Self-Organization

Autowaves and Structures Far from Equilibrium

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Foreword

According to its definition, Synergetics is concerned with systems that produce macroscopic spatial, temporal, or functional structures. Autowaves are a specific, yet very important, case of spatio-temporal structures. The term "autowave" was coined in the Soviet Union in analogy to the term "auto-oscillator". This is a - perhaps too literal - translation of the Russian word "avto-ostsillyatory" (= self-oscillator) which in its proper translation means "self-sustained oscillator". These are oscillators, e.g., clocks, whose internal energy dissipation is compensated by a (more or less) continuous power input. Similarly, the term "autowaves" denotes propagation effects - including waves - in active media, which provide spatially distributed energy sources and thus may compensate dissipation. An example which is now famous is represented by spiral or concentric waves in a chemically active medium, undergoing the Belousov-Zhabotinsky reaction.

This book provides the reader with numerous further examples from physics, chemistry, and biology - e.g., autowaves of the heart. While the Belousov-Zhabotinsky reaction is now widely known, a number of very important results obtained in the Soviet Union are perhaps less well known. I am particularly glad that this book may help to make readers outside the Soviet Union acquainted with these important experimental and theoretical findings which are presented in a way which elucidates the common principles underlying this kind of propagation effects. Professor V. Krinsky has taken great care in editing this book to which prominent scientists from the Soviet Union and from abroad contribute and I wish to congratulate him for his successful efforts.

Hermann Haken

Preface

During recent years remarkably universal mechanisms have been found for the development of order from random distributions in active systems of quite different natures. These mechanisms are linked to the propagation of strongly nonlinear waves, the so-called autowaves which are spatio-temporal analogs of auto-oscillations. While the auto-oscillation theory is a well-developed branch of science, the study of autowaves is still only in an embryonic state.

Among interesting examples of self-organization discovered in the study of the propagation of autowaves in active media is the occurrence of dissipative wave structures in quite different fields: the morphogenesis of the simplest multicellular organisms, the optic cortex, the retina, heart tissue, the Belousov-Zhabotinsky chemical reaction. The autowave processes are also important in phase transitions, in particular for the disappearance of superconductivity, for movements of the domain walls in magnetic or magnetoelectric media, and for phenomena of critical boiling. Different types of chaos in active media, including cardiac arrhythmias, proved to be connected with the initiation and reproduction of autowave vortices. In 1983, a symposium devoted to these problems took place at the Scientific Centre of Biological Research of the USSR Academy of Sciences in Pushchino. This volume contains short reviews by invited authors, written after the symposium, with account taken of results presented there.

The camera-ready volume was prepared at the Biological Research Centre of the USSR Academie of Sciences, Institute of Biological Physics, Phushchino, USSR.

Pushchino
August 1984

V. I. Krinsky

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Part I

Introduction

Synergetics - Some Basic Concepts and Recent Results

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1. INTRODUCTION

In my contribution I should like to give a brief outline of some basic ideas of synergetics [1], [2] . Then I shall present some of our recent results obtained by an application of our mathematical methods. The word SYNERGETICS is composed of two greek words and means COOPERATION. What we study in this field is the cooperation of individual parts of a system so that a self-organized formation of spatial, temporal, or functional structures on macroscopic scales becomes possible. In particular we shall ask whether there are general principles which govern self-organization irrespective of the nature of the individual subsystems which may be electrons, molecules, photons, biological cells, or animals. Or, to use an idea expressed by Danilov and Kadomtsev [3], synergetics can be considered as a search for universal mathematical models (of self-organization). In particular, we wish to develop an operational approach in the sense of general systems theory. Such kind of approach has been persued in the Soviet Union by Lyapunov, Mandelstam, Andronov, Vitt, Chaikin, and many others.

