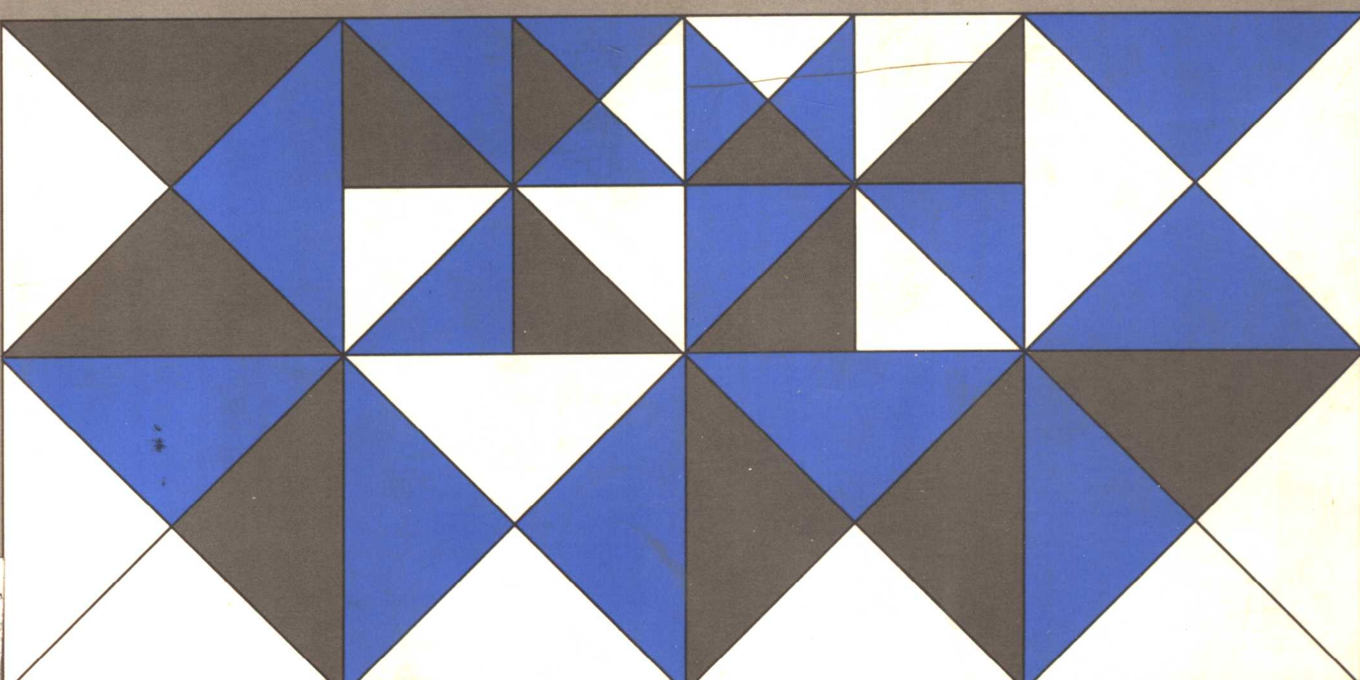
The companion activity book for *A Guide to Teaching Mathematics in the Primary Grades*

Elementary Mathematics Activities

A Teacher's Guidebook



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A Teacher's Guidebook

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Allyn and Bacon
Boston • ~~London~~ • Sydney • Toronto

A Division of Simon & Schuster, Inc.
160 Gould Street
Needham Heights, Massachusetts 02194

ISBN 0-205-11831-3

Printed in the United States of America

10 9 8 7 6 5 4 3 2 1 95 94 93 92 91 90 89

Acknowledgements

This book is based, in part, on research supported by Public Health Service Grant HD16757 from the National Institute of Child Health and Human Development (National Institutes of Health) and work supported by National Science Foundation Grant MDR-8470191.

We are deeply grateful to Karen Andrews and Gerry Swift who carefully read drafts of the book. Thanks are due to Sharon C. Baroody for her help. We are also indebted to Donna Auble, Lana Bates, June Chambliss, Selena Douglass, Teri Frerichs, and Teresa Jarvis whose word processing work made this book possible.

Preface

This manual supplements the 1989 Allyn and Bacon text, A Guide to Teaching Mathematics in the Primary Grades, in three ways. (1) It describes additional activities for some of the skills and concepts outlined in the text. (2) It provides photocopy masters for many of the exercises and game materials described in the text. (3) It helps the classroom teacher sequence the games, activities, and exercises described in the text and this manual. Chapter 1 discusses the issue of an instructional sequence. Chapter 2 provides some practical tips for preparing materials and designing instruction. Chapters 3 to 12 focus on specific areas of instruction. Each begins with a suggested instructional sequence, followed by descriptions of instructional ideas and materials for implementing many of the teaching suggestions.

Because no single source can provide an ideal and complete program, primary educators need to create their own program by drawing on the best ideas from a variety of sources. The activities described in the text and this manual are samples of what can be done. They provide a basis for getting started on a particular topic. The material described in the text and this manual provides a core primary curriculum--the basis for designing a complete program. Teachers may find that they may need to adapt some suggestions to fit better their situation or students. Undoubtedly many will find ways to improve upon suggestions or find new uses for others. Indeed, teachers are encouraged to construct their own activities, games, and exercises in order to provide more variety or practice or to more closely match instruction to their students' interests and needs. Moreover, teachers will want to draw upon other rich sources of instructional ideas such as the Arithmetic Teacher and the yearbooks published by the National Council of Teachers of Mathematics. These two sources are particularly valuable for ideas regarding the teaching of measurement and algebra, and the use of computers--topics not discussed by the text or this manual.

Problem solving is required by a number of the activities described in the text and this manual. Many of the word problems included herein use a nonstandard format that requires thoughtful analysis. For example, some problems require more than one step (e.g., a child may have to add two numbers in order to determine the difference between the sum and a third number). Some problems include extraneous information that students should disregard, and some do not have sufficient information to answer the questions posed. Other nonstandard problems involve missing-term problems. Note that for problems involving single-digit numbers, number names such as two are used instead of the numeral. This requires children to read the problem carefully, rather than simply retrieving the numerals and, for example, adding them. For weak readers, the problems can be read aloud. To design supplemental story problems, see the guidelines outlined in the first three references listed below. The last three references are particularly rich sources of problem-solving activities.

Baroody, A. J. (1987). Children's mathematical thinking: A developmental framework for preschool, primary, and special education teachers (Chapter 14). New York: Teachers College Press.

Burns, M. (1984). The math solution: Teaching for mastery through problem solving. Sausalito, CA: Burns Education Associates.

Krulik, S., & Reys, R. E. (Eds.). (1980). Problem solving in school mathematics, 1980 Yearbook (see Chapters 9 and 10, in particular). Reston, VA: National Council of Teachers of Mathematics.

Krulik, S., & Rudnick, J. A. (1988). Problem solving: A handbook for teachers (second edition). Boston: Allyn and Bacon.

In brief, the important work of making mathematics instruction meaningful, interesting, and thought-provoking can be an ongoing challenge. It is our hope this volume contributes to your excitement about teaching mathematics and search for improving it.

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Margaret Hank

Table of Contents

Chapter 1: INTRODUCTION

This manual outlines a theory- and research-based core curriculum for teaching mathematics to children in preschool to third grade. It organizes the materials described in A Guide to Teaching Mathematics in the Primary Grades and this manual into modules. Each module is numbered and focuses on a particular cognitive objective. The sample entry below is for seventeenth module listed in Chapter 8. The title or name of the module follows the module number. The next line indicates the aim of the module. Some modules, such as the sample, have notes for comments or suggestions. The last part of the entry indicates relevant activities, games, or exercises (in bold print). The information in the parenthesis indicates where this material is described in the text (T) or in the manual (M).

8.17 Application of Commutativity to Symbolic Addition.
To relate informal knowledge that addend order does not affect a sum to written addition, and to use additive commutativity as a shortcut.
Note that this module should be introduced after Module 9.4 and before Module 9.6.
Math Detective (T: 26, 34 [Example 2-10]; M: 174) and/or **Album of Addition Combination Families** (T: 255-256 [Example 9-4]; M: 223).

Table 1-1 outlines a suggested scope and sequence of activities, games, and exercises. For example, the series of modules 3.1 to 3.2 is entered in the table and should be introduced before the series 4.1 to 4.4 or 5.5 to 5.6. Skills or concepts that can be taught simultaneously are listed together. This applies to competencies from different chapters (e.g., Modules 3.1 to 3.2 and 5.1 to 5.4) as well as those from the same chapter (e.g., the series of modules 4.10 to 4.15).

The recommended order or pacing of modules should not be viewed as immutable. Indeed, teachers are encouraged to work out a sequence that fits their goals, situation, and students. The suggested order takes small steps appropriate for children with learning problems. Thus, classroom teachers may find that some modules can be combined or skipped without loss of understanding. The grade levels noted provide a general indication as to when material might be introduced. For children with learning problems, the indicated sequence may be too fast; for above average students it is probably too slow.

In general, work on a module should continue until a child has mastered the skill or concept. Most of the games and activities can be used a number of times without loss of interest. Many modules consist of more than one game, activity and/or exercise, and a teacher can choose among these as the need and interest of students dictate. Moreover, teachers are encouraged to supplement the sample lesson(s) of a module with games or activities from other sources.

Table 1-1
Scope-and-Sequence Chart
of the Modules Described in This Manual

Level	Module(s)	
PK	3.1 to 3.2 5.1 to 5.4	(count by ones 1 to 10); (perception of "same" & "more")
	4.1 to 4.4 5.5 to 5.6	(enumeration of sets 1 to 5); (perception of fine differences 1 to 4)
	3.3 to 3.4	(count by ones 1 to 10, continued)
	4.5 to 4.6	(counting 1 to 10 and enumeration of sets 1 to 5, continued; cardinality rule)
	4.7	(identity-conservation principle)
	4.8	(recognition of sets 1 to 3)
	4.9	(order-irrelevance principle)
	3.5 to 3.9	(number after 1 to 9)
	5.7	(same number 1 to 5)
	4.10 to 4.15 5.8	(finger patterns; production of sets 1 to 6); (gross comparisons 1 to 10)
	5.9 to 5.10	(fine comparisons 1 to 5)
	5.11 to 5.14	(practice to foster <u>mental</u> comparison skills)
	3.10 to 3.12	(count by ones 11 to 19)
	[7.1 to 7.5	(recognition of 1-digit numerals)]*
K	4.16 to 4.18	(enumeration of sets 6 to 10)
	4.19 5.15 to 5.17 7.1 to 7.5	(recognition of sets 4 to 6); (same number 6 to 10); (recognition of 1-digit numerals)
	4.20 to 4.22 7.6 to 7.7	(enumeration of sets 6 to 10, continued; production of sets 6 to 10); (reading 1-digit numerals)
	5.18 to 5.22	(fine comparisons 6 to 10)

*Optional at the prekindergarten level. Depending upon need, these modules can be used in kindergarten.

	5.23 to 5.28	(gross comparisons of less)
	3.13 to 3.15	(number before 1-10)
	3.16 to 3.20 4.23	(count by ones 20 to 29; number after 10 to 28); (enumeration and production of sets 11 to 20)
	6.1 to 6.4	(count all: sets 1 to 5)**
	6.5 to 6.6	(mentally add one more)**
	6.7 to 6.10	(take away with objects)**
	6.11 to 6.12	(mentally take away one)**
	7.8 to 7.9 8.1 to 8.2	(copying one-digit numerals); (cardinal value)
	7.10 to 7.11 8.3	(writing one-digit numerals); (cardinal value, continued)
	3.21 to 3.23	(count backward from 10)
1	4.24	(discussion of math terms)
	4.25 to 4.26	(finger patterns 6 to 10)
	5.29	(matching and number conservation)
	8.4 to 8.5	(representation of equivalence and inequivalence relationships)
	8.6 to 8.8	(representation of magnitude relationships)
	8.9 to 8.10	(representation of ordinal relationships)
	6.13	(count all: sets more than 5)
	8.11 to 8.12	(informal same-sum-as and commutativity of addition)
	8.13 to 8.15	(linking informal and symbolic addition)
	8.16 to 8.17 9.1 to 9.6	(symbolic addition; count from one to add 2 to 5; application of commutativity); ($\underline{n}$ + 1, 1 + $\underline{n}$, $\underline{n}$ + 0, and 0 + $\underline{n}$ addition facts)
	6.14 to 6.15 8.18 to 8.19	(count on 2 to 5 from larger addend); (union of sets and part-part-whole)

**In many programs, these basic arithmetic skills are introduced in first grade, not kindergarten. If introduced in kindergarten, they probably should be reviewed in first grade.

	6.16	(take away: 2 to 5)
	8.20 to 8.22	(symbolic subtraction);
	9.7 to 9.9	(n - 1, n - 0, and n - n subtraction facts)
	8.23 to 8.24	(addition-subtraction inverse principle);
	9.10 to 9.12	(n + 2, 2 + n, and small n + n addition facts)
	9.13 to 9.15	(n - 2 subtraction facts)
	8.25 to 8.26	(associativity of addition)
	3.24 to 3.27	(count by tens to 100; decade after 10 to 90)
	3.28 to 3.29	(number before 11 to 29)
	3.30 to 3.34	(count by ones 30 to 100; number after 29 to 99)
	5.30 to 5.33	(gross comparisons 11 to 100)
	5.34	(fine comparisons of less 1 to 10)
	3.35 to 3.37	(count by five to 100 and by two to 20)
	5.35	(fine comparisons 11 to 100)
	5.36	(fine comparisons of less 11 to 100)
	6.17	(mentally add on 6 to 9 more);
	10.1 to 10.8	(reading and writing teen terms; place recognition: ones and tens, ones and tens notation; base 10 equivalents: ones for 10);
	11.1 to 11.3	(10 + n and n + 10 facts)
	10.9 to 10.16	(reading and writing 2-digit terms; ones, tens, and hundreds notation; base 10 equivalents: tens for 100);
	12.1 to 12.5	(concrete and semiconcrete 2-digit addition and subtraction);
	11.4 to 11.5	(decade + n and teen - 10 facts)
	10.17 to 10.18	(smallest/largest 1- and 2-digit term; concrete model for combining two 2-digit numbers);
	11.6 to 11.8	(decade + 10 and decade - 10)
	11.9 to 11.10	(front-end estimates with 2-digit terms)
2	3.38 to 3.39	(count odd numbers 1 to 19)
	3.40 to 3.42	(count backwards from 20)
	3.43 to 3.46	(count by tens 100 to 200; decade after 100 to 190)

3.47 to 3.51	(number before 30 to 100; decade before 20 to 100)
3.52 to 3.55	(count by ones 101 to 200; number after 100 to 199)
6.18	(take away: 6 to 9)
6.19	(take away: teens)
8.27 to 8.29	(missing part; representation of addition-subtraction inverse principle)
8.30 to 8.31	(representation of missing addend)
8.32 to 8.38	(additive-subtraction and difference concepts; representation of other names for a number and commutativity of addition)
6.20 to 6.23 8.39 to 8.40	(adding 2 to 5 like sets); (representation of associativity of addition)
9.16 to 9.18	(small miscellaneous addition facts)
9.19 to 9.20	(small $\underline{m} - \underline{n} = \underline{n}$; difference of one subtraction facts)
9.21 to 9.24	(large $\underline{n} + \underline{n}$ addition facts; equals 10 addition facts)
9.25	(small miscellaneous subtraction facts)
9.26 to 9.28	($\underline{n} + 8$, $8 + \underline{n}$, $\underline{n} + 9$, and $9 + \underline{n}$ addition facts)
9.29 to 9.30	(large $\underline{m} - \underline{n} = \underline{n}$ subtraction facts)
10.19 to 10.24 12.6 to 12.10	(reading and writing 3-digit terms; place recognition: thousands and hundreds notation; base 10 equivalents: hundreds for 1000); (concrete and semiconcrete 3-digit addition and subtraction)
11.11 to 11.12	(decade + decade; decade - decade)
11.13 to 11.17	(2-digit $\underline{n}$ + 10; 2-digit $\underline{n}$ - 10)
11.18	(2-digit $\underline{n}$ + decade; 2-digit $\underline{n}$ - decade)
11.19 to 11.21	(parallel addition and subtraction facts)
11.22 to 11.23	(100 + $\underline{n}$ and $\underline{n}$ + 100)
11.24 to 11.25	(hundreds + 100; hundreds + hundreds)
11.26	(front-end estimates with 3-digit terms)

	12.11 to 12.14	(2- and 3-digit addition with and without renaming)
	12.15 to 12.18	(2- and 3-digit subtraction with and without renaming)
3	9.30 to 9.33	(larger miscellaneous addition facts)
	9.34 to 9.36	(10 - _n_ subtraction facts)
	9.38 to 9.39	(teen - 8, teen - 9 and large miscellaneous subtraction facts)
	10.25	(smallest/largest 3-digit term)
	10.26	(flexible enumeration ones and tens)
	10.27 to 10.29	(reading and writing 4-digit numerals); (smallest/largest 4-digit terms)
	10.30	(flexible enumeration: ones, tens, and hundreds)
	10.31	(notation with renaming)
	11.27 to 11.28	(3-digit _n_ + 10)
	11.29 to 11.31	(3-digit _n_ + decade; practice with 3-digit mental arithmetic)
	11.32 to 11.33	(rounding with 2- and 3-digits)
	8.41 to 8.44	(representation of missing augend)
	6.24 to 6.27	(adding more than 5 like sets)
	8.45 to 8.51	(symbolic repeated addition and multiplication)
	8.52	(commutativity of multiplication)
	9.40	(_n_ x 1, 1 x _n_, _n_ x 0, and 0 x _n_ times fact)
	9.41	(_n_ x 2 and 2 x _n_ times fact)
	9.42 to 9.43	(_n_ x 5 and 5 x _n_ times fact)

Most of the materials for the games and activities listed in the text and this manual can be easily made with inexpensive supplies. The number sticks can be constructed from wood with some effort. An easier alternative is to make them from interlocking blocks and label them with a permanent marker. For example, a two-stick can be made by putting or even gluing two red interlocking blocks together and labelling the second with the numeral 2; a three-stick can be constructed from three orange blocks, and so forth.

After photocopying the cards, game pieces, and gameboards for a game, the use of clear contact on both sides will greatly extend the life of these materials. To increase sturdiness, paste game boards to oaktag or cardboard and then cover the boards with clear contact. Game cards can be photocopied onto mailing labels, the mailing labels can be peeled off and placed on 3-inch x 5-inch (3 x 5) cards, and the cards can then be covered with clear contact. To improve the aesthetics of the materials, a teacher may wish to enhance some of the photocopied material by adding color, pasting on stickers, and so forth. For example, the cards for Module 4.5 (the Hidden Stars Game: 1 to 5) can be improved by using large stick-on stars.

Instruction can be usefully organized in a variety of ways. A "centers approach" is especially useful for providing small-group instruction (e.g., when a teacher needs to work with children of different ability levels). While a teacher is working with one group (e.g., introducing a new concept with manipulatives), the other groups are occupied at centers. One center might be a math game; a second, a math-worksheet area; a third, a computer station; a fourth, a problem-solving activity, and so forth. At prescribed times, a new group would join the teacher and the remaining groups would rotate to a new center. Depending on the amount of time needed for each group, a teacher might see all or just some of the groups in one day.

A centers approach has a number of advantages. One is that the teacher has the opportunity to work with small groups of children and observe the progress of each pupil. A second is that it provides a way of delivering individualized instruction and practice. A center could have separate instructions and "jobs" for each group. For example, the math-worksheet station could have a separate folder for each group. The centers and group-specific assignments with centers can be color-, picture-, or symbol-coded to help children find the right place or work with a minimum of supervision.

A centers approach does require considerable planning and set-up time. The instructions for an unsupervised center have to be clear and easily understood. For complicated instructions or with weak readers, use a tape-recorded message and headphones. A teacher needs to plan each center carefully so that all the groups will finish at the same time. This can be facilitated by using a time limit rather than a point limit for math games. At first, it may take some experimenting and juggling to get a centers approach to work efficiently.

The use of centers or any child-centered approach is greatly facilitated if a teacher has competent help. Indeed, efforts to individualize instruction are not practical for most without assistance. Help can come in the form of a teachers aide, a student teacher, a parent volunteer, an older or more advanced pupil and so forth. (Using pupils to help with instruction can greatly benefit the student tutors. They learn public speaking, teaching, and child-management skills, not to mention the boost to self-esteem that comes from useful service.) Helpers, of course, must be trained in order to provide competent assistance. For example, a helper can be taught the rules of a math game, what kinds of problems to anticipate, and how to help children overcome difficulties. Though it requires time and effort, the reward is a more effective and interesting program.

Many children find competition exciting and will find math games very interesting. Some children do not care for competition and may find the math games uninteresting or even threatening. In such cases, encourage a child to compete against his or her own record, de-emphasize the winning-and-losing aspect of the game, or simply use the game as a learning activity (e.g., do not bother to keep score). To help such a child deal with the issues of competition, a teacher may introduce competitive games gradually.

The use of prizes is optional. Prizes should not be used all the time or be the prime reason for a child's involvement. Indeed, for many children, the enjoyment of playing and learning will often be sufficient. Occasional and careful use of prizes may increase the excitement of participation for many children.

It is important to develop constructive beliefs. By teaching mathematics in an integrated and meaningful context, children may see it as something useful--as something that is supposed to make sense. By discussing mathematics, children may come to believe that mathematics involves asking questions, analyzing information--in short, thinking. Experiential lessons may teach children that they can have command over and fun with mathematics. Teachers should also <u>explicitly point out</u> constructive beliefs about mathematical learning and thinking such as those listed below:

(a) Asking questions is a sign of intelligence, not stupidity.

(b) Mistakes can provide valuable information and are a natural part of learning and solving problems. An intelligent person is someone who learns from his or her mistakes, not someone who does not make mistakes in the first place.

(c) Quick, correct answers are not as important as understanding why the answer is correct.

(d) Counting or using fingers is a sensible way of figuring out arithmetic questions.

Chapter 3: ORAL COUNTING*

3.1 Listening to Stories: Oral Counting to 10.
To hear the number sequence in an interesting context.
Counting Stories: 1 to 10 (T: 53-55 [Example 3-1]; M: 17-18).

3.2 Number Patterns: Oral Counting 1 to 10.
To learn the count sequence to 10 by using an error-detection activity: discussing errors and the correct order of terms.
Good-or-Bad Counter and/or **Absent-Minded Counter** (T: 55-56 [Example 3-2]; M: 19).

3.3 Oral Counting: 1 to 10.
To refine and practice the count-by-ones sequence to 10 with the aid of feedback.
Snake Game (T: 56 [Example 3-3]), **Taller Tower Game** (T: 56-57 [Example 3-4]), and **Tape Recording: 1 to 10** (T: 57 [Example 3-5]).

3.4 Oral-Counting Practice: 1 to 10 With Objects.
To refine and practice the count-by-ones sequence to 10 in an interesting and meaningful object-counting context.
Note that these games do not require a child to enumerate sets but simply to count orally as an adult points to or puts out the objects. Developmentally more advanced children can be encouraged to enumerate the sets themselves.
Spinner Game (T: 58 [Example 3-6]), **Star Race** (T: 58-59 [Example 3-7]), and/or **Train Game** (T: 63-64 [Example 3-12]).

3.5 Semiabstract Number-After 1 to 4.**
To learn number after up to 4 with the aid of a number list and a "running start."
Note that this version of the game uses a simplified domino: one with a single set of dots or numerals.
Basic Number-After Dominoes: 1 to 4 (T: 69-71 [Example 3-18]; M: 25).

*Though written exercises, In-Out Machine lessons encourage the discovery of patterns that can be useful in expanding children's oral counting skills. See pages 26-34 [Example 2-9] of the text for a general description of their use. Module entries list the manual page numbers of exercises that highlight specific patterns. Note that In-Out Machine exercises can be put on the board or an overhead and done verbally with a class. They can also be acted out. For example, a child can sit inside a box and serve as the machine.

**For children who cannot read numerals but who have great difficulty enumerating sets in this range, use numeral dominoes and read the numbers to the children. (Note that children functioning at this level may not be ready for number-after training.) For children who cannot read numerals but have at least some skill at enumerating sets required, use dot dominoes. For children who can read numerals (and who do not need enumeration practice), use numeral "dominoes."

3.6 Oral-Counting Practice: Number After 1 to 4.**
To practice number after up to 4 in random order.
Note that this version of the game uses a domino with two sets of dots or numerals.
Advanced Number-After Dominoes: 1 to 4 (T: 69-71 [Example 3-18]; M: 25), Rings for a King or Queen: 1 to 4 (T: 71-72 [Example 3-19]; M: 25), Number-After Race: 1 to 4 (T: 69, 72 [Example 3-20]; M: 25-27), and/or Number-After Quiz Game: 1 to 4 (T: 69, 72-73 [Example 3-21]; M: 25).

3.7 Oral-Counting Practice: Semiabstract Number-After 1 to 9.**
To practice number after up to 9 with the aid of a number list and a "running start."
Note that this version of the game uses a simplified domino: one with a single set of dots or numerals.
Basic Number-After Dominoes: 1 to 9 (T: 69-71 [Example 3-18]; M: 25).

3.8 Oral-Counting Practice: Number-After 1 to 9.**
To practice number after up to 9 in random order.
Note that this version of the game uses a domino with two sets of dots or numerals.
Advanced Number-After Dominoes: 1 to 9 (T: 69-71 [Example 3-18]; M: 25), Rings for a King or Queen: 1 to 9 (T: 71-72 [Example 3-19]; M: 25), Number-After Race: 1 to 9 (T: 69, 72 [Example 3-20]; M: 25-27), and/or Number-After Quiz Game: 1 to 9 (T: 69, 72-73 [Example 3-21]; M: 25).

3.9 Oral-Counting Practice: Symbolic Number-After 1 to 9.
To practice number after 1 to 9 in random order using written symbols.
Note that this exercise should be used as independent seatwork only with children who can read and write numerals to 10. For most preschoolers (or many kindergarteners), the activity should be done with the assistance of a teacher. The teacher can read a number to the child or group, solicit an answer, verbally note the correct response, and record the answer.
In-Out Machine (M: 28).

3.10 Number Patterns: Oral Counting 1 to 19.
To discover and discuss the rules underlying the count sequence to 19 by using error-detection activities.
Good-or-Bad Counter and/or Absent-Minded Counter (T: 55-56 [Example 3-2], 57; M: 19).

**For children who cannot read numerals but who have great difficulty enumerating sets in this range, use numeral dominoes and read the numbers to the children. (Note that children functioning at this level may not be ready for number-after training.) For children who cannot read numerals but have at least some skill at enumerating sets required, use dot dominoes. For children who can read numerals (and who do not need enumeration practice), use numeral "dominoes."