luminance increment Δ _L Transform decorrelation efficient η _C Transform decorrelation efficiency η _E Transform decorrelation efficiency η _E Transform decorrelation matical intensional correlation coefficient </th <th></th> <th></th>		
Y Particular value assumed by y Y Luminance signal Y Transform coefficient vector α Parameter in exponential form of the first-order Markov correlation relation α One-step prediction coefficient β Parameter in sinusoidal transform development γ System (non)linearity parameter Γ(·) Gamma function Δ Reconstruction error Δh,v Mean square differences between adjacent horizontal or vertical elements ΔL Luminance increment Δx,z,z Inter-elements ΔL Luminance increment Δx,z,z Inter-element spacing in axial directions ηc Transform decorrelation efficiency ηc Transform decorrelation efficiency ηc Transform decorrelation efficiency ηc Quantisation/bit allocation parameter λ Optimisation multiplier λ ith eigenvector [Λ] Diagonal matrix of eigenvalues [Λ] Diagonal matrix of eigenvalues [Λ] Diagonal matrix of eigenvalues <		
Y Luminance signal Y Transform coefficient vector α Parameter in exponential form of the first-order Markov correlation relation α One-step prediction coefficient β Parameter in sinusoidal transform development γ System (non)linearity parameter Γ(·) Gamma function Δ Reconstruction error $\Delta_{h,v}$ Mean square differences between adjacent horizontal or vertical elements Δ_L Luminance increment $\Delta_{x,y,z}$ Inter-element spacing in axial directions η_C Transform decorrelation efficiency η_E Transform energy packing efficiency θ Quantisation/bit allocation parameter θ Subtended angle θ Distortion parameter λ Optimisation multiplier λ_i ith eigenvector [Λ] Diagonal matrix of eigenvalues [Λ] DCT matrix ρ Inter-element correlation coefficient ρ_A One-step diagonal correlation coefficient ρ_B Cone-step horizontal correlation coefficient ρ_h One-step horizontal correlation coefficient ρ_h Cone-step vertical correlation coefficient ρ_N Single letter fidelity criterion ρ_V One-step vertical correlation coefficient ρ_R Single letter fidelity criterion ρ_V Vertical correlation coefficient as a function of ν $\rho(x, \nu)$ Two-dimensional correlation function σ^2 Variance (appropriately subscripted) τ Element delay ω Angular frequency ω_h Horizontal angular frequency (two-dimensional case) ω_V Vertical angular frequency (two-dimensional case)		
Transform coefficient vector α Parameter in exponential form of the first-order Markov correlation relation α One-step prediction coefficient β Parameter in sinusoidal transform development γ System (non)linearity parameter Γ(·) Gamma function Δ Reconstruction error Δ _{h,ν} Mean square differences between adjacent horizontal or vertical elements Δ _L Luminance increment Δ _{x,y,z} Inter-element spacing in axial directions η _C Transform decorrelation efficiency η _E Transform energy packing efficiency θ Quantisation/bit allocation parameter θ Subtended angle θ Distortion parameter λ Optimisation multiplier λ _i ith eigenvector [Λ] Diagonal matrix of eigenvalues [Λ] DCT matrix ρ Inter-element correlation coefficient ρ _h One-step diagonal correlation coefficient ρ _h One-step horizontal correlation coefficient ρ _h Nore-step horizontal correlation coefficient ρ _n Single letter fidelity criterion ρ _ν One-step vertical correlation coefficient ρ _γ Vy) Vertical correlation coefficient as a function of y ρ(x, y) Two-dimensional correlation function σ² Variance (appropriately subscripted) τ Element delay ω Angular frequency ω _h Horizontal angular frequency (two-dimensional case) Vertical angular frequency (two-dimensional case)		• •