2. OUTLINE OF THE GENERAL APPROACH

Let me take an example from physics. We may describe the behavior of a fluid at three different levels. At the microscopic level we deal with the motion of individual atoms or molecules. At the mesoscopic level we lump many molecules together into droplets so that we may speak of densities, temperature etc., but so that at this level no macroscopic structure is visible. At the macroscopic level we deal with the formation of structures e.g. rolls, hexagons etc. While e.g. in laser physics we directly proceed from the microscopic to the macroscopic level [4], in this lecture we shall adopt the following attitude. We assume that the transition from the microscopic to the mesoscopic level has been achieved by statistical mechanics or that adequate equations have been formulated at the mesoscopic level in a more or less phenomenological manner. An example is provided by the Navier Stokes equations, or by rate equations for chemical reactions. We then wish to study the evolution of patterns at the macroscopic level.

The state of the system is described by a set of variables $q_1 \ldots q_n$ which we lump together into a state vector q. Because in general the processes depend on space and time, q is a function of x and t also. The following list gives a number of interpretations of the various components of q

numbers or densities of atoms or molecules	fluids, solidification chemical reactions
velocity fields	flames, lasers, plasmas
electromagnetic fields	electronic devices
electrons	solid state
firing rates of neurons	neural nets
numbers of specific cells	morphogenesis
monetary flows etc.	economy
numbers of animals	ecology

The processes may take place in various geometries e.g.in the plane, in threedimensional space, but also on a sphere. For instance pattern formation on spherical shells in biology have been studied by Velarde [5] or pattern formation in the atmosphere of planets by Busse and others. Also one may think of more complicated manifolds or even evolving manifolds. The concept of approach of synergetics rests on a number of paradigms, to use a word en vogue, namely

a) evolution equations
b) instability
c) slaving
d) order parameters
e) formation of structures
f) instability hierarchies

3. A BRIEF OUTLINE OF THE MATHEMATICAL APPROACH

a) Evolution equations

These equations deal with the temporal evolution of q, i.e. we have to study $\dot{q} = N(q)$. The r.h.s. is a nonlinear function of the components q_j, e.g. q_1^2, q_1q_2 etc. The systems under consideration are dissipative i.e. they contain equations of the form

$$\dot{q}_1 = - \gamma q_1 + \dots \tag{3.1}$$

They may contain transport terms describing

$$\begin{array}{lll} \text{convection:} & v\nabla v, & v\text{:velocity} \\ \text{diffusion:} & \Delta & \\ \text{waves:} & \Delta & \end{array} \tag{3.2}$$

The systems are controlled from the outside, e.g. by changing the energy input. This control is described by control parameters, e.g. by α in the eq.

$$\dot{q} = (\alpha - \gamma)q + \dots \tag{3.3}$$

Finally, close to transition points of nonequilibrium phase transitions fluctuations play a decisive role. These fluctuations stem from fluctuating forces which represent the action of the microscopic "underworld" on the physical quantities q of the mesoscopic level. Lumping all the different terms together, we are led to consider coupled nonlinear stochastic partial differential equations of the type

$$dq(x,t) = N(q,\nabla,x,\alpha,t)dt + dF \tag{3.4}$$

where we may use the Stratonovich calculus. Without fluctuations the equations reduce to

$$\dot{q} = N(q,\nabla,x,\alpha,t) \tag{3.5}$$

A special case treated in chemistry has the form

$$\dot{q} = R(q) + D\nabla^2 q \tag{3.6}$$

where the first term R describes the reactions whereas the second describes diffusion processes. For sake of completeness we mention that as long as we deal with Markov processes we may also invoke other types of equations, e.g. the Chapman-Kolmogorov equation. Finally we mention that the methods we shall present below, including the slaving principle, possess a quantum mechanical analogue, where the evolution equations are replaced by Heisenberg's operator equations which contain damping terms and fluctuating operator forces.

