Laurence D. Hoffmann

Gerald L. Bradley

FINITE MATHEMATICS WITH CALCULUS, 2/E

FINITE MATHEMATICS WITH CALCULUS

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Preface

Finite Mathematics with Calculus, Second Edition is intended for use primarily in a two-term course taken by students majoring in business, economics, technological sciences, or the life and social sciences. This new edition is an extensive revision based on numerous user and reviewer suggestions. However, it still retains the straightforward style, intuitive approach, and applications orientation of the first edition, *Mathematics with Applications*.

Features

Applications The text is applications-oriented. Each new concept is applied to a variety of practical situations. The techniques and strategies needed to solve applied problems are stressed. The applications are drawn from the social, managerial, environmental and life sciences, and business and economics areas. All applications are titled for reference.

Problems Each section in this text is followed by an extensive set of problems. Many involve routine computation and are designed to help the student master new techniques. Others ask the student to apply the new techniques to practical situations.

Algebra Review If a student needs to brush up on his or her algebra skills, there is an extensive review of algebra (optional) that includes worked-out examples and practice problems. The student is advised throughout the text when it is appropriate to consult this material.

Color Pedagogical use of a full-color design makes the book more visually appealing and benefits students who think geometrically rather than linearly.

Study Aids Each chapter opens with a list of objectives identifying the skills to be learned within the chapter. The chapter ends with a chapter summary that includes important terms, symbols, and formulas. There is also a set of review problems. Answers to the odd-numbered problems and to all of the review problems are provided at the back of the book.

To the Student

If you are preparing for a career in business, economics, psychology, sociology, architecture, or biology, and if you have taken high school algebra, then this book was written for you. Its primary goal is to teach you the techniques used in decision making and differential and integral calculus that you are likely to encounter in undergraduate courses in your major and in your subsequent professional activities. The exposition is designed to give you a sound, intuitive understanding of the basic concepts without sacrificing mathematical accuracy. Thus, the main results are stated carefully and completely, and whenever possible, explanations are intuitive or geometric.

Supplementary Materials

Student Supplements

Student's Solutions Manual: This manual by Henri Feiner (West Los Angeles College) contains complete step-by-step solutions to the odd-numbered end-of-section exercises as well as solutions to all review exercises.

Student Tutorial: This tutorial reinforces topics and provides opportunities to review concepts and to practice problem solving. It requires virtually *no* computer training and is available for IBM, IBM compatible, and Macintosh computers.

Graphing Calculator Enhancement Manual: This manual presents an integrated approach that utilizes calculator-based graphing to enhance understanding and development. It includes calculator exercises and examples as well as appendixes on how to use the most popular calculators.

Instructor Supplements

Instructor's Manual: This manual contains the answers to all of the exercises in the text as well as step-by-step solutions to the even-numbered end-of-section exercises. Sample Tests are also included. These are multiple-choice and short answer tests for each chapter, midterm tests, and final examinations.

The Professor's Assistant: This is a unique computerized test generator available to instructors. This system allows the instructor to create tests using algorithmically generated test questions and those from a standard test bank. This testing system enables the instructor to choose questions either manually or randomly by section, question type, difficulty level, and other criteria. This system is available for IBM, IBM compatible, and Macintosh computers.

Print Test Bank: This is a printed and bound version of the questions found in the standard test bank.

For further information, please contact your local McGraw-Hill sales representative.