α Parameter in exponential form of the first-order Markov correlation relation α One-step prediction coefficient β Parameter in sinusoidal transform development γ System (non)linearity parameter Γ(·) Gamma function Δ Reconstruction error Δ _{h,v} Mean square differences between adjacent horizontal or vertical elements Δ _L Luminance increment Δ _{x,y,z} Inter-element spacing in axial directions η _C Transform decorrelation efficiency η _E Transform decorrelation efficiency θ Quantisation/bit allocation parameter θ Subtended angle θ Distortion parameter λ Optimisation multiplier λ _i ith eigenvector [Λ] Diagonal matrix of eigenvalues [Λ] DCT matrix ρ Inter-element correlation coefficient ρ _h One-step diagonal correlation coefficient ρ _h Horizontal correlation coefficient as a function of x ρ _k k-step correlation coefficient coefficient ρ _h		•
β Parameter in sinusoidal transform development γ System (non)linearity parameter Γ(·) Gamma function Δ Reconstruction error Δ _{h,v} Mean square differences between adjacent horizontal or vertical elements Δ _L Luminance increment Δ _{x,y,z} Inter-element spacing in axial directions η _C Transform decorrelation efficiency η _E Transform decorrelation efficiency θ Quantisation/bit allocation parameter θ Quantisation/bit allocation parameter θ Subtended angle θ Distortion parameter λ Optimisation multiplier λ _i ith eigenvector [Λ] Diagonal matrix of eigenvalues [Λ] DCT matrix ρ Inter-element correlation coefficient ρ _A One-step diagonal correlation coefficient ρ _h One-step horizontal correlation coefficient ρ _h k-step correlation coefficient ρ _h k-step correlation coefficient ρ _h Vertical correlation coefficient as a function of y ρ _h Vertical correlation co	α	Parameter in exponential form of the first-order Markov cor-
$\begin{array}{lll} \gamma & & \text{System (non)linearity parameter} \\ \Gamma(\cdot) & & \text{Gamma function} \\ \Delta & & \text{Reconstruction error} \\ \Delta_{\text{h,v}} & & \text{Mean square differences between adjacent horizontal or vertical elements} \\ \Delta_{\text{L}} & & \text{Luminance increment} \\ \Delta_{\text{x,y,z}} & & \text{Inter-element spacing in axial directions} \\ \eta_{\text{C}} & & \text{Transform decorrelation efficiency} \\ \eta_{\text{E}} & & \text{Transform energy packing efficiency} \\ \theta & & \text{Quantisation/bit allocation parameter} \\ \theta & & \text{Subtended angle} \\ \theta & & \text{Distortion parameter} \\ \lambda & & \text{Optimisation multiplier} \\ \lambda_i & & \text{ith eigenvector} \\ [\Lambda] & & \text{Diagonal matrix of eigenvalues} \\ [\Lambda] & & \text{DCT matrix} \\ \rho & & \text{Inter-element correlation coefficient} \\ \rho_{\theta} & & \text{One-step diagonal correlation coefficient} \\ \rho_{h} & & \text{One-step horizontal correlation coefficient} \\ \rho_{h} & & \text{One-step horizontal correlation coefficient} \\ \rho_{\eta} & & \text{Single letter fidelity criterion} \\ \rho_{V} & & \text{One-step vertical correlation coefficient} \\ \rho_{V} & & \text{One-step vertical correlation coefficient} \\ \rho_{V}(y) & & \text{Vertical correlation coefficient as a function of } y \\ \rho(x,y) & & \text{Two-dimensional correlation function} \\ \sigma^2 & & \text{Variance (appropriately subscripted)} \\ \tau & & \text{Element delay} \\ \omega & & \text{Angular frequency} \\ \omega_{\text{h}} & & \text{Horizontal angular frequency (two-dimensional case)} \\ \text{Vertical angular frequency (two-dimensional case)} \\ \end{array}$	α	One-step prediction coefficient