b. Instability

We assume that we have found a solution of the nonlinear equations for given control parameters $\alpha=\alpha_0$. In practical cases such a solution may describe, for instance, a quiescent and homogeneous state, but our treatment may also include spatially inhomogeneous and oscillatory states. We denote the corresponding solution by q_0. When we change the control parameter that solution q_0 may loose its stability. To study the stability (or instability) we put

$$q(x,t,\alpha) = q_0(x,t,\alpha) + w(x,t,\alpha) \tag{3.7}$$

and insert it into (3.5). Assuming that w is a small quantity we may linearize (3.5) and study the resulting equations of the form

$$\dot{w} = L(q_0(x,t),\nabla,x,\alpha)w, \quad w = w(t) \tag{3.8}$$

If L is independent of t or depends on t periodically, or in a large class of systems in a quasiperiodic fashion, the solutions can be written in the form

$$w^{(j)}(t) = \exp(\lambda_j t)v^{(j)}(t) \tag{3.9}$$

where v(t) is bounded. Thus the global behavior of w is determined by the exponential function in (3.9). We call those solutions, whose real part of λ is positive, unstable, and those whose real part of λ is negative, stable. In order to solve the nonlinear equation (3.5) (or, more generally, its stochastic counterpart (3.4)) we make the hypothesis

$$q(x,t) = q_0(x,t,\Phi(t)) + \sum_j u_j(t)v^{(j)}(x,t,\Phi(t)) + \sum_k s_k(t)v^{(k)}(x,t,\Phi(t)) \tag{3.10*}$$

where Φ is a set of certain phase angles in case we deal with quasiperiodic motion. For details I refer the reader to my book ADVANCED SYNERGETICS. Here it may suffice to note that by inserting

* j and k run over the unstable and stable mode indices, respectively.

the hypothesis (3.10) into our original nonlinear equations (3.5) we find after some mathematical manipulations the following equations

$$\dot{u}_j = \lambda_j u_j + N_j^{(u)}(u,\Phi,t,s), \tag{3.11}$$

$$\dot{s}_k = \lambda_k s_k + N_k^{(s)}(u,\Phi,t,s), \tag{3.12}$$

$$\dot{\Phi}_1 = N_1^{(\Phi)}(u,\Phi,t,s). \tag{3.13}$$

Similarly, starting from (3.4) we obtain stochastic equations for u,s, Φ. Though in general one may not expect to simplify a problem by means of a trans-formation, the new equations (3.11)-(3.13) can be considerably simplified when a system is close to instability points, where the real parts of some λ's change their sign from negative to positive.

c) The slaving principle

For the situations just mentioned we have derived the slaving principle for stochastic differential equations and discrete noisy maps. The slaving principle states that we may express the amplitudes s of the damped modes by means of u and Φ at the same time, so that

$$s = f(u,\Phi,t) \tag{3.14}$$

We shall call u and Φ order parameters. We have studied numerous cases of dissipative systems and have found that in practically all of them there occur only few order parameters while there are still very many slaved modes. As a consequence we achieve an enormous reduction of the degrees of freedom because we may express all damped modes s by the order parameters. In this way we obtain a closed set of equations of the form

$$\dot{u} = N(u,\Phi,t), \tag{3.15}$$

$$\dot{\Phi} = N'(u,\Phi,t). \tag{3.16}$$

Some applications of these equations will be discussed below in section 5.

4. GENERALIZED GINZBURG-LANDAU EQUATIONS

When the dimensions of continuously extended systems are large compared to the fundamental length of developing patterns, the spectrum λ is practically continuous. In such a case particular mathematical difficulties arise because it is no more possible to distinguish clearly between undamped and damped modes. A way out of this difficulty can be found when we resort to the formation of wave packets. This in turn necessitates that the order parameters, which we shall call ξ, depend not only on time but now also on space (in a slowly varying fashion). Therefore our hypothesis reads

$$q(x,t) = q_0 + \sum_{k_c} \xi_{k_c}(x,t) v_{k_c}(x) + \sum \text{slaved modes} \tag{4.1}$$

where k_c runs over a discrete set of critical wave vectors at which the instabilities occur. For simplicity let us again consider a case in which no phase angles occur and let us furtheron be satisfied with an expansion of the nonlinear terms up to third order. The order parameter equations then acquire the form

$$\xi_{k_c}(x,t) = \lambda_{k_c}(\nabla)\xi_{k_c}(x,t) + \sum_{k_1,k_2} A\ldots \xi_{k_1}\xi_{k_2} + \sum_{k_1,k_2,k_3} B\ldots \xi_{k_1}\xi_{k_2}\xi_{k_3} + F_{k_c}. \tag{4.2}$$

