CONTROL SYSTEMS

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To the memory of my father.

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Preface

Teaching control theory has always been a challenging task. Although real control systems are often quite complicated, they must be presented in a simplified form so that they are mathematically tractable. In the past there has often been a tendency to oversimplify, mainly because real-life problems require a great deal of computation. This is no longer a limitation now that engineers and engineering students have easy access to microcomputers. The main problem in taking full advantage of the situation has been the lack of availability of computer programs suitable for use by students of control theory. This book is an attempt to remedy this situation by providing such programs as well as several challenging problems to which they can be applied in the process of mastering the theory.

This book is intended as an introduction to control systems. The object is to provide the reader with the basic concepts of control theory as developed over the years in both the frequency domain and the time domain. An attempt has been made to retain the classical concepts while introducing modern ideas. The effect of the availability of inexpensive microcomputers has been kept in mind.

The book contains a large number of worked out examples in each chapter in order to help explain the theory and methods developed. Chapters 2 and 3 present a unified treatment of modeling of dynamic systems, including transfer function and state-space models. Chapter 10, on the compensation of control systems, has been presented in a logical and unified manner. In particular, the trial-and-error approach to the design of lead compensators, as found in most books, has been replaced by a direct method developed recently. Moreover, the design of a pole-placement compensator using transfer functions, which is the counterpart of the combined observer and state feedback controller, has been included for the first time in a book appropriate for undergraduates and practicing engineers. Chapter 11, or digital control, is an up-to-date treatment of a rapidly developing and popular area, presented in a manner suitable for a first examination of the subject, with many practical examples to provide motivation. Chapter 12 provides a good treatment of the state-space approach, written in a way that will make it attractive to both undergraduates and professionals. The concepts of controllability and observability are introduced using the theory of discrete-time systems already developed in Chapter 11. This is followed by the theory of state feedback. Both state-space and transfer function approaches are used for determining the state feedback vector. The asymptotic state observer is then introduced and its design described. The combined state feedback and observer design is also discussed and related to the transfer function approach presented in Chapter 10.

It is appreciated that in most engineering disciplines one has to solve many numerical problems in order to learn the various details and subtleties and to be able to handle engineering design problems. To provide motivation, a number of realistic problems have been included at the end of each chapter. Appendix D discusses computational aspects. Program listings have been given for many cases, with thorough documentation. It may be added that in the past most books on control systems were limited to including problems of second-order systems due to the fact that computing facilities were not available. Until about 1978, if a problem required finding the roots of a polynomial of degree higher than 2, one generally gave up. In recognition of the fact that at present all engineers have access to either a programmable calculator or a microcomputer, an attempt has been made to remedy this situation and prepare the reader for real-life problems by including program listings suitable for use on either an HP-41C programmable calculator or an IBM Personal Computer.

The book is intended for all engineering students at the senior level and practicing engineers, without any bias towards a particular branch of engineering. Although it is expected that the reader will have some background in Laplace transforms, z transforms, and matrices, these topics are reviewed in the appendixes. An attempt has been made to retain some mathematical rigor while providing motivation by giving many practical examples.

Earlier versions of this book have been used for a one-semester senior level course on control systems at McMaster University for the past three years. Since our seniors have completed a course on circuit and systems theory at the junior level, they have sufficient background in Laplace transforms, z transforms, state equations for electrical circuits, Bode plots, and theory of matrices. As a result, it has been possible to teach all the material in this book in a 13-week semester. If it is necessary to review some of this background material, there may be insufficient time to cover the entire book. In that case, Chapter 11, Chapter 13, and parts of Chapter 12 may be left out.

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Just as one does not thank himself, expressing gratitude to one's wife in public is not a Hindu custom. The wife is considered part of the husband and her coauthorship is tacitly assumed in any book that her husband writes. There is little doubt that without Meena's help, patience, and encouragement, this book would not have been possible.

Naresh K. Sinha

Contents

7. THE ROOT LOCUS METHOD 115

8. FREQUENCY RESPONSE PLOTS 152

9. STABILITY FROM FREQUENCY RESPONSE 185

10. DESIGN AND COMPENSATION OF CONTROL SYSTEMS 217

I

Introduction

The subject of control systems is of great inportance to all engineers. The objective is to free human beings from boring repetitive chores that can be done easily and more economically by automatic control devices. The recent developments in the large-scale integration of semiconductor devices and the resulting availability of inexpensive microprocessors has made it practical to use computers as integral parts of control systems, making them cheaper as well as more sophisticated.

Historically, the first automatic control device used in the industry was the Watt fly-ball governor, invented in 1767 by James Watt, who was also the inventor of the steam engine. The object of this device was to keep the speed of the engine nearly constant by regulating the supply of steam to the engine. A schematic diagram is shown in Fig. 1.1. The two fly balls in the governor rotate about a vertical axis at a speed proportional to the speed of the engine. Due to the centrifugal force acting on them, they tend to move out. This movement controls the supply of steam to the engine through a mechanical linkage to the steam flow valve in such a manner that the steam supply is reduced when the speed is high and increased when the speed is low.

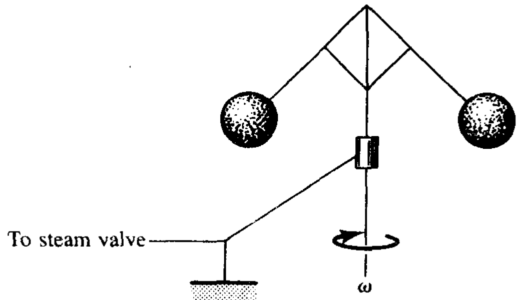

FIGURE 1.1. The Watt fly-ball governor.

It was found that by a proper design of the governor the speed could be kept within narrow limits of a specified value. It was also observed that if one tried to increase the sensitivity of the governor by increasing the gear ratio between the engine shaft and the governor, it tended to "hunt" or oscillate about the desired setting. It was about 100 years later that a complete mathematical analysis was made by James Clerk Maxwell (more well known for his contributions to electromagnetic field theory).

Much later it was realized that all automatic control systems worked on the principle of feedback. By a coincidence, about the same time the theory of feedback amplifiers had been developed by electrical engineers who had been concerned with transmitting telephone signals over long distances. In particular, one may mention the Nyquist theory of stability developed about 1930. A great impetus to the theory of automatic control came during World War II when servomechanisms were utilized for the control of anti-aircraft guns. After World War II many peacetime applications followed. Some of these are the "autopilot" for aircraft, automatic control of machine tools, automatic control of chemical processes, and automatic regulation of voltage at electric power plants. Although originally the theory was based on frequency response and Laplace transform methods, in the 1960s the impact of the digital computer led to the development of time-domain theory using state variables. This was especially useful as more sophisticated multivariable control systems were developed for more complex processes. As computers have become cheaper and more compact, they have been utilized as components of more advanced control systems.

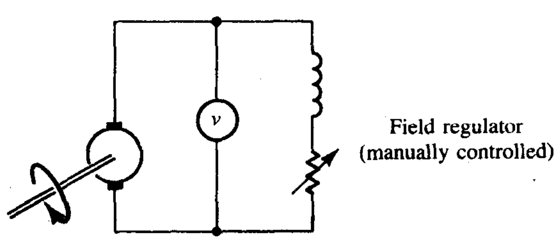

FIGURE 1.2. A voltage control system.

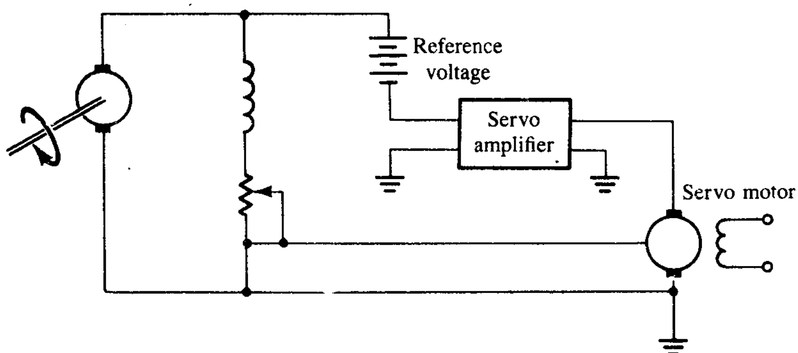

FIGURE 1.3. Automatic voltage regulator.

Let us consider some simple examples of control systems. Figure 1.2 shows the scheme for controlling the voltage at an electric power station in the 1940s. A human operator was required to watch a voltmeter connected to the busbars and adjust the field rheostat in order to keep the voltage close to the specified value. A scheme for automatic voltage regulation is shown in Fig. 1.3 and shows that it works by comparing the actual value of the voltage with the desired value. The difference or "error" is applied to a servomotor, after suitable amplification. This servomotor drives a shaft coupled to the field rheostat to alter the resistance in the field winding in such a manner that the error is reduced. Hence, it may be said that "feedback" is utilized to obtain automatic control. As a matter of fact, all automatic control systems use feedback and can be represented by the block diagram shown in Fig. 1.4. It can be seen that the controlled output is fed back and compared with the reference input. The difference, called the "error," is then utilized to drive the system in such a manner that the output approaches the desired value (i.e., the reference input).

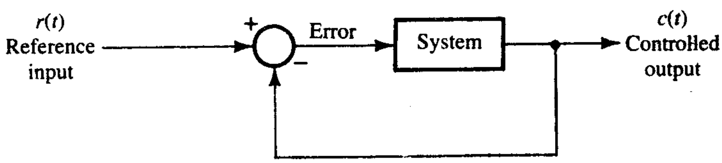

FIGURE 1.4. Block diagram of automatic control system.

Another example is the home heating system. A thermostat senses the temperature and if it is lower than a set value, the furnace is turned on. The furnace is turned off when the temperature exceeds another set value. The block diagram is shown in Fig. 1.5. Although it is similar to Figs. 1.3 and 1.4, it may be noted that this is an on/off-type control system, whereas the voltage regulator is a continuous-type system.

It was noticed at the very outset that if one tried to improve the accuracy of a control system by increasing the loop gain, it led to instability,

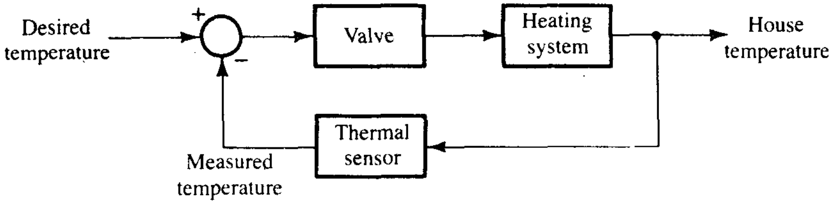

FIGURE 1.5. A home heating system.

or hunting. It is caused by the fact that although the system is designed with negative feedback, due to inherent time lags, it may change into positive feedback at some frequency. Therefore, oscillations may be produced at this frequency if the gain is increased sufficiently. The Nyquist criterion of stability, developed for feedback amplifiers, provides a good understanding of this. We shall later see how one can increase the sensitivity (or accuracy) without causing instability.

The components used for control systems are usually of a wide variety. For example, these may be electromechanical, electronic, thermal, hydraulic, or pneumatic. In order to analyze the response of the various components, we replace them by their mathematical models. Although the input and the output of these devices are generally related through nonlinear differential equations, it is customary to obtain simplified linear models about the operating points because such models are easier to analyze. Transfer function and state-variable models are most commonly employed.

In our development of control theory, we shall generally be carrying out the analysis and design in terms of the mathematical models. Although this approach may sometimes appear abstract, one must appreciate that these models represent real systems. To a certain extent this abstraction is necessary for developing a unified theory of automatic control systems despite the great variety of components. One important aspect is the problem of obtaining mathematical models for different types of physical systems. This will be discussed in Chapter 2. It will be assumed that the reader is familiar with the theory of Laplace transforms. For the sake of completeness, a review of Laplace transforms is given in Appendix A.

We shall close this chapter by mentioning some areas in which the theory of control systems has been applied. These include robotics, automatic control of large-scale power systems, numerical control of machine tools, autopilots for aircraft, prosthetic devices for handicapped persons, and the steering control of ocean liners. An important consequence of the development of control theory has been the increased use of automation in the industry with a view to increasing productivity. The concept of modeling developed by control engineers has been applied to many diverse areas, including biomedical systems, socioeconomical systems, and ecological systems. With the rapid advances in the area of microelectronics and the exciting possibility of using inexpensive computers as parts of control systems, it can truly be said that the applications of control theory are limited only by human imagination.

2

Mathematical Models of Physical Systems

2.1 INTRODUCTION

A crucial problem in engineering design and analysis is the determination of a mathematical model of a given physical system. This model must relate in a quantitative manner the various variables in the system. A model may be defined as "a representation of the essential aspects of a system which presents knowledge of that system in a usable manner." To be useful, a model must not be so complicated that it cannot be understood and thereby be unsuitable for analysis; at the same time it must not be oversimplified and trivial to the extent that predictions of the behavior of the system based on this model are grossly inadequate.

The systems that we shall be concerned with are dynamic in nature, and their behavior will be described in the form of differential equations. Although these will normally be nonlinear, it is customary to linearize them about an operating point to obtain linear differential equations. This is done in order that the analysis can be carried out conveniently. It should, however, be borne