FOURTH EDITION

Strength of Materials

Andrew Pytel

Ferdinand L. Singer

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Preface

Today, more than ever, engineering applications are often interdisciplinary, involving the interrelationship of several of the basic engineering sciences (mechanical, electrical, chemical, etc.). Therefore the modern engineer must have a fundamental knowledge in each of these areas. An understanding of how bodies respond to applied loads, the main area of emphasis in *Strength of Materials*, is a part of this knowledge. Furthermore, for successful machine or structural design, a thorough mastery of strength of materials is a must.

The unique feature of this fourth edition, as compared with previous editions, is that it uses both SI* and U.S. Customary Units. Since the United States has yet to adopt the SI system as its standard, there remains a need for engineers here to be trained in both sets of units. In this edition, the problems to be solved are divided almost evenly between SI and U.S. Customary Units, thus allowing the instructor to determine the proper balance for his or her students.

This edition retains the general plan and features of the earlier editions, with the major emphasis still on elastic analysis, and, in addition, a chapter devoted to inelastic response. The importance of beam deflections in structural design warranted keeping the fairly complete treatment of this topic which includes energy methods, double-integration, area-moment, and moment distribution. However, since each of these topics is discussed separately, the instructor can easily choose only those methods that are relevant to his or her own presentation.

Other features that are pertinent to this edition include an expanded discussion of plane stress, with a more thorough consideration of absolute maximum shearing stress; a revision of the chapter on connections to explain more fully

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* SI is the official abbreviation for the international system of units, Le Système International d'Unités.

the distinctions between bearing-type and friction-type connections; and an updating of several topics due to changes in various design codes.

Keeping in mind the special problems of students, we have, as in previous editions, endeavored to explain the fundamental concepts by using clear and concise language. The relatively large number of illustrative problems is intended to help the student bridge the gap between theory and application. The equations or principles used in the solutions of these problems are usually first stated in brackets; then the numerical values are substituted in the order in which the symbols appear in the equation. This technique enables the reader to follow the analysis more easily.

The almost 1000 problems in this text have been carefully chosen to illustrate the fundamental concepts without overburdening the student with tedious numerical computation, wherever possible. The importance of free-body diagrams in strength of materials continues to be emphasized. The problems have been arranged largely in their order of difficulty; and answers to about two-thirds of them accompany the appropriate problem statements.

We continue to use a numbering plan that enables the reader to locate any cross reference quickly. In this scheme, all articles, figures, equations, tables, and problem statements, which are preceded by the number of the chapter in which they appear, are numbered consecutively throughout each chapter. The scheme is further simplified by having the numbers of the problems coincide with the numbers of the appropriate problem figures.

The valuable suggestions and advice received from colleagues all over the world are sincerely acknowledged. To identify each contributor here would result in too lengthy a list, with the possibility of an inadvertent omission; each of these people has been thanked individually. However, a special debt is owed to Dr. Jean Landa Pytel, whose assistance in the preparation of this manuscript is greatly appreciated.

Andrew Pytel

List of Symbols and Abbreviations

A	area
A'	partial area of beam section
$\bar{a}$, $\bar{b}$	coordinates of centroid of moment diagram caused by simply supported loads
b	breadth, width
c	distance from neutral axis to extreme fiber
C	centroid of area
D, d	diameter
E	modulus of elasticity in tension or compression
e	eccentricity, base of natural logarithms
f	frequency
f_c	unit compressive stress in concrete
f_s	unit tensile stress in reinforcing steel
G	modulus of rigidity (i.e., modulus of elasticity in shear)
g	gravitational acceleration (32.2 ft/s^2; 9.81 m/s^2)
h	height, depth of beam
I	moment of inertia of area
I_{NA}	moment of inertia with respect to neutral axis
$\bar{I}$	centroidal moment of inertia
J	polar moment of inertia
$\bar{J}$	centroidal polar moment of inertia
K	stress concentration factor
k	spring constant, radius of gyration
L	length
L_e	effective length for columns
M	bending moment

m	mass
N	normal force, factor of safety
n	ratio of moduli of elasticity
P	force, concentrated load, hoop tension
$\mathcal{P}$	power
P_{cr}	critical load for columns
P_{uv}, P_{xy}	products of inertia
p	pressure per unit area
Q	first moment of area
q	shear flow
R	reaction, resultant force, radius
r	radius, radius of gyration
S	section modulus (I/c)
σ	unit stress, normal stress
σ_1, σ_2, σ_3	principal stresses
σ_b	unit bearing stress
σ_c	unit compressive stress
σ_{cr}	critical unit stress in column formula
σ_f	unit flexural stress
σ_r	unit radial stress
σ_w	allowable or working stress
σ_t	unit tensile stress, unit tangential stress
σ_x, σ_y, σ_z	unit normal stress in x, y, and z directions, respectively
σ_{yp}	stress at yield point
T	torque, temperature
t	thickness, tangential deviation
τ	unit shearing stress
τ_{xy}	unit shearing stress in x-y plane
u, v, w	rectangular coordinates
V	vertical shearing force
v	velocity
W	total weight or load
w_0	weight or load per unit of length
x, y, z	rectangular coordinates
$\bar{x}$, $\bar{y}$, $\bar{z}$	coordinates of centroid or center of gravity
y	deflection of beam
α	temperature coefficient of linear expansion
α, β, γ . . .	angles
γ	unit shearing strain
δ	total elongation or contraction; deflection of beam; maximum deflection of column
δ_{st}	static deflection
ϵ	unit tensile or compressive strain
ϵ_1, ϵ_2	principal strains
ϵ_x, ϵ_y, ϵ_z	unit tensile or compressive strain in the x, y, and z direction, respectively