I have called these equations, which I derived some time ago "Generalized Ginzburg-Landau-equations", because they are strongly reminescent of the famous Ginzburg-Landau-equations. But two important distinctions should be noted. While the original Ginzburg-Landau-equations refer to a system in thermal equilibrium my Generalized Ginzburg-Landau-equations refer to systems far from thermal equilibrium. Furthermore the original Ginzburg-Landau-equations were derived in a heuristic fashion, whereas here the Generalized Ginzburg-Landau-equations have been derived rigorously. Because of the double and triple sums these equations are quite clumsy. However, under well justified assumptions these equations can be simplified as I have shown recently. To this end I define a new function

$$\Psi(x,t) = \sum_{k_c} e^{ik_c x}\xi_{k_c}(x,t). \tag{4.3}$$

After a few elementary manipulations and under specific assumptions on λ, A and B eq.(4.2) can be cast into the form

$$\dot{\Psi}(x,t) = (a + b(k_0^2 - \nabla^2)^2)\Psi + a\Psi^2 + B\Psi^3 + F, \tag{4.4}$$

where I have chosen an explicit example for $\lambda(k)$ which refers to the eigenvalues of the convection instability. We have solved this equation on a computer to study the temporal evolution of patterns. A typical result is shown in Fig. 1.

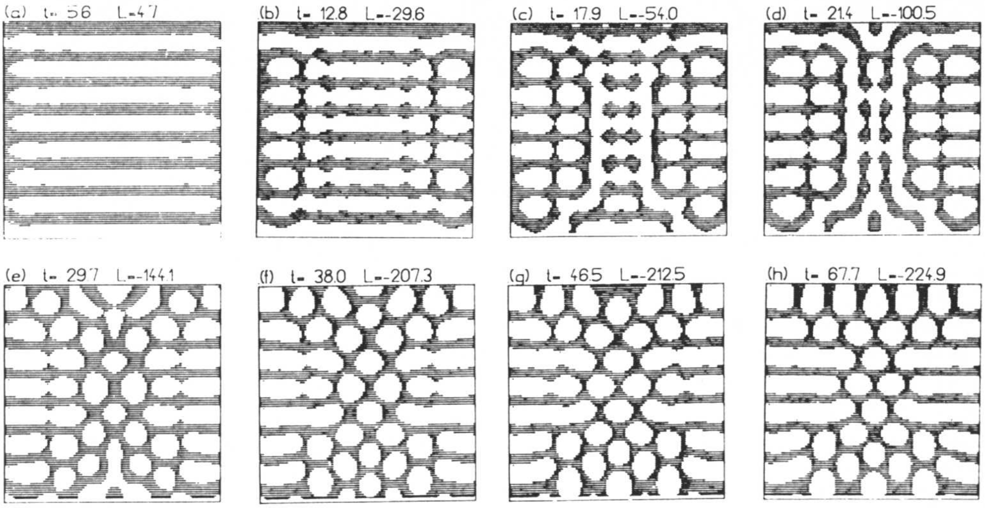
5. SOME FURTHER APPLICATIONS

By means of the mathematical methods we have outlined above my coworkers and I have treated a number of explicit cases over the recent years. I present a few of them in order to demonstrate the applicability of the mathematical method I have briefly sketched in the beginning of my lectures.

a) Pattern formation of an MHD plasma which is heated from below and is subjected to a vertical constant magnetic field. The boundary conditions in the horizontal directions are chosen periodic. Several patterns could be found. In the single mode case rolls appear, well known from fluid dynamics. However, also two or three mode cases are possible. A typical velocity distribution is shown in Fig. 2.

b) Running waves in the positive column of a gas discharge in neon

By means of a nonlinear treatment it has been possible to derive the corresponding spatio-temporal pattern in good qualitative and semiqualitative agreement with experiments.