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Laurence D. Hoffmann

Gerald L. Bradley

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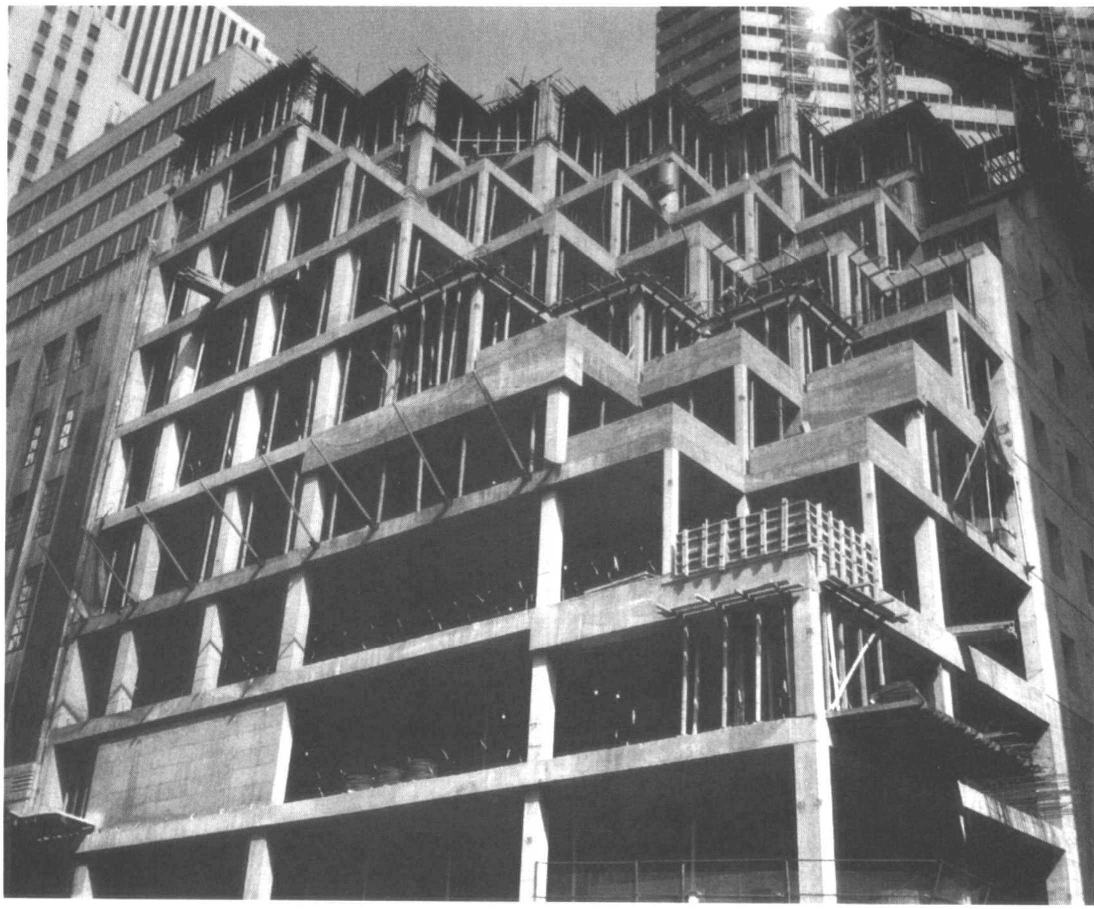
CHAPTER 1
PRELIMINARIES: ALGEBRAIC REVIEW



1 The Real Numbers

This chapter contains a review of algebra and a few miscellaneous topics from earlier courses that you will need in this text. We begin by examining the real number system.

Whenever you balance your checkbook, calculate your income tax return, or determine how much change you should receive at the grocery checkout stand, you are using the **real number system.** Real numbers include both rational and irrational numbers.

A **rational number** is a number that can be expressed as the quotient $\frac{a}{b}$ of two integers, where $b \neq 0$. For example,

$$\frac{4}{3} \qquad \frac{8}{5} \qquad -6\frac{1}{2} = \frac{-13}{2} \qquad \text{and} \qquad 0.25 = \frac{25}{100} = \frac{1}{4}$$

are all rational numbers. When expressed in decimal form, rational numbers are either terminating or infinitely repeating decimals. For example,

$$\frac{5}{8} = 0.625 \qquad \frac{1}{3} = 0.33333\ldots \qquad \frac{13}{11} = 1.181818\ldots$$