	β	Parameter in sinusoidal transform development
	γ	System (non)linearity parameter
	$\Gamma(\cdot)$	
tical elements Δ_1 Luminance increment $\Delta_{x,y,z}$ Inter-element spacing in axial directions η_C Transform decorrelation efficiency η_E Transform energy packing efficiency θ Quantisation/bit allocation parameter θ Subtended angle θ Distortion parameter λ Optimisation multiplier λ_i ith eigenvector [Λ] Diagonal matrix of eigenvalues [Λ] DCT matrix ρ Inter-element correlation coefficient ρ_d One-step diagonal correlation coefficient ρ_h One-step horizontal correlation coefficient ρ_h Horizontal correlation coefficient as a function of x ρ_k k -step correlation coefficient ρ_V One-step vertical correlation coefficient ρ_V One-step vertical correlation coefficient ρ_V One-step vertical correlation coefficient ρ_V Two-dimensional correlation function σ^2 Variance (appropriately subscripted) σ^2 Variance (appropriately subscripted) σ^2 Urical angular frequency (two-dimensional case) σ_V Vertical angular frequency (two-dimensional case)	Δ	Reconstruction error
	$\Delta_{ m h,v}$	•
η_C Transform decorrelation efficiency η_E Transform energy packing efficiency θ Quantisation/bit allocation parameter θ Subtended angle θ Distortion parameter λ Optimisation multiplier λ_i i th eigenvector $[\Lambda]$ Diagonal matrix of eigenvalues $[\Lambda]$ DCT matrix ρ Inter-element correlation coefficient ρ_d One-step diagonal correlation coefficient ρ_h One-step horizontal correlation coefficient $\rho_H(x)$ Horizontal correlation coefficient as a function of x ρ_k k -step correlation coefficient ρ_n Single letter fidelity criterion ρ_V One-step vertical correlation coefficient $\rho_V(y)$ Vertical correlation coefficient as a function of y $\rho(x, y)$ Two-dimensional correlation function σ^2 Variance (appropriately subscripted) τ Element delay ω Angular frequency ω_h Horizontal angular frequency (two-dimensional case) ω_V Vertical angular frequency (two-dimensional case)	$\Delta_{\mathtt{L}}$	Luminance increment
$\eta_{\rm C}$ Transform decorrelation efficiency $\eta_{\rm E}$ Transform energy packing efficiency θ Quantisation/bit allocation parameter θ Subtended angle θ Distortion parameter λ Optimisation multiplier λ_i ith eigenvector $[\Lambda]$ Diagonal matrix of eigenvalues $[\Lambda]$ DCT matrix ρ Inter-element correlation coefficient $\rho_{\rm d}$ One-step diagonal correlation coefficient $\rho_{\rm h}$ One-step horizontal correlation coefficient $\rho_{\rm h}$ Horizontal correlation coefficient as a function of x $\rho_{\rm h}$ K-step correlation coefficient $\rho_{\rm v}$ One-step vertical correlation coefficient $\rho_{\rm v}$ One-step vertical correlation coefficient $\rho_{\rm v}$ Vertical correlation coefficient as a function of y $\rho_{\rm c}$ Vertical correlation function σ^2 Variance (appropriately subscripted) τ Element delay ω Angular frequency $\omega_{\rm h}$ Horizontal angular frequency (two-dimensional case) $\omega_{\rm v}$ Vertical angular frequency (two-dimensional case)	$\Delta_{x,y,z}$	Inter-element spacing in axial directions