▲Fig.1. A roll pattern is prescribed but the parameter values of the equation are chosen such that hexagons should be formed. The sequence a - h shows the formation of hexagons but the final state is reached only at infinitely large time (critical slowing down)

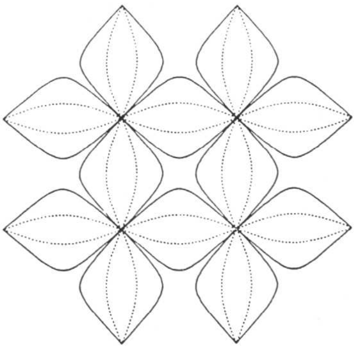


Fig.2. Lines of constant vertical velocity in a plasma heated from below and subject to vertical constant magnetic field

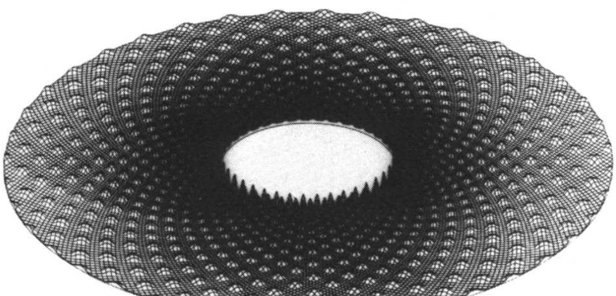


Fig.3. Embossed map on model calculation on fructification of sunflower

c) Prepattern formation of the spiral wave pattern of a sunflower head

We have adopted reaction diffusion equations of the form (cf.3.6) and have used the Gierer-Meinhardt-model for the reaction terms. Since the specific form of the reaction terms is not so important we don´t discuss them here. However, it was assumed that the diffusion is space dependent. Results of the nonlinear analysis are shown in Fig.3. They clearly exhibit two counter rotating sets of spirals in good agreement with the observed fructification of the sunflower head.

6. OUTLOOK

By means of the systematic approach of synergetics it has been possible to classify a number of spatial and temporal patterns which occur over and over again. In addition it has become possible to study the dynamics close to transition points in detail because the dynamics is governed by few order parameters only. A few words of future problems may be in order and I will list only three of them:

1) So far we have assumed that we start from a spatially homogeneous state. The whole approach works also if the original state is spatially inhomogeneous but time independent or time periodic. However, in order to solve the linearized equations and to derive the order parameter equations in most cases computer calculations may be needed.

2) When we go away from instability points, the patterns remain qualitatively the same as is known from numerous experiments. However, as it seems to me a rigorous theory far away from instability points is still lacking.

3) A rich field of further study is provided by chaos and it seems to me that we are just at the beginning of classifying and understanding chaotic motion.

In conclusion it might be worth pointing out that in the field of synergetics we need not only a further development of mathematical methods but also the corresponding experiments must be performed and a close interaction between experimentalists and theoreticians is needed. We believe that the approaches so far have not only given us fundamental insights into the way new patterns evolve at instability points but have also led to a number of practical applications by exploiting analogies between different systems. These analogies become apparent through the order parameter equations. I am sure that this will lead to a development of new devices, especially in solid state physics and quantum electronics.

References:

1. H. Haken, "Synergetics. An Introduction. Nonequilibrium Phase Transitions and Self-Organization in Physics, Chemistry and Biology". Third Revised and Enlarged Edition, Springer Verlag, Berlin, Heidelberg, New York, Tokyo 1983
2. H. Haken, "Advanced Synergetics. Instability Hierarchies of Self-Organizing Systems and Devices", Springer Verlag, Berlin, Heidelberg, New York, Tokyo 1983
3. Yu. A. Danilov, B.B. Kadomtsev, in "Nonlinear Waves, Self-Organization" edited by A.V. Gaponov-Grechov, M.I. Rabinovich, Moscow 1983
4. H. Haken "Laser Theory" Encyclopedia of Physics Vol.XXV/2c, Springer Verlag Berlin, Heidelberg, New York 1970, reprinted 1983
5. J.L. Ibanez, M.G. Velarde, J.Non-Equilib.Thermodyn. Vol.3. 63 (1978)
Ch. Berding and H. Haken, J. Math.Biolog.14, 133 (1982)