θ	total angle of twist, slope angle for elastic curve
ρ	radius of curvature, variable radius, mass density
ν	Poisson's ratio
ω	angular velocity
CG	center of gravity
deg	degrees
DF	distribution factor
FS	factor of safety
FEM	fixed end moment
NA	neutral axis
PL	proportional limit
YP	yield point

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Chapter 1

Simple Stress

1-1 INTRODUCTION

Three fundamental areas of engineering mechanics are statics, dynamics, and strength of materials. Statics and dynamics are devoted primarily to the study of the external effects of forces on rigid bodies, that is, bodies for which the change in shape (deformation) can be neglected.

In contrast, strength of materials deals with the relations between externally applied loads and their internal effects on bodies. Moreover, the bodies are no longer assumed to be rigid; the deformations, however small, are of major interest. In mechanical design, the engineer must consider both dimensions and material properties to satisfy requirements of strength and rigidity. When loaded, a machine part or structure should neither break, nor deform excessively.

The differences between rigid-body mechanics and strength of materials can be further emphasized by considering the following example. For the bar in Fig. 1-1, it is a simple problem in statics to determine the force required to support the load W. A moment summation about the pin support determines P. This statics solution assumes the bar to be both rigid and strong enough to support the load. In strength of materials, however, the solution must extend further. We must investigate the bar itself to be sure that it will neither break nor be so flexible that it bends without supporting the load.

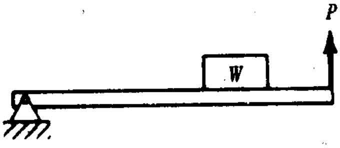


Figure 1-1 Bar must neither break nor bend excessively.

Throughout this text we study the principles that govern the two fundamental concepts, strength and rigidity. In this first chapter we start with simple axial loadings; later, we consider twisting loads and bending loads; and finally, we discuss simultaneous combinations of these three basic types of loadings.

1-2 ANALYSIS OF INTERNAL FORCES

Consider a body of arbitrary shape acted upon by the forces shown in Fig. 1-2. In statics, we would start by determining the resultant of the applied forces to determine whether or not the body remains at rest. If the resultant is zero, we

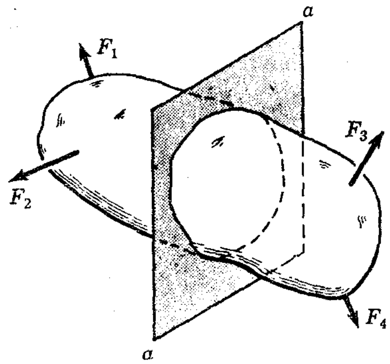


Figure 1-2 Exploratory section *a-a* through loaded member.

have static equilibrium—a condition generally prevailing in structures. If the resultant is not zero, we may apply inertia forces to bring about dynamic equilibrium. Such cases are discussed later under dynamic loading. For the present, we consider only cases involving static equilibrium.

In strength of materials, we make an additional investigation of the internal distribution of the forces. This is done by passing an exploratory section *a–a* through the body and exposing the internal forces acting on the exploratory section that are necessary to maintain the equilibrium of either segment. In general, the internal forces reduce to a force and a couple that, for convenience, are resolved into components that are normal and tangent to the section, as shown in Fig. 1-3.

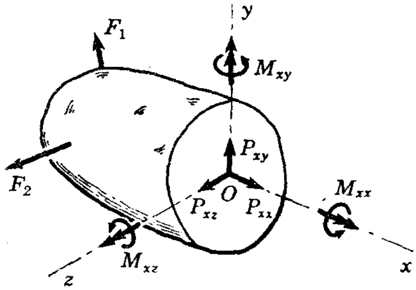


Figure 1-3 Components of internal effects on exploratory section *a-a*.