$\begin{array}{lll} \theta & \text{Quantisation/bit allocation parameter} \\ \theta & \text{Subtended angle} \\ \theta & \text{Distortion parameter} \\ \lambda & \text{Optimisation multiplier} \\ \lambda_i & i\text{th eigenvector} \\ \hline [\Lambda] & \text{Diagonal matrix of eigenvalues} \\ \hline [\Lambda] & \text{DCT matrix} \\ \rho & \text{Inter-element correlation coefficient} \\ \rho_d & \text{One-step diagonal correlation coefficient} \\ \rho_h & \text{One-step horizontal correlation coefficient} \\ \rho_h & \text{One-step horizontal correlation coefficient} \\ \rho_k & k\text{-step correlation coefficient as a function of } x \\ \rho_k & k\text{-step correlation coefficient} \\ \rho_{\nu} & \text{One-step vertical correlation coefficient} \\ \rho_{\nu} & \text{One-step vertical correlation coefficient} \\ \rho_{\nu} & \text{One-step vertical correlation coefficient} \\ \sigma_{\nu} & \text{Vertical correlation coefficient as a function of } y \\ \rho(x,y) & \text{Two-dimensional correlation function} \\ \sigma^2 & \text{Variance (appropriately subscripted)} \\ \tau & \text{Element delay} \\ \omega & \text{Angular frequency} \\ \omega_h & \text{Horizontal angular frequency (two-dimensional case)} \\ \omega_{\nu} & \text{Vertical angular frequency (two-dimensional case)} \\ \end{array}$		Transform decorrelation efficiency
θSubtended angleθDistortion parameterλOptimisation multiplier λ_i ith eigenvector[Λ]Diagonal matrix of eigenvalues[Λ]DCT matrixρInter-element correlation coefficient ρ_d One-step diagonal correlation coefficient ρ_h One-step horizontal correlation coefficient ρ_h Horizontal correlation coefficient as a function of x ρ_k k -step correlation coefficient ρ_n Single letter fidelity criterion ρ_V One-step vertical correlation coefficient $\rho_V(y)$ Vertical correlation coefficient as a function of y $\rho(x, y)$ Two-dimensional correlation function σ^2 Variance (appropriately subscripted) τ Element delay ω Angular frequency ω_h Horizontal angular frequency (two-dimensional case) ω_v Vertical angular frequency (two-dimensional case)	η_{E}	Transform energy packing efficiency
$\begin{array}{lll} \theta & \text{Distortion parameter} \\ \lambda & \text{Optimisation multiplier} \\ \lambda_i & \text{ith eigenvector} \\ \hline [\Lambda] & \text{Diagonal matrix of eigenvalues} \\ \hline [\Lambda] & \text{DCT matrix} \\ \rho & \text{Inter-element correlation coefficient} \\ \rho_d & \text{One-step diagonal correlation coefficient} \\ \rho_h & \text{One-step horizontal correlation coefficient} \\ \rho_h & \text{Horizontal correlation coefficient as a function of } x \\ \rho_k & k\text{-step correlation coefficient} \\ \rho_n & \text{Single letter fidelity criterion} \\ \rho_V & \text{One-step vertical correlation coefficient} \\ \rho_V(y) & \text{Vertical correlation coefficient as a function of } y \\ \rho(x,y) & \text{Two-dimensional correlation function} \\ \sigma^2 & \text{Variance (appropriately subscripted)} \\ \tau & \text{Element delay} \\ \omega & \text{Angular frequency} \\ \omega_h & \text{Horizontal angular frequency (two-dimensional case)} \\ \omega_V & \text{Vertical angular frequency (two-dimensional case)} \\ \end{array}$	$oldsymbol{ heta}$	Quantisation/bit allocation parameter
$\begin{array}{lll} \lambda & \text{Optimisation multiplier} \\ \lambda_i & \text{ith eigenvector} \\ [\Lambda] & \text{Diagonal matrix of eigenvalues} \\ [\Lambda] & \text{DCT matrix} \\ \rho & \text{Inter-element correlation coefficient} \\ \rho_d & \text{One-step diagonal correlation coefficient} \\ \rho_h & \text{One-step horizontal correlation coefficient} \\ \rho_h & \text{Horizontal correlation coefficient as a function of } x \\ \rho_k & k\text{-step correlation coefficient} \\ \rho_n & \text{Single letter fidelity criterion} \\ \rho_V & \text{One-step vertical correlation coefficient} \\ \rho_V(y) & \text{Vertical correlation coefficient as a function of } y \\ \rho(x,y) & \text{Two-dimensional correlation function} \\ \sigma^2 & \text{Variance (appropriately subscripted)} \\ \tau & \text{Element delay} \\ \omega & \text{Angular frequency} \\ \omega_h & \text{Horizontal angular frequency (two-dimensional case)} \\ \omega_v & \text{Vertical angular frequency (two-dimensional case)} \\ \end{array}$	$oldsymbol{ heta}$	Subtended angle
