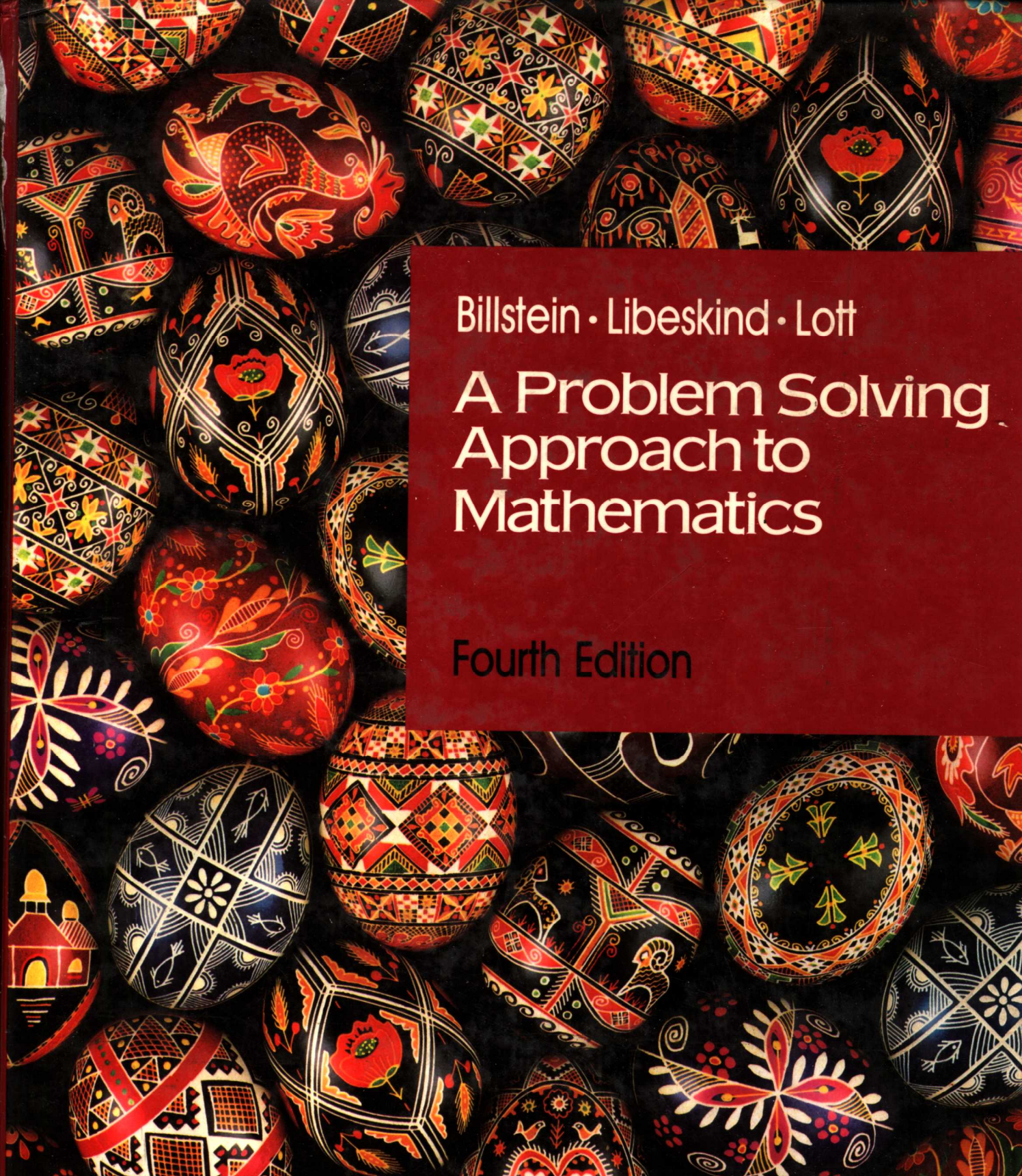

Billstein · Libeskind · Lott

A Problem Solving Approach to Mathematics

Fourth Edition

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P R E F A C E

The fourth edition of *A Problem Solving Approach to Mathematics* retains the goals of the previous editions, but extends them to meet the requests of users of the third edition as well as to meet new challenges of the 1990s.

Standards of the NCTM

In particular, we focus on the 1989 publication by the National Council of Teachers of Mathematics (NCTM), *Curriculum and Evaluation Standards of School Mathematics* (hereafter referred to as the *Standards*) which states, "Prospective teachers must be taught in a manner similar to how they are to teach—by exploring, conjecturing, communicating, reasoning, and so forth." In addition, "all teachers need an understanding of both the historical developments and current applications of mathematics. Further, they should be familiar with the power of technology."

To that end, the fourth edition allows instructors a variety of approaches to teaching and encourages "doing" as well as lecturing. We kept the elementary mathematics student of the future in mind and have produced a text which allows prospective teachers to examine, represent, transform, prove, apply, solve problems, and communicate in mathematics as they study.

Continuing Goals

In the fourth edition our goals remain:

- To present the appropriate mathematics in an intellectually honest and mathematically correct manner.
- To use the heuristics of problem solving as an integral part of mathematics.
- To approach the mathematics in a sequence which initially instills confidence, then challenges students as they complete the exercises and problem sets.

New Goals

For this edition, we had two additional goals:

- To present the topics in the context of the *Standards*.
- To significantly revise and expand the geometry coverage.

Problem Solving in the Fourth Edition

We showcase problem solving skills by:

- Devoting two chapters (1 and 15) to problem solving.

Chapter 1 has been expanded. Chapter 15 includes a long problem set that utilizes many concepts presented throughout the book.

- Beginning Chapters 1 through 14 with a preliminary problem which poses a question that students can answer with the skills they've mastered from the chapter.

We encourage teachers to introduce the problem at the beginning of the chapter, then spiral back after covering the chapter to show how the techniques therein are necessary to solve the problem.

- Using a four-step problem solving process to solve problems in each chapter.

The four steps are: Understanding the Problem, Devising a Plan, Carrying out the Plan, and Looking Back.

Computer Corner

Features Retained in this Edition

Wherever possible, we present topics in ways that could be used in actual classrooms. Further, we have incorporated various study aids and features to facilitate learning.

- Historical notes add context and humanize the mathematics.
- Cartoons teach or emphasize important material and add levity.
- Problems emphasizing computer usage are indicated by a computer symbol with special computer corners where appropriate.
- Problems emphasizing calculator usage are indicated by a calculator symbol .
- Brain Teasers, solved in the Instructor's Guide, may be assigned or used by the teacher to challenge students.
- Key terms are presented in the margins for quick review.
- Review problems are at the end of each nonoptional section.
- Questions from the Classroom have been expanded. We strongly recommend that instructors use these when building a course syllabus. Instructors

Brain Teaser

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may require students to write two answers to the questions—one mathematical and one pedagogical—using student texts and professional journals for research.

- Selected Bibliographies have been updated and revised. They are in the Instructor's Guide.
- Chapter tests allow students to test themselves.
- Chapter outlines help students review the chapter.
- Optional problems are marked with an asterisk (*); more difficult problems are marked with a star (★). Problems numbered in color have answers at the back of the book.
- Definitions are either set off in text or presented as key terms in the margin for quick review.
- Color highlights key mathematical concepts.

Features New to this Edition

- van Hiele-type Laboratory Activities in each non-optional geometry section.
- Optional Logo sections in each geometry chapter.
- Problem sets are reorganized first to follow the order of the chapter and second to proceed from easy to difficult.
- Accompanying Activities book by Daniel Dolan and James Williamson parallels the chapter content from the text.

Content

The material has been rewritten and the problem sets revised so that students with diverse backgrounds will find the material accessible. Flexibility is built in for instructors who wish to adapt the text to a variety of course lengths and organizations. Sections marked with asterisks (*) are optional and may be omitted without loss of continuity.

Geometry

In this edition, special attention was paid to revising the geometry chapters:

Chapter 10: Introduces material necessary to study geometry including topological notions, and sets the stage for van Hiele-type activities.

Chapter 11: Includes various types of construction activities involving congruent and similar figures. Activities use tracing paper, paperfolding, Mira, and compass and straightedge.

Chapter 12: Teaches a transformational approach to congruence and similarity.

Chapter 13: Presents the rudiments of the English and metric measuring systems and their uses.

Chapter 14: Covers Coordinate geometry and the relationship between algebra and geometry.

Appendix II: Covers basics of the Logo computer language, a tool for learning geometry. Optional sections and problems in each chapter also use Logo.

Calculator usage

As prescribed in the *Standards*, coverage of calculators is necessary and timely

Chapter 1: Introduces use of calculators as a problem solving tool. Problems involving calculators are marked with an icon in most sections throughout the book.

Chapters 7 and 9: Incorporate the use of calculators to investigate a variety of problems.

Numerations systems, whole numbers, and integers

Historical significance of numerations sytems and conceptual development of whole numbers and integers are highlighted.

Chapters 3 and 4: Emphasize use of different models for presenting the mathematics. Include new sections on mental mathematics and estimation.

Sets, relations, functions, and logic

These topics provide a background for later formal development of mathematics.

Chapter 2: Includes more problems and applications of functions.

Number theory

Number theory has long been one of the most exciting areas in mathematics.

Chapter 5: Develops divisibility rules and presents unsolved problems.

Real numbers

Decimals, fractions, and computation with these numbers are typically among the most difficult concepts elementary teachers are required to teach.

Chapters 6 and 7: Provide necessary mathematical tools for understanding relationships among these sets of numbers.

Probability

Topics in probability are now common in elementary school texts.

Chapter 8: Now includes geometric probability and more on simulations as well as tree diagrams and Monte Carlo methods, to aid in an understanding of probability topics.

Statistics

Understanding statistics is key to becoming an educated person.

Chapter 9: Presents new and expanded material using such techniques as stem and leaf graphs, box plots, and line plots for presenting and understanding data.

The content of the book is compatible with the recommendations from the *Standards*. We strongly believe that most people learn mathematics first intuitively and then formally. However we recognize that propspective teachers must learn both the content and the methods of teaching at the same time. As a result, we present the material with motivation for what a concept is, why it is important and how it may be used. To this same end, we encourage users to incorporate the laboratory activities and to use the accompanying activities book by Dolan and Williamson.

Supplements

- Instructor's Guide includes: Answers to all problems; Solutions to problems in Chapter 15, answers for Appendices I and II; Two sample chapter tests for each chapter; Selected Bibliography for each chapter; Suggested answers to Questions from the Classroom; Solutions to Brain Teasers; Solution to the problem appearing on the cover.
- *Mathematics Activities for Elementary School Teachers: A Problem Solving Approach*, by Daniel Dolan and James Williamson.
- Computerized Test Generator contains the questions from the Instructor's Guide, and allows instructors to create the tests to fit their own needs.

Acknowledgements

Our students and users of this text from across the nation have provided us with valuable feedback in revising this text. We have welcomed and continue to welcome constructive comments concerning topics in the text. Comments have led to many positive changes in the book. Many reviewers have contributed time to helping us with the book. One person in particular whose name was omitted from the last edition, but who has taught courses using the book and who has been a friend, colleague, and critic, is Professor Don Loftsgaarden of the University of Montana. We appreciate his help along with the latest reviewers of the manuscript. We could not have succeeded without your help.

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CHAPTER 1

Tools for Problem Solving

PRELIMINARY PROBLEM

Sara and David were reading the same novel. When Sara asked David what page he was reading, he replied that the product of the number of the page he was reading and the next page number was 98,282. What page was David reading?

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