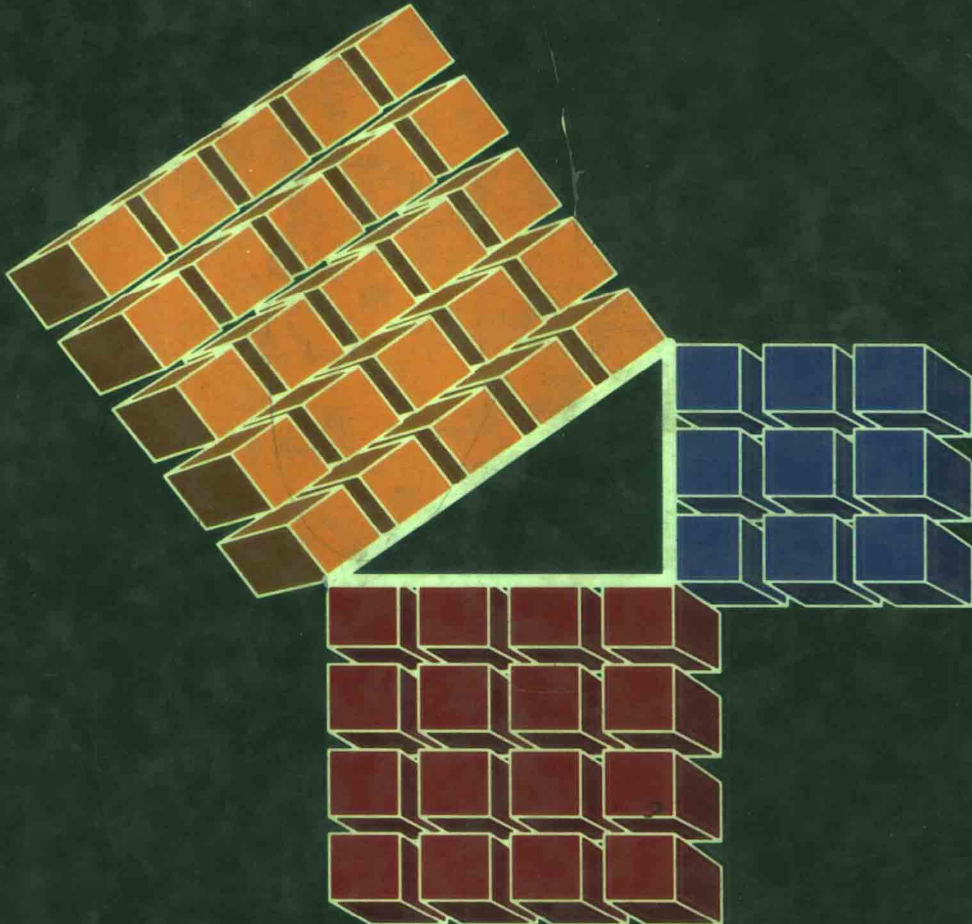

Second Edition

# PRECALCULUS MATHEMATICS

## A Problem–Solving Approach

WALTER FLEMING • DALE VARBERG

SECOND EDITION

# Precalculus Mathematics

## A PROBLEM-SOLVING APPROACH

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# Preface

As its title indicates, we designed this book to prepare students for calculus. After a chapter on basic algebra (which many instructors will either omit or review quickly), we plunge into what must be the heart of any precalculus course—the study of functions and their graphs. We look in turn at linear functions, quadratic functions, polynomial functions, rational functions, exponential functions, logarithmic functions, and trigonometric functions with occasional asides to consider more special functions such as the absolute value function and the greatest integer function. Our concern is to develop in students both a geometric feeling for these functions and a solid understanding of their properties.

With our eye on the calculus course that students will take later, we attend to such topics as difference quotients (pages 30 and 110), composition and decomposition of functions (page 107), inverse functions (page 113), the natural logarithm function (page 154), and partial fractions (page 341). Because complex numbers play almost no role in a first calculus course, some instructors will prefer to omit discussion of these numbers in a precalculus course. We make this possible by postponing treatment of the complex numbers until Chapter 7 and by labeling thereafter all problems that use these numbers with the symbol [i].