λ_i ith eigenvector[Λ]Diagonal matrix of eigenvalues[Λ]DCT matrix ρ Inter-element correlation coefficient ρ_d One-step diagonal correlation coefficient ρ_h One-step horizontal correlation coefficient as a function of x ρ_k k -step correlation coefficient ρ_n Single letter fidelity criterion ρ_V One-step vertical correlation coefficient $\rho_V(y)$ Vertical correlation coefficient as a function of y $\rho(x,y)$ Two-dimensional correlation function σ^2 Variance (appropriately subscripted) τ Element delay ω Angular frequency ω_h Horizontal angular frequency (two-dimensional case) ω_v Vertical angular frequency (two-dimensional case)	$\boldsymbol{\theta}$	Distortion parameter
[Λ] Diagonal matrix of eigenvalues [Λ] DCT matrix ρ Inter-element correlation coefficient $\rho_{\rm d}$ One-step diagonal correlation coefficient $\rho_{\rm h}$ One-step horizontal correlation coefficient $\rho_{\rm h}(x)$ Horizontal correlation coefficient as a function of x ρ_{k} k -step correlation coefficient ρ_{n} Single letter fidelity criterion $\rho_{\rm V}$ One-step vertical correlation coefficient $\rho_{\rm V}(y)$ Vertical correlation coefficient as a function of y $\rho(x,y)$ Two-dimensional correlation function σ^{2} Variance (appropriately subscripted) τ Element delay ω Angular frequency $\omega_{\rm h}$ Horizontal angular frequency (two-dimensional case) $\omega_{\rm v}$ Vertical angular frequency (two-dimensional case)		Optimisation multiplier
$ \begin{array}{lll} [\Lambda] & \text{DCT matrix} \\ \rho & \text{Inter-element correlation coefficient} \\ \rho_{\rm d} & \text{One-step diagonal correlation coefficient} \\ \rho_{\rm h} & \text{One-step horizontal correlation coefficient} \\ \rho_{\rm h} & \text{Horizontal correlation coefficient as a function of } x \\ \rho_{k} & k\text{-step correlation coefficient as a function of } x \\ \rho_{n} & \text{Single letter fidelity criterion} \\ \rho_{V} & \text{One-step vertical correlation coefficient} \\ \rho_{V}(y) & \text{Vertical correlation coefficient as a function of } y \\ \rho(x,y) & \text{Two-dimensional correlation function} \\ \sigma^{2} & \text{Variance (appropriately subscripted)} \\ \tau & \text{Element delay} \\ \omega & \text{Angular frequency} \\ \omega_{\rm h} & \text{Horizontal angular frequency (two-dimensional case)} \\ \omega_{\rm v} & \text{Vertical angular frequency (two-dimensional case)} \\ \end{array}$	λ_i	ith eigenvector
$\begin{array}{lll} \rho & & & & & \\ \rho_{\rm d} & & & & \\ One-step \ diagonal \ correlation \ coefficient \\ \rho_{\rm h} & & & & \\ One-step \ horizontal \ correlation \ coefficient \\ \rho_{\rm H}(x) & & & & \\ Horizontal \ correlation \ coefficient \ as \ a \ function \ of \ x \\ \rho_k & & & \\ k\text{-step \ correlation \ coefficient} \ single \ letter \ fidelity \ criterion \\ \rho_{\rm V} & & & \\ One-step \ vertical \ correlation \ coefficient \\ \rho_{\rm V}(y) & & & \\ Vertical \ correlation \ coefficient \ as \ a \ function \ of \ y \\ \rho(x,y) & & & \\ Two-dimensional \ correlation \ function \\ \sigma^2 & & & \\ Variance \ (appropriately \ subscripted) \\ \tau & & & \\ Element \ delay \\ \omega & & & \\ Angular \ frequency \\ \omega_h & & & \\ Horizontal \ angular \ frequency \ (two-dimensional \ case) \\ \omega_{\rm v} & & & \\ Vertical \ angular \ frequency \ (two-dimensional \ case) \\ \end{array}$	[Λ]	Diagonal matrix of eigenvalues
$\begin{array}{lll} \rho_{\rm d} & {\rm One\text{-}step\ diagonal\ correlation\ coefficient} \\ \rho_{\rm h} & {\rm One\text{-}step\ horizontal\ correlation\ coefficient} \\ \rho_{\rm H}(x) & {\rm Horizontal\ correlation\ coefficient\ as\ a\ function\ of\ x} \\ \rho_{k} & k\text{-}step\ correlation\ coefficient\ as\ a\ function\ of\ x} \\ \rho_{n} & {\rm Single\ letter\ fidelity\ criterion} \\ \rho_{V} & {\rm One\text{-}step\ vertical\ correlation\ coefficient\ }} \\ \rho_{V}(y) & {\rm Vertical\ correlation\ coefficient\ as\ a\ function\ of\ y} \\ \rho(x,y) & {\rm Two\text{-}dimensional\ correlation\ function} \\ \sigma^{2} & {\rm Variance\ (appropriately\ subscripted)} \\ \tau & {\rm Element\ delay\ }} \\ \omega & {\rm Angular\ frequency\ }} \\ \omega_{\rm h} & {\rm Horizontal\ angular\ frequency\ (two\text{-}dimensional\ case)} \\ \omega_{\rm v} & {\rm Vertical\ angular\ frequency\ (two\text{-}dimensional\ case)} \\ \end{array}$	[Λ]	DCT matrix
$\begin{array}{lll} \rho_h & \text{One-step horizontal correlation coefficient} \\ \rho_H(x) & \text{Horizontal correlation coefficient as a function of } x \\ \rho_k & k\text{-step correlation coefficient} \\ \rho_n & \text{Single letter fidelity criterion} \\ \rho_V & \text{One-step vertical correlation coefficient} \\ \rho_V(y) & \text{Vertical correlation coefficient as a function of } y \\ \rho(x,y) & \text{Two-dimensional correlation function} \\ \sigma^2 & \text{Variance (appropriately subscripted)} \\ \tau & \text{Element delay} \\ \omega & \text{Angular frequency} \\ \omega_h & \text{Horizontal angular frequency (two-dimensional case)} \\ \omega_v & \text{Vertical angular frequency (two-dimensional case)} \\ \end{array}$	ρ	Inter-element correlation coefficient
$ \rho_{\rm H}(x) $ Horizontal correlation coefficient as a function of x $ \rho_{k} $ k -step correlation coefficient $ \rho_{n} $ Single letter fidelity criterion $ \rho_{V} $ One-step vertical correlation coefficient $ \rho_{V}(y) $ Vertical correlation coefficient as a function of y $ \rho(x, y) $ Two-dimensional correlation function $ \sigma^{2} $ Variance (appropriately subscripted) $ \tau $ Element delay $ \omega $ Angular frequency $ \omega_{h} $ Horizontal angular frequency (two-dimensional case) $ \omega_{v} $ Vertical angular frequency (two-dimensional case)	$ ho_{\mathbf{d}}$	One-step diagonal correlation coefficient
$ \rho_k $ $ k$ -step correlation coefficient $ \rho_n $ Single letter fidelity criterion $ \rho_V $ One-step vertical correlation coefficient $ \rho_V(y) $ Vertical correlation coefficient as a function of y $ \rho(x, y) $ Two-dimensional correlation function $ \sigma^2 $ Variance (appropriately subscripted) $ \tau $ Element delay $ \omega $ Angular frequency $ \omega_h $ Horizontal angular frequency (two-dimensional case) $ \omega_v $ Vertical angular frequency (two-dimensional case)	$ ho_{\mathtt{h}}$	One-step horizontal correlation coefficient
$ \rho_n $ Single letter fidelity criterion $ \rho_V $ One-step vertical correlation coefficient $ \rho_V(y) $ Vertical correlation coefficient as a function of y $ \rho(x, y) $ Two-dimensional correlation function $ \sigma^2 $ Variance (appropriately subscripted) Element delay $ \omega $ Angular frequency $ \omega_h $ Horizontal angular frequency (two-dimensional case) $ \omega_v $ Vertical angular frequency (two-dimensional case)	$\rho_{\rm H}(x)$	Horizontal correlation coefficient as a function of x
$ \rho_{\rm V} $ One-step vertical correlation coefficient $ \rho_{\rm V}(y) $ Vertical correlation coefficient as a function of y $ \rho(x, y) $ Two-dimensional correlation function $ \sigma^2 $ Variance (appropriately subscripted) $ \tau $ Element delay $ \omega $ Angular frequency $ \omega_{\rm h} $ Horizontal angular frequency (two-dimensional case) $ \omega_{\rm v} $ Vertical angular frequency (two-dimensional case)	$ ho_{k}$	k-step correlation coefficient
$ \rho_{\rm V}(y) $ Vertical correlation coefficient as a function of y $\rho(x, y)$ Two-dimensional correlation function σ^2 Variance (appropriately subscripted) τ Element delay ω Angular frequency $\omega_{\rm h}$ Horizontal angular frequency (two-dimensional case) $\omega_{\rm v}$ Vertical angular frequency (two-dimensional case)	ρ_n	Single letter fidelity criterion
$ \rho(x, y) $ Two-dimensional correlation function $ \sigma^2 $ Variance (appropriately subscripted) $ \tau $ Element delay $ \omega $ Angular frequency $ \omega_h $ Horizontal angular frequency (two-dimensional case) $ \omega_v $ Vertical angular frequency (two-dimensional case)	$ ho_{ m V}$	One-step vertical correlation coefficient
σ^2 Variance (appropriately subscripted) τ Element delay ω Angular frequency ω_h Horizontal angular frequency (two-dimensional case) ω_v Vertical angular frequency (two-dimensional case)	$\rho_{\mathbf{v}}(y)$	Vertical correlation coefficient as a function of y
$ \omega $ Element delay $ \omega $ Angular frequency $ \omega_{\rm h} $ Horizontal angular frequency (two-dimensional case) $ \omega_{\rm v} $ Vertical angular frequency (two-dimensional case)	$\rho(x, y)$	Two-dimensional correlation function
ω Angular frequency ω_h Horizontal angular frequency (two-dimensional case) ω_v Vertical angular frequency (two-dimensional case)	σ^2	Variance (appropriately subscripted)
ω_h Horizontal angular frequency (two-dimensional case) ω_v Vertical angular frequency (two-dimensional case)	τ	Element delay
$\omega_{\rm v}$ Vertical angular frequency (two-dimensional case)	ω	
	$\omega_\mathtt{h}$,
$\omega_{\rm r}$ Radial angular frequency	$\omega_{ extsf{v}}$	
	$\omega_{ m r}$.	Radial angular frequency

Contents

rrejace Acknowledge Symbols	ements .	•	xi xiii
Chapter 1	Image Data Compression: An Introduction		
	1.1 Introduction		1
	1.2 Statistical image data compression	•	2
	1.3 Predictive coding		. 2 3 4
	1.4 Transform coding		
	1.5 Hybrid coding		5
	1.6 Chapter contents		5
	1.7 Further reading		7
Chapter 2	Statistical Properties of Images		
•	2.1 Introduction		8
	2.2 Image properties in the data domain		. 9
	2.3 Image properties in the spectral domain		31
	2.4 Application of the theory to practical images		39
	2.5 Summary		69
Chapter 3	Orthogonal Transforms for Image Coding		
	3.1 Introduction		72
	3.2 The transform operation in one dimension		73
	3.3 The transform operation in two dimensions		80

Vi	Contents	
:	3.4 Determination of transform efficiency	87
•	3.5 Specific transforms for image processing	91
	3.6 General comparison of transforms	123
	3.7 Transform comparison based on image data	130
	3.8 Summary	133
Chapter 4	Transform Coefficient Quantisation	
	and Bit Allocation	
	4.1 Introduction	135
	4.2 Transform coefficient energy distribution	136
	4.3 The dynamic range of transform coefficients	139
	4.4 Theoretical models for coefficient probability	
	density functions	143
	4.5 The coefficient distribution of the test images	148
	4.6 Coefficient quantisation	163
	4.7 Bit assignment procedures	175
	4.8 Summary	183
Chapter 5	Practical Coding Techniques	
	5.1 Intraframe coding	185
	5.2 Interframe coding	232
	5.3 Transform coding of colour data	248
Chapter 6	The Human Visual Response	
	6.1 Introduction	259
	6.2 Properties of the eye	263
	6.3 Modelling the human visual system	270
	6.4 Application of visual models to the assessment	
	of image quality	278
	6.5 Transform coding and the visual response	282
	6.6 Conclusions	289
Chapter 7	Fast Transforms and System Implementation	
	7.1 Introduction	291
	7.2 Fast Fourier transform	294
	7.3 Fast Walsh - Hadamard transform	301
	7.4 Fast Haar transform	305
	7.5 Fast slant transform	307
	7.6 Fast Karhunen-Loève transform	310
	7.7 Fast discrete cosine transform	311
	7.8 Fast discrete sine transform	318
	7.9 Fast high-correlation transform	320
	7.10 General comments on fast transforms	322

	Contents	vii
	7.11 Implementation of transform coding schemes	323
·	7.12 General comments on implementation	335
Chapter 8	Error and Noise Effects	
	8.1 Introduction	336
	8.2 Requirements of implementation	338
	8.3 Coefficient deletion errors	340
	8.4 Coefficient quantisation errors	341
	8.5 Channel error effects	344
	8.6 Source-channel coding	353
	8.7 Error correction techniques	354
	8.8 Summary	357
Chapter 9	Rate Distortion Theory and Image Coding	
	9.1 Introduction	359
	9.2 Rate distortion function	361
	9.3 One-dimensional spectral representation	301
	and the rate for the first-order Markov source	371
	9.4 Two-dimensional rate distortion representation	376
	9.5 Quantisation and rate distortion theory	379
	9.6 Application to practical coding schemes	384
	9.7 Summary	387
Appendix 1	Calculation of First- and Second-Order Entropies	38 9
Appendix 2	Statistical Relationships	391
Appendix 3	The Inverse Transform	395
Appendix 4	Gradients of Vector and Matrix Functions	398
Appendix 4	Gradients of Vector and Matrix Panetions	376
Appendix 5	O	
	of the Walsh-Hadamard Transform	400
Appendix 6	Inner and Outer Products	403
Appendix 7	Mean-Square-Error Measures	405
•		
References		407
Supplementa	ary Material	422
Supplementa	ary References	423
Index		429

Chapter 1

Image Data Compression: An Introduction

1.1 Introduction

It is a truism nowadays to say that the advent of the digital computer has revolutionised almost every technological discipline. Not least amongst those affected has been electrical engineering (the very field which itself has been responsible for the myriad advances in computer technology), and there are many who, though working as electrical engineers, now hardly come into the close vicinity of moving electrons in a professional capacity from one year to the next. Within that sphere of interest, arguably one of the most radical changes has taken place in communications, in which the accent is now heavily placed upon representation of the input information in discrete (sampled and quantised) form, leading to a much greater flexibility in the scope and nature of the operations which can subsequently be carried out on the data. In more general terms, the information 'revolution' appears to have created a demand for a wide range of new services, and it is just such a diversity of requirements that can be catered for by the now rapidly evolving networks of digital communications channels. Many of the new services envisaged (or already in operation) require the transmission and/or storage of image detail, and the days when the only pictures to be seen 'at a distance' were those of broadcast television are long past.

Given the more or less continual demand for increased bandwidth (or channel capacity) to enable more users to benefit from communication systems, and also the great demands made upon such systems by the desire to

8850152