

DESIGN OF FEEDBACK CONTROL SYSTEMS

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Holt, Rinehart and Winston 000166

New York Chicago San Francisco
Philadelphia Montreal Toronto London
Sydney Tokyo Mexico City
Rio de Janeiro Madrid

Library of Congress Cataloging in Publication Data

Hostetter, G. H., 1939–

Design of feedback control systems.

Includes bibliographies and index.

I. Feedback control systems. I. Savant, C. J.

II. Stefani, R. T. III. Title.

TJ216.H63 629.8'312 81-6371

ISBN 0-03-057593-1 AACR2

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Address correspondence to:

383 Madison Avenue

New York, N.Y. 10017

Printed in the United States of America

1 2 3 4 038 9 8 7 6 5 4 3 2 1

CBS COLLEGE PUBLISHING

Holt, Rinehart and Winston

The Dryden Press

Saunders College Publishing

Preface

This is a design-oriented control systems text intended for use in an introductory academic course and for reference and self-study by electrical and mechanical engineers in industry. Laplace transforms and electrical and/or mechanical network analysis are the prerequisite subjects upon which this text builds. It is especially well suited for a one-term junior- or early senior-level first course in control systems.

The manuscript evolved over several years of combined effort to provide an interesting, relevant, and effective introductory control system design class at California State University, Long Beach. Much of the understanding and skill we once taught specifically as it applied to networks is here presented in a more general context, with a wider immediate applicability. When used as a prerequisite for a sequence of modern control courses, this material also greatly reduces the large amount of time and effort that would otherwise be expended in establishing (then parting from) classical concepts. And it serves to encourage broad interests, perspectives, and skills at an early stage.

The greatly increased availability of digital computers naturally poses questions as to the proper and best role of computers in design. As the creative aspect of system design continues to involve the *directed* use of analytical tools, the emphasis here is upon the understanding, practical experience, and judgment necessary to be a creative designer. The manner in which the analytic tools are employed (hand calculation, pocket calculator, or computer) is taken to be of secondary concern at this introductory stage.

This text is designed to guide the reader in gaining the following:

1. A review of the fundamentals of electrical, translational mechanical, rotational mechanical, and electromechanical networks
2. Confidence in the use of Laplace transform methods in system response calculation and an understanding of commonly used response components
3. Familiarity with the use of transfer functions for linear, time-invariant systems, including asymptotic stability concepts and multivariable relations
4. Capability with block diagram manipulations, signal flow graphs, and the use of Mason's gain rule
5. Thorough acquaintance with Routh-Hurwitz polynomial testing and the ability to determine root distributions, to test adjustable systems, and to axis-shift to find relative stability
6. Appreciation of the feedback concept and its importance to tracking and other systems; familiarity with steady state error concepts and calculation and an acquaintance with

system parameter sensitivity, susceptibility to disturbances, and the use of performance indices

7. A thorough understanding of root locus methods, including those for adjustable systems other than the unity subtractive feedback type
8. Experience with classical compensation methods and real understanding of the principles guiding compensator design
9. Good comprehension and ability with frequency response methods, including those for systems involving delay elements and unstable components; appreciation of the powerful approach of incorporating experimental data
10. Competence with polar response plots and Nyquist methods
11. Familiarity with basic state variable concepts, including simulation diagrams, time-domain vector-matrix equations and solutions, transfer function matrices, stability, diagonalization, observability, and controllability; an introduction to state feedback design
12. Acquaintance with digital control concepts, sampling, and discrete-time system models at a level suitable for easy transition to study of computer and microprocessor-based

Quarter System Schedule

<i>Week</i>	<i>Chapters</i>	<i>Topics</i>
1	1	Introduction to the course System equations and terminology Review of Laplace transform Transfer functions
2	2-3	Block diagrams and signal flow graphs System response Stability and Routh-Hurwitz testing
3	4	Steady state errors Sensitivity and disturbance rejection Performance indices, optimality, and design
4, 5	5	Root locus construction and examples System compensation Design using root locus methods
Midterm Examination		
6, 7	6	Bode plot construction Frequency response examples Gain and phase margins Design using frequency response methods The Nyquist criterion
8, 9	7	State variable system models Controllability and observability Time-domain response Response computation
10	8	Digital control concepts Sampling Discrete-time system models Introduction to digital control system design

Semester System Schedule

Week	Chapters	Topics
1, 2	1	Introduction to the course System equations and terminology Review of Laplace transform Transfer functions
3	2	Block diagrams and signal flow graphs Response of first-, second-, and higher-order systems
4	3	Stability and Routh-Hurwitz testing
5, 6	4	Steady state errors Sensitivity and disturbance rejection Performance indices, optimality, and design
7, 8	5	Root locus construction and examples System compensation Design using root locus methods Midterm Examination
9, 10	6	Bode plot construction Frequency response examples Gain and phase margins Design using frequency response methods The Nyquist criterion
11, 12	7	State variable system models Controllability and observability Time-domain response Response computation
13, 14	8	Digital control concepts Sampling Discrete-time system models Introduction to digital control system design

real-time systems such as that given in B. C. Kuo, *Digital Control Systems* (New York: Holt, Rinehart and Winston, 1980), G. F. Franklin and J. D. Powell, *Digital Control of Dynamic Systems* (Reading, Mass.: Addison-Wesley, 1980), and similar texts

Along the way, it is hoped that the reader will learn much about the iterative process of engineering design. We have found the large number of example systems included here to be invaluable to this learning process.

Suggested class textbook schedules for quarters and semesters are given in the accompanying tables. For some classes—for example, for an introductory controls course early in an engineering program—the range of material available here is more than should be covered in a single term. The text is designed so that it is easy to abbreviate or delete topics to achieve a desired emphasis. In a course emphasizing transition to modern control theory, the compensation material of Secs. 5.7 through the end of Chap. 5 may be omitted with no penalty in understanding of the later topics. In a course emphasizing the classical viewpoint, Chap. 7 on state space methods and Sec. 8.8, which ties these ideas to the digital domain, may

be omitted without disturbing the flow of topics. Chapter 8 may be easily omitted or abridged as desired.

We sincerely hope that you or your students will enjoy reading and using this text as much as we have enjoyed its development. Our students have enthusiastically contributed to it as have our colleagues, especially L. Bailin, G. H. Cain, E. N. Evans, H. J. Lane, C. S. Lindquist, M. Santina, R. Rountree, and S. Wolf. Special thanks are due to Cynthia Klepadlo, who supervised the manuscript typing and the drafting of many of the original figures and to Mohammed Santina, who tirelessly compiled the problem solutions.

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INTRODUCTORY CONCEPTS

1.1 PREVIEW

This study begins by introducing control system terminology. The feedback concept is very important to control system design, and examples are given to demonstrate associated ideas and properties in Sec. 1.3.

Control system components are typically electrical, electronic, mechanical, and electromechanical devices. Mathematical models for selected components are then introduced and the relations are summarized in several tables in Sec. 1.4.

The manipulation and solution of system equations is greatly aided by Laplace transform methods which are reviewed and summarized in Sec. 1.5. Transfer functions, both for single-input, single-output and for multivariable systems are then defined and explained. The decomposition of system response into zero-state/zero-input and into forced/natural components is discussed carefully, and stability for linear, time-invariant systems is defined.

A position servo system is then discussed and used to reinforce understanding of the feedback principle. DC and AC control motors are analyzed, and transfer functions, including the coupling of a disturbance input, are computed from the system equations.

It is expected that much of the material of this first chapter will involve subjects known to the reader from previous study and experience. Our initial purpose is to bring together these introductory topics from several areas, relating them to systems and control.

1.2 CONTROL SYSTEMS AND TERMINOLOGY

Control systems influence every facet of modern life. Automatic washers and dryers, microwave ovens, chemical process plants, navigation and guidance systems, space satellites, pollution control, mass transit, and economic regula-

tion are a few examples. A control system is, in the broadest sense, any interconnection of components to provide a desired function.

The portion of a system which is to be controlled is called the *plant* or the *process*. It is affected by applied signals, called *inputs*, and produces signals of particular interest, called *outputs*, as indicated in Fig. 1-1.

A *controller* may be used to produce a desired behavior of the plant, as shown in Fig. 1-2. The controller generates plant input signals designed to produce desired outputs. Some of the plant inputs are accessible to the designer and some are generally not available. The inaccessible input signals are often disturbances to the plant. The double lines in the figure indicate that several signals of each type may be involved. This system is termed *open-loop* because the control inputs are not influenced by the plant outputs; that is, there is no *feedback* around the plant.

Such an open-loop control system has the advantage of simplicity, but its performance is highly dependent upon the properties of the plant, which may vary with time. The disturbances to the plant may also create unwanted response which it would be desirable to reduce.

As an example, suppose a gasoline engine is used to drive a large pump, as depicted in Fig. 1-3. The carburetor and engine comprise a common type of control system wherein large-power output is controlled with a small-power input. The carburetor is the controller in this case, and the engine is the plant. The fuel rate is the control input, and the pump load is a disturbance signal. The

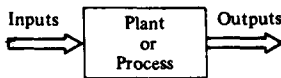


FIGURE 1-1
A plant or process to be controlled.

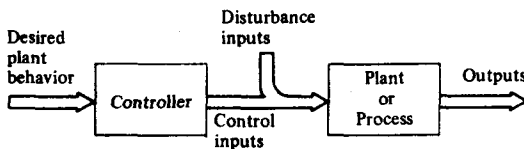


FIGURE 1-2
An open-loop control system.

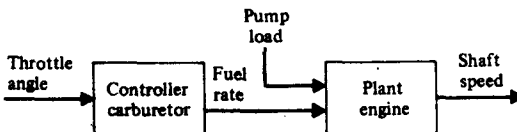


FIGURE 1-3
Example of an open-loop control system.

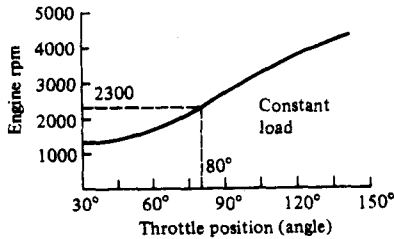


FIGURE 1-4

An engine speed vs. throttle angle curve.

desired plant output, a certain engine shaft speed, may be obtained by adjusting the throttle angle. The single lines in the figure indicate individual signals.

A representative plot of engine speed versus throttle angle is sketched in Fig. 1-4. This "calibration curve" gives the engine speed for a given throttle setting, at constant load on the engine. To produce an engine speed of 2300 rpm, for example, set the throttle angle to 80°. If the engine should become untuned (a change in the plant) or if the load should change (a disturbance), the calibration curve would change, and an 80° throttle angle would no longer produce a 2300-rpm engine speed.

In applications such as automatic washing machines and variable-speed hand drills, maintaining an accurate calibration curve is of little importance, within bounds. In other applications, such as laboratory instrumentation systems, it suffices to calibrate the system, reestablishing knowledge of the input-output relation often enough to obtain the desired accuracy. In systems such as the automobile, the human operator is capable of adjusting to changes and disturbances in the plant. In driving another's automobile for the first time, a new sense of "feel" must be established because no two automobiles produce exactly the same engine performance with the same accelerator setting.

1.3 THE FEEDBACK CONCEPT

If the requirements of the system cannot be satisfied with an open-loop control system, a closed-loop or feedback system is desirable. A path (or loop) is provided from the output back to the controller. Some or all of the system outputs are measured and used by the controller, as indicated in Fig. 1-5. The controller may then compare a desired plant output with the actual output and act to reduce the difference between the two.

Suppose that the system comprising a gasoline engine driving a pump is arranged in a closed-loop manner. One possible feedback control configuration is shown in Fig. 1-6. A tachometer produces a voltage proportional to the engine shaft speed. The input voltage, which is proportional to the desired speed, is set with a potentiometer. The tachometer voltage is subtracted from the input

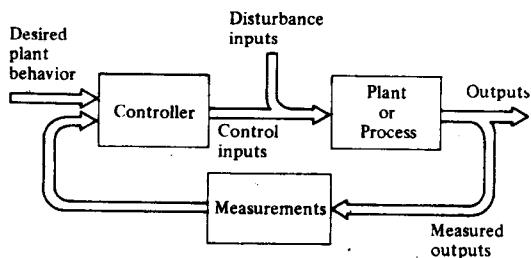


FIGURE 1-5
Closed-loop or feedback control.

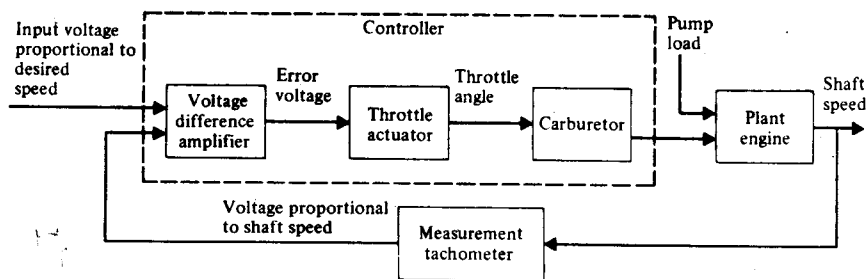


FIGURE 1-6
A closed-loop engine control system.

voltage, giving an error voltage which is proportional to the difference between the actual speed and the desired speed.

The error voