Graph Theory Application to the

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Application to the Calculation of Electrical Networks

ISTVÁN VÁGÓ

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PREFACE

A means of describing the connections of electrical networks is provided by graph theory. Its application yields a method for solving network analysis problems, by means of a systematic derivation of an appropriate number of linearly independent equations. Digital computers are readily utilized for writing the necessary relationships and solving them. It is for this reason that the application of graph theory to the calculation of electrical networks has gained widespread use in recent decades.

After the earliest work on graph theory (that of L. EULER published in 1736 [17]) it was G. KIRCHHOFF, as far as we know, who was the first to deal with this subject in his treatise issued in 1847 [30], examining primarily the laws of electrical networks. The first comprehensive book to discuss graph theory was that by D. KÖNIG [31] and it was published in 1936. A detailed study of the application of graph theory to electrical networks is presented in the book of S. SESHU and M. B. REED [42], published in 1961.

Lectures on electrical networks at the Faculty of Electrical Engineering of the Technical University of Budapest have included the applications of graph theory for more than a decade. On the basis of the experience gained in teaching this subject, a book in the Hungarian language was published by the Technical Publishing House, Budapest, in 1976. The present volume is a revised and enlarged, English language edition of this book.

I wish first of all to express my thanks to Academician Prof. OTTÓ P. GESZTI, who did pioneer work by introducing graph theory methods into education. The suggestions made by Prof. GESZTI during the writing of the first four chapters, by Prof. ANDRÁS AMBRÓZY on the completion of chapters 5 and 6, and by Assistant Professor MIKLÓS BOHUS during the writing of chapters 7 and 8 were invaluable to me. My discussions with Prof. GYÖRGY FODOR on the subject were extremely useful. In writing the theoretical parts and in the collection of examples I was helped by my immediate colleagues: assistants ISTVÁN BÁRDI and OSZKÁR BIRÓ and principal assistants EDIT HOLLÓS and IMRE SEBESTYÉN. I herewith express my thanks to all of them.

István Vágó

NOTATIONS

$\mathbf{X}$ column matrix
$\boldsymbol{X}$ matrix
$\mathbf{X}^+$ transpose of $\mathbf{X}$, row matrix
$\boldsymbol{X}^+$ transpose of $\boldsymbol{X}$
$\boldsymbol{X}^*$ conjugate of $\boldsymbol{X}$
$\dot{x}$ time derivative of x
$\langle x_1 x_2 \ldots x_n \rangle$ diagonal matrix, $x_1, x_2, \ldots, x_n$ being the elements on the main diagonal
$\boldsymbol{A}$ basis incidence matrix
$\mathbf{A}^+$ row matrix of vertex
$\boldsymbol{A}_t$ non-basis incidence matrix
$\boldsymbol{A}_0$ basis incidence matrix of undirected graph
$\mathbf{B}^+$ row matrix of loop
$\boldsymbol{B}_t$ non-basis loop matrix
$\boldsymbol{B}$ basis loop matrix
C capacitance
$\mathbf{C}^+$ row matrix of path
$\boldsymbol{F}$ a block in the normal form of the loop matrix
G conductance
$\boldsymbol{G}$ matrix of conductances
I current
$\mathbf{I}$ column matrix of currents
I_g source-current
J loop current
$\mathbf{J}$ column matrix of loop currents
L inductance
$\boldsymbol{L}$ incidence matrix of self-loops
$\boldsymbol{L}_i$ matrix Lagrange-polynomials
$\mathscr{L}$ symbol for Laplace transformation
$\mathscr{L}^{-1}$ symbol for inverse Laplace transformation
$\boldsymbol{P}$ matrix formed by the parameters p_i or $\boldsymbol{p}_i$ of a transmission line network
$\mathbf{Q}^+$ row matrix of cutset

$\boldsymbol{Q}_t$	non-basis cutset matrix
$\boldsymbol{Q}$	basis cutset matrix
R	resistance
$\mathbf{R}$	column matrix of excitation signals
$\boldsymbol{R}$	matrix of resistances
$\boldsymbol{R}$	matrix formed by the parameters r_i or $\boldsymbol{r}_i$ of a transmission line network
$\boldsymbol{S}$	reflection matrix
T	period
U	voltage
$\mathbf{U}$	column matrix of voltages
U_g	source-voltage
V	cutset-voltage
$\mathbf{V}_Q$	column matrix of cutset-voltages
$\mathbf{V}$	column matrix of internal signals
$\boldsymbol{W}_0$	transfer matrix
$\boldsymbol{W}_t$	vertex transfer matrix
Z	impedance
$\boldsymbol{Z}$	impedance matrix
$\boldsymbol{Z}_B$	loop-impedance matrix
$\mathscr{Z}$	symbol for z-transformation
$\mathscr{Z}^{-1}$	symbol for inverse z-transformation
Y	admittance
Y_0	characteristic admittance
$\mathbf{Y}$	column matrix of response signals
$\boldsymbol{Y}$	admittance matrix
$\boldsymbol{Y}_A$	vertex-admittance matrix
$\boldsymbol{Y}_Q$	cutset-admittance matrix
$\boldsymbol{Y}_0$	characteristic admittance matrix
b	number of edges
c	number of components
d_{ik}	inverse hybrid parameter
$f^*(t)$	sampling of $f(t)$
h_{ik}	hybrid parameter
i	current
$\mathbf{i}$	column matrix of currents
i_g, i_0	source-current
m	number of chords
n	number of vertices, nullity
$p_i, \boldsymbol{p}_i$	an admittance parameter of the i-th transmission line section
r	rank
$r(t)$	time function of excitation signal
$r_i, \boldsymbol{r}_i$	an admittance parameter of the i-th transmission line section
s	variable of Laplace transform
t	time

u	voltage
$\mathbf{u}$	column matrix of voltages
u_g, u_0	source-voltage
$v(t)$	time function of internal signal
$y(t)$	time function of response signal
$\delta(t)$	Dirac delta
γ	propagation coefficient
$\boldsymbol{\Gamma}$	matrix of propagation coefficients
Φ	node-potential
$\mathbf{\Phi}$	column matrix of node-potentials
ω	angular frequency
$1(t)$	unit step function
$\boldsymbol{I}$	unit matrix
$\boldsymbol{0}$	zero matrix

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CHAPTER 1

BASIC CONCEPTS OF GRAPH THEORY

Graphs and subgraphs

The currents and voltages in electrical networks and the relationships between them depend upon the characteristics of the network elements constituting the network and on the way these elements are interconnected. The relation between the current and voltage of a two-terminal element as well as between the currents and voltages of *multi-terminal* elements is expressed with the aid of the element characteristics. Such characteristics are the resistance R, inductance L, capacitance C, source-voltage U_g, source-current I_g of linear *two-terminal* elements, the impedance and admittance parameters of linear two-ports, the characteristics of nonlinear elements, etc. The network elements form one or more branches of the network, which connect at nodes. The *graph* of the network represents the manner in which the branches and nodes are interconnected in the network independently of the network elements forming the branches. Thus, graph theory is applicable to network analysis [26, 28, 33, 36, 42, 44].

Instead of the rigorous mathematical definition of graphs the following illustrative description suffices for our discussion [9, 27].

Graphs are formed from two types of element: *edges* and *vertices*. The terms *branch* instead of *edge* and *node* instead of *vertex* are often used, particularly in an engineering, as opposed to mathematical context. The graph is a union of sets of edges and vertices with two vertices associated with each edge. One vertex may be associated with several edges. The graph can be illustrated as follows. An edge is indicated by a line or curve, and a vertex by a small circle. Each edge has two distinct endpoints, called vertices, which belong to the edge. Any two edges in the graph may only have vertices as common points. Graphs consisting of a finite number of edges and vertices are called *finite graphs*. For the solution of electrical network problems we shall only consider the use of finite graphs.

Let us examine networks made up of coupled and non-coupled two-terminal elements only. A graph of the network is drawn by representing the two-terminal elements of the network by lines, with the intersections of the lines corresponding to the connection points between the elements, these lines having no other common points. In the case of a planar or two-dimensional representation of the graph, any other intersections of the lines are not considered common points of the edges (as is customary for connection diagrams of networks). The edges of the graph

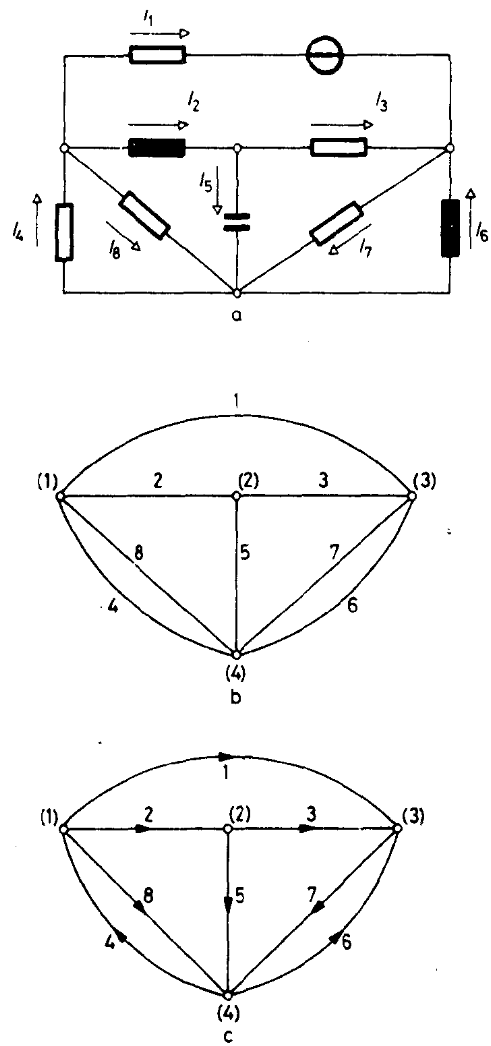

Fig. 1.1

correspond to the branches of the network, and the vertices of the graph to the joining points of edges, i.e. to the nodes. A graph of the network shown in Fig. 1.1, a is drawn in Fig. 1.1, b. The edges and vertices of the graph are numbered, using parentheses for the vertex numbers.

A direction may be assigned to the edges of the graph (Fig. 1.1, c). The direction of edge *k* between vertices (*i*) and (*j*) may point from (*i*) to (*j*) or from (*j*) to (*i*). If every edge of the graph is assigned a direction*, a *directed graph* is obtained. As is well

* The terms *orientation* and *oriented graph* are also commonly used.