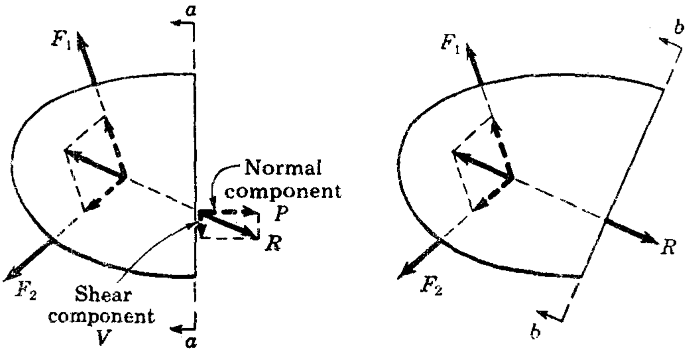
The origin of the reference axes is always taken at the centroid which is the key reference point of the section. Although we are not yet ready to show why this is so, we shall prove it as we progress; in particular, we shall prove it for normal forces in the next article. If the x axis is normal to the section, the section is known as the x surface or, more briefly, the x face.

The notation used in Fig. 1-3 identifies both the exploratory section and the direction of the force or moment component. The first subscript denotes the face on which the component acts; the second subscript indicates the direction of the particular component. Thus P_{xy} is the force on the x face acting in the y direction.

Each component reflects a different effect of the applied loads on the member and is given a special name, as follows:

P_{xx}	*Axial force.* This component measures the pulling (or pushing) action perpendicular to the section. A pull represents a tensile force that tends to elongate the member, whereas a push is a compressive force that tends to shorten it. It is often denoted by P.
P_{xy}, P_{xz}	*Shear forces.* These are components of the total resistance to sliding the portion to one side of the exploratory section past the other. The resultant shear force is usually designated by V, and its components by V_y and V_z to identify their directions.
M_{xx}	*Torque.* This component measures the resistance to twisting the member and is commonly given the symbol T.
M_{xy}, M_{xz}	*Bending moments.* These components measure the resistance to bending the member about the y or z axes and are often denoted merely by M_y or M_z

From the preceding discussion, it is evident that the internal effect of a given loading depends on the selection and orientation of the exploratory section. In particular, if the loading acts in one plane, say, the xy plane as is frequently the case, the six components in Fig. 1-3 reduce to only three, namely, the axial force P_{xx} (or P), the shear force P_{xy} (or V), and the bending moment M_{xz} (or M). Then, as shown in Fig. 1-4a, these components are equivalent to the single



(a) Normal and shear components on arbitrary section $a-a$.

(b) When exploratory section $b-b$ is perpendicular to resultant R of applied loads, only normal forces are produced.

Figure 1-4 (a) Normal and shear components on arbitrary section *a-a;* (b) when exploratory section *b-b* is perpendicular to resultant R of applied loads, only normal forces are produced.

resultant force R. A little reflection will show that if the exploratory section had been oriented differently, like b–b in Fig. 1-4b where it is perpendicular to R, the shearing effect on the section would reduce to zero and the tensile effect would be at a maximum.

The purpose of studying strength of materials is to ensure that the structures used will be safe against the maximum internal effects that may be produced by any combination of loading. We shall learn as our study proceeds that it is not always possible or convenient to select an exploratory section that is perpendicular to the resultant load; instead, we may have to start by analyzing the effects acting on a section like a–a in Figs. 1-2 and 1-4a, and then learn how these effects combine to produce maximum internal effects like those on section b–b in Fig. 1-4b. We shall study this procedure later in Chapter 9, which deals with combined stresses. For the present, we restrict our study to conditions of loading in which the section of maximum internal effect is evident by inspection.

1-3 SIMPLE STRESS

One of the basic problems facing the engineer is to select the proper material and proportion it to enable a structure or machine to perform its function efficiently. For this purpose, it is essential to determine the strength, stiffness, and other properties of materials. A tabulation of the average properties of common metals is given in Appendix B, Table B-1, on page 552.

Let us consider two bars of equal length but different materials, suspended from a common support as shown in Fig. 1-5. If we knew nothing about the bars except that they could support the indicated maximum axial loads [500 N (newtons) for bar 1 and 5000 N for bar 2], we could not tell which material is stronger. Of course, bar 2 supports a greater load, but we cannot compare strengths without having a common basis of comparison. In this instance, the cross-sectional areas are needed. So let us further specify that bar 1 has a cross-sectional area of 10 mm^2 and bar 2 has an area of 1000 mm^2. Now it is simple to compare their strengths by reducing the data to load capacity per unit area. Here we note that the unit strength of bar 1 is

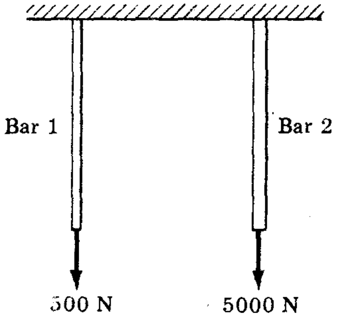


Figure 1-5 Bars supporting maximum loads.