We have made two significant content changes for this edition. First, we have added a chapter titled "Systems of Equations and Inequalities." This chapter includes an introduction to matrices and determinants. Second, the last chapter, retitled "Analytic Geometry," has been rewritten and enlarged. It features a complete treatment of the conic sections (now defined in the traditional way) and considerable material on polar coordinates and parametric equations.

Our organization and format is similar to that in our other books: *Algebra and Trigonometry* (third edition), *College Algebra* (third edition), and *Plane Trigonometry* (second edition). Here are some of the special features.

## LIVELY OPENING DISPLAYS

Each section begins with a challenging problem, a historical anecdote, a famous quotation, or an appropriate cartoon. These displays are designed to spark the readers' curiosity, to draw them into the section.

## INFORMAL WRITING STYLE

We avoid the ponderous theorem-proof style found in many mathematics books. We are more interested in explaining than in proving and we shun unneeded technical jargon like the plague. Yet we are careful to state definitions and theorems correctly and we do proceed in a reasoned logical manner. Our sections are broken into smaller subsections with descriptive titles but we do not chop the text into pieces with the constant use of headings like theorem, proof, definition, example, remark, and so on. We want students to read our book, so our goal was to make it readable.

## EXAMPLES

We believe that students learn mathematics by studying examples and doing problems. Our book is full of examples. First there are plenty of examples within the textual discussion itself though they are not explicitly labeled as such. Then in the problem sets we usually offer several more examples, always accompanied by a group of related problems. We think that placing examples and problems together is good pedagogy. In any case, *the examples in the problem sets are an integral part of the text and should be studied carefully.*

## PROBLEM-SOLVING EMPHASIS

We think the activity that most characterizes mathematics is problem solving. Problem solving is more than mere answer finding; it is much more than simply substituting in a formula or following a recipe. As we use the term, it involves formulating a problem clearly, organizing the given information (perhaps in a diagram), collecting the tools that will be needed, using one's wits to develop a strategy, following a reasoned process to a solution, and writing the results in a clear organized manner. It is the activity that George Polya (1887–1985) wrote about so wisely in his many books and from whom we offer this quotation.

> "Solving a problem is similar to building a house. We must collect the right material, but collecting the material is not enough; a heap of stones is not yet a house. To construct the house or the solution, we must put together the parts and organize them into a purposeful whole." From *Mathematical Discovery* (vol. 1, p. 66).

If emphasis on problem solving characterized the first edition of this book, this emphasis is even more evident in the present edition. Every section of the book ends with an extensive problem set, and each of these is in two parts. First there is a set of basic problems, problems intended to develop the skills and reinforce the main ideas of the section. Following the basic problems, there is a set of miscellaneous problems. *These have been completely reworked for this edition.* Our aim for the miscellaneous problems is to give the student an exciting and challenging tour through the applications of the ideas of the section. We begin with a few easy review type problems, move in a carefully graded manner to harder and more interesting application, and conclude with a teaser problem.

## THE TEASER PROBLEMS

In our combined seventy years of teaching, we have collected a large number of intriguing problems. Many of them are our own creation; some are part of current mathematical lore; others come from mathematical history. As a special attraction for this edition, we have inserted one of these problems at the end of each section. These teasers may appear difficult at first glance but in most cases become surprisingly easy when looked at the right way. In each case, the teaser relates to the ideas of the section in which it appears. As a group, the teasers form a collection of problems that we think would please even George Polya.

How should the teasers be used in class? We suggest that teachers might select some of their favorite teasers and offer a prize to the student offering the best set of solutions at the end of the term. Or teachers might use these problems as the basis for a weekly problem-solving session (perhaps as an addition to the regular class sessions). Or a teacher might select from them a problem of the week, offering a small prize for the best solution. Or they can simply be treated as extra stimulation for the very best students.

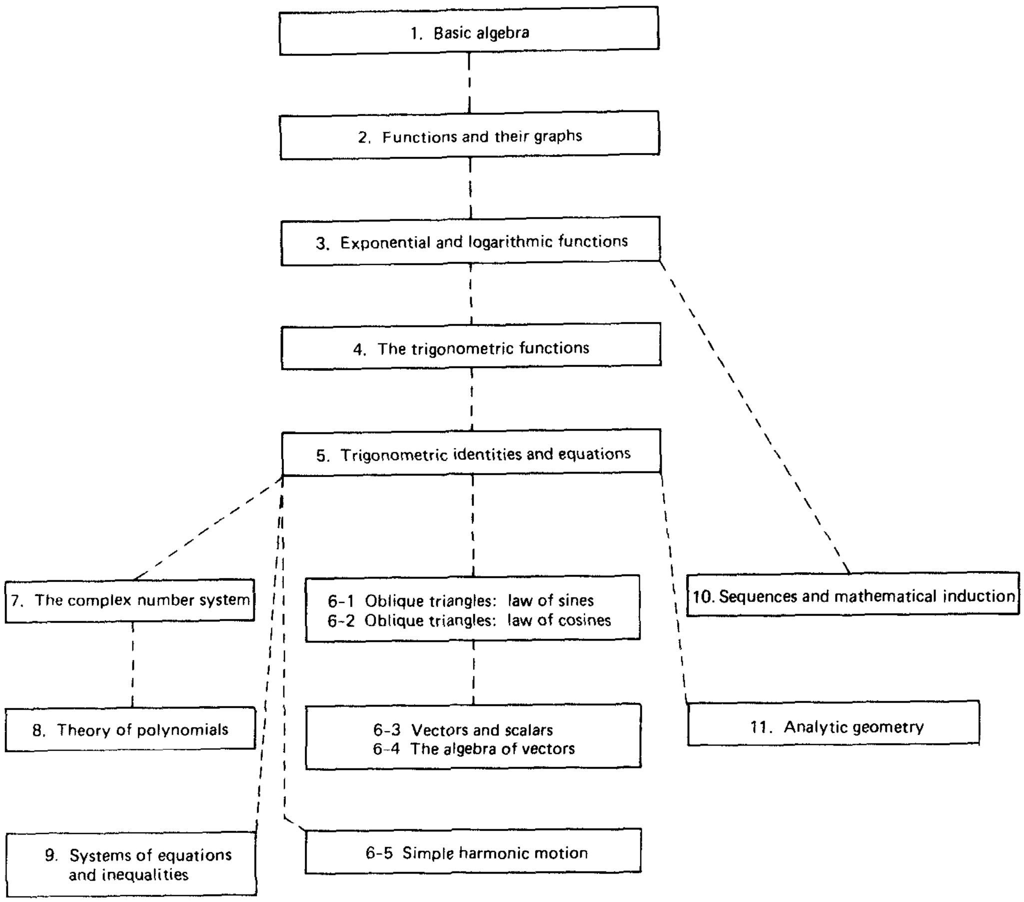
## CALCULATORS

Calculators are by now standard equipment for most college students. They greatly aid in the solution of problems that involve heavy calculations. In trigonometry, use of these devices allows us to largely dispense with the traditional emphasis on use of tables. However, we warn students that calculators can never substitute for clear thinking, and we hope our problem sets will make this abundantly clear. Problems whose solution is substantially eased by use of a calculator are labeled with the symbol Ⓒ.

## FLEXIBILITY

This book can be used in either a one or a two term course. To aid in the designing of a syllabus, we include the following dependence chart.

DEPENDENCE CHART




## SUPPLEMENTARY MATERIALS

An extensive variety of instructional aids is available from Prentice Hall.

***Instructor's Manual*** The instructor's manual was prepared by the authors of the textbook. It contains the following items.

(a) Answers to all the even-numbered problems (answers to the odd problems appear at the end of the textbook).
(b) Complete solutions to the last four problems in each problem set. This includes the teaser problem.
(c) Six versions of a chapter test for each chapter together with an answer key for these tests.
(d) A test bank of more than 1300 problems with answers designed to aid an instructor in constructing examinations.
(e) A set of transparencies that illustrate key ideas.

***Prentice Hall Test Generator*** The test bank of more than 1300 problems is available on floppy disk for the IBM PC. This allows the instructor to generate examinations by choosing individual problems, editing them, and if desired by creating completely new problems.

***Videotapes*** Approximately five hours of videotaped lectures covering selected topics in college algebra are available with a qualified adoption. Contact your local Prentice Hall representative for details.

***Student Solutions Manual*** This manual has worked-out solutions to every third problem (not including teaser problems).

***Function Plotter Software*** A one-variable function plotter for the IBM PC is available with a qualified adoption. Contact your local Prentice Hall representative for details.

***"How to Study Math"*** Designed to help your students overcome math anxiety and to offer helpful hints regarding study habits, this useful booklet is available free with each copy sold. To request copies for your students in quantity, contact your local Prentice Hall representative.

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The staff at Prentice Hall is to be congratulated on another fine production job. The authors wish to express appreciation especially to Priscilla McGeehon (mathematics editor), Zita de Schauensee (project editor), and Christine Gehring-Wolf (designer) for their exceptional contributions.

Walter Fleming
Dale Varberg

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*As the sun eclipses the stars by its brilliancy, so the man of knowledge will eclipse the fame of others in the assemblies of the people if he proposes algebraic problems, and still more if he solves them.*

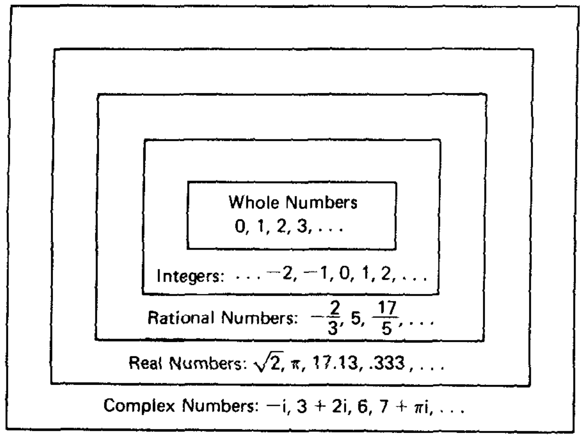
*Brahmagupta*

# CHAPTER 1

# Basic Algebra

Let us make no mistake about it: mathematics is and always has been the numbers game par excellence.

Philip J. Davis




# 1-1 The Real Number System

Mathematics students are number crunchers. That popular image is both profoundly right and terribly wrong. Most mathematicians disdain adding up long columns of numbers, finding square roots, or doing long division. They have relegated such tasks to electronic calculators and computers. But it is still true that numbers play a fundamental role in most of mathematics. Certainly this is true in precalculus.

It would take too long to describe the long, tortuous road traveled by mankind in going from the whole numbers to the complex numbers. It is enough to call your attention to the five classes of numbers in our opening display, to suggest that they were developed roughly in the order listed, and to recall a few facts about the real numbers. The real numbers will, in fact, be the principal characters in this book until Chapter 7, where we give a thorough discussion of the complex numbers.

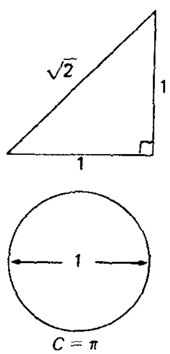


Figure 1

## THE REAL NUMBERS

Given a prescribed unit of length, we can (at least theoretically) measure the length of any line segment. The set of all numbers that can measure lengths, together with their negatives and zero, constitute the **real numbers.** The number $\sqrt{2}$ is a real number since it measures the length of the hypotenuse of a right triangle with legs of unit length (Figure 1). So is $\pi$; it measures the circumference of a circle of unit diameter ($C = \pi d$). Every **rational number** (a number which can be expressed as a ratio of two integers $p/q$) is a real number.

The best way to visualize the system of real numbers is as a set of labels for points on a line. Consider a horizontal line and select an arbitrary point to

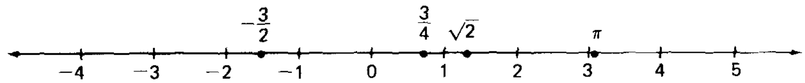



Figure 2

be labeled with 0. Then label a point one unit to the right with 1, a point one unit to the left with −1, and so on. The process is so familiar that we omit further details and draw a picture (Figure 2).

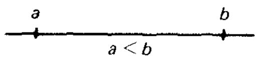



Figure 3

Of course, we cannot show all the labels, but we want you to imagine that each point has a number label, or **coordinate**, that measures its distance to the right or left of 0. We refer to the resulting coordinate line as the **real line.**

If $b - a$ is positive, we say that $a$ is less than $b$. We write $a < b$, which is called an **inequality**. On the real line, $a < b$ simply means that $a$ is to the left of $b$ (Figure 3). Similarly, if $b - a$ is positive or zero, we say $a$ is less than or equal to $b$ and write $a \le b$. It is correct to say $5 < 6$, $5 \le 6$, and $5 \le 5$.

The symbol $|a|$, read the **absolute value** of $a$, is defined by

$$|a| = \begin{cases} a & \text{if } 0 \le a \\ -a & \text{if } a < 0 \end{cases}$$

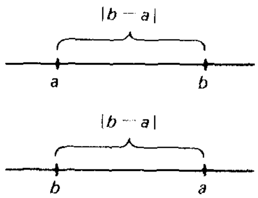



Figure 4

Geometrically, $|a|$ is the (undirected) distance from 0 to $a$. Some would say that $|a|$ is the magnitude of $a$ without regard to its sign. Be careful with this: It is correct to say $|+5| = 5$ and $|-5| = 5$, but it is not necessarily true that $|-x| = x$ (try $x = -2$). Finally, note that the distance between $b$ and $a$ on the real line is $|b - a|$; this is correct whether $a$ is to the left or to the right of $b$ (Figure 4).

## DECIMALS

```
    .875
 8) 7.000
    6 4
      60
      56
       40
       40
```

Figure 5

There is another important way to describe the real numbers. We must first review a basic idea. Recall that

$$.4 = \frac{4}{10}$$

$$.42 = \frac{4}{10} + \frac{2}{100} = \frac{40}{100} + \frac{2}{100} = \frac{42}{100}$$

$$.731 = \frac{7}{10} + \frac{3}{100} + \frac{1}{1000} = \frac{700}{1000} + \frac{30}{1000} + \frac{1}{1000} = \frac{731}{1000}$$

```
    .18181
11) 2.00000
    1 1
      90
      88
       20
       11
        90
        88
         20
         11
```

Figure 6

Clearly, each of these decimals represents a rational number.

Conversely, if we are given a rational number, we can find its decimal expansion by long division. For example, the division in Figure 5 shows that $\frac{7}{8} = .875$. When we try the same procedure on $\frac{2}{11}$, something different happens (Figure 6). The decimal just keeps on going; it is a **nonterminating decimal.**

Actually, the decimal .875 can be thought of as nonterminating if we adjoin zeros. Thus

$$\frac{7}{8} = .8750000\ldots \qquad = .875\overline{0}$$

$$\frac{2}{11} = .181818\ldots \qquad = .\overline{18}$$

$$\frac{2}{7} = .285714285714\ldots = .\overline{285714}$$

Note that in each case, the decimal has a repeating pattern. This is indicated by putting a bar over the group of digits that repeat. Now we state a remarkable fact about the rational numbers (ratios of integers).

*The rational numbers are precisely those numbers that can be represented as repeating nonterminating decimals.*

What about nonrepeating decimals like

$$.12112111211112\ldots$$

They represent the **irrational numbers,** of which $\sqrt{2} = 1.4142135\ldots$ and $\pi = 3.1415926\ldots$ are the best-known examples. Together, the rational numbers and the irrational numbers make up the real numbers. Thus we may say that:

*The real numbers are those numbers that can be represented as nonterminating decimals.*

THE REAL NUMBERS

| Rational numbers (the repeating decimals) | Irrational numbers (the nonrepeating decimals) |
|---|---|

While it is true that the real numbers are the fundamental numbers of precalculus (and calculus), in practical situations we work with a very small subset of them. Who can calculate with nonterminating decimals? Neither humans nor electronic calculators. Calculators are, in fact, restricted to decimals of a certain length (perhaps 8 or 10 digits). Thus in practical calculations, most real numbers must be **rounded**. For example, $\pi$ is often rounded to 3.141593, or perhaps to 3.14159. Our rule for rounding is to round down if the first discarded digit is 4 or less and round up if it is 5 or more. Thus $\pi$ is 3.1416 rounded to four decimal places, 3.142 rounded to three decimal places, and 3.14 rounded to two decimal places.