A number that cannot be expressed as the quotient of two integers is called an **irrational number**. For example,

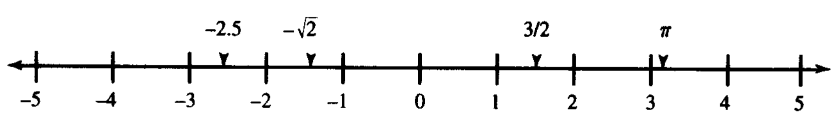
$$\sqrt{2} = 1.41421356\ldots \qquad \text{and} \qquad \pi = 3.14159265\ldots$$

are irrational numbers.

Integers are part of the set of rational numbers. An **integer** may be a natural (whole) number, zero, or a negative of a natural number. For example 1, 875, 0, and -83 are all integers, whereas $\frac{2}{3}$, 8.71, and $\sqrt{2}$ are not.

All real numbers can be plotted on a **number line**, as illustrated in Figure 1.1. The **coordinate** of a point is the number associated with it.

Figure 1.1 A number line.



Basic Properties of Real Numbers

There are several basic algebraic properties that allow us to convert algebraic expressions into equivalent expressions. The following box presents these properties.

Basic Properties of Real Numbers

The following properties hold for all real numbers a, b, c, and d:

Name of Property	
Closure	Addition: the sum $a + b$ is a real number. Multiplication: The product ab is a real number.
Associative Laws	Addition: $a + (b + c) = (a + b) + c$. Multiplication: $a(bc) = (ab)c$.
Commutative Laws	Addition: $a + b = b + a$. Multiplication: $ab = ba$.
Identity Laws	Addition: $a + 0 = 0 + a = a$. Multiplication: $a \cdot 1 = 1 \cdot a = a$.
Distributive Laws	$a(b + c) = ab + ac$ $(a + b)c = ac + bc$
Inverse Laws	For every real number a, there is a (unique) real number denoted by $-a$ such that $a + (-a) = 0$. For every real number $b \neq 0$, there is a (unique) real number denoted by $\frac{1}{b}$ such that $b\left(\frac{1}{b}\right) = 1$. The number $-a$ is called the **additive inverse** of a, and the number $\frac{1}{b}$ is the **multiplicative inverse** of b.
Zero Properties	For every real number a, $a \cdot 0 = 0$. $ab = 0$ if and only if $a = 0$ or $b = 0$ (or both).

EXAMPLE 1.1

Here are a few numerical illustrations of these laws:

(a) Associative laws: $4 + (3 + 7) = (4 + 3) + 7 = 14$
$$7 \cdot (3 \cdot 4) = (7 \cdot 3) \cdot 4 = 84$$
(b) Commutative laws: $3 + (-5) = (-5) + 3 = -2$
$$6 \cdot 2 = 2 \cdot 6 = 12$$
(c) Distributive laws: $6 \cdot (2 + 5) = (6 \cdot 2) + (6 \cdot 5) = 42$
$$(3 + (-2)) \cdot 9 = (3 \cdot 9) + ((-2) \cdot 9) = 9$$
(d) Identity laws: $4 + 0 = 0 + 4 = 4$
$$7 \cdot 1 = 1 \cdot 7 = 7$$
(e) Inverse laws: $4 + (-4) = (-4) + 4 = 0$
$$\frac{1}{5}(5) = (5)\frac{1}{5} = 1$$
(f) Zero property: $9 \cdot 0 = 0 \cdot 9 = 0$

We can also define subtraction and division in terms of addition and multiplication, respectively. Specifically, the **difference** $a - b$ is defined as

$a + (-b)$, and the **quotient** $\frac{a}{b}$ is defined as $\frac{a}{b}$ if $b \neq 0$. It is important to remember that division by 0 is not defined. The main properties of subtraction and division are presented in the following box.

Reduction Rules

If a, b, c, d are real numbers, with $b \neq 0$ and $d \neq 0$, then:

1. ***Negative rules*: $-(-a) = a$ and $(-a)b = -ab$.**
2. ***Cross-multiplication rule*: $\frac{a}{b} = \frac{c}{d}$ if and only if $ad = bc$.**
3. ***Cancellation rule*: $\frac{ka}{kb} = \frac{a}{b}$ if $k \neq 0$.**
4. ***Common denominator rule*: $\frac{a}{b} + \frac{c}{b} = \frac{ad + bc}{bd}$.**

EXAMPLE 1.2

Simplify the following expressions using the reduction rules:

(a) $\frac{55}{15}$ (b) $\frac{2}{5} + \frac{4}{3}$ (c) $\frac{5x}{3x}$, for any number $x \neq 0$ (d) $3 - (-2)(-7)$

Solution

(a) By the cancellation rule: $\frac{55}{15} = \frac{5}{3}\frac{11}{5} = \frac{11}{3}$.

(b) By the common denominator rule: $\frac{2}{5} + \frac{4}{3} = \frac{(2)(3) + (4)(5)}{15} = \frac{26}{15}$.

(c) By the cancellation rule: $\frac{5x}{3x} = \frac{5}{3}$.

(d) By the negative rules: $3 - (-2)(-7) = 3 - (2)(7) = 3 - 14 = -11$.

Order and Intervals

The real numbers are ordered by their relative position on the number line. In particular, we say ***a* is greater than *b*** if a is to the right of b on the number line, and we denote this relationship by writing $a > b$. Similarly, ***a* is less than *b*** if a is to the left of b, and we write $a < b$. These order relationships are illustrated in Figure 1.2.

A few basic properties of order are presented in the following box. Note especially property 3, which states that the sense of an inequality is *preserved*

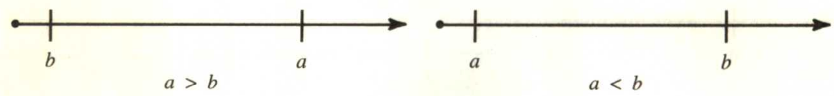


Figure 1.2 Inequalities.

if both sides are multiplied by a positive number, but it is *reversed* if the multiplier is negative.

Properties of Order

1. *Transitive property*: If $a > b$ and $b > c$, then $a > c$.
2. *Additive property*: If $a > b$ and $c > d$, then $a + c > b + d$.
3. *Multiplicative properties*: If $a > b$ and $c > 0$, then $ac > bc$, and if $a > b$ and $c < 0$, then $ac < bc$.

For example, since $7 > 3$, we have $7 - 9 > 3 - 9$ or $-2 > -6$. Since $5 > 2$ and $3 > 0$, it follows that $5 \cdot 3 > 2 \cdot 3$, or $15 > 6$. Since $5 > 2$ and $-2 < 0$, we have $5(-2) < 2(-2)$, or $-10 < -4$.

The symbol $\geq$ stands for **greater than or equal to**, and the symbol $\leq$ stands for **less than or equal to**. Thus, for example,

$$-3 \geq -4 \qquad -3 \geq -3 \qquad -4 \leq -3 \qquad \text{and} \qquad -4 \leq -4$$

A set of real numbers that can be represented on the number line by a line segment is called an **interval**. Inequalities can be used to describe intervals. For example, the interval $a \leq x < b$ consists of all real numbers x that are between a and b, including a but excluding b. This interval is shown in Figure 1.3. The numbers a and b are known as the **endpoints** of the interval. The square bracket at a indicates that a is included in the interval, while the rounded bracket at b indicates that b is excluded.

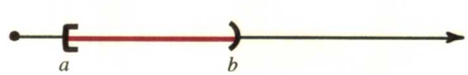


Figure 1.3 The interval $a \leq x < b$.

Intervals may be **finite** or **infinite** in extent and may or may not contain any endpoints. The possibilities (including customary notation and terminology) are illustrated in Figure 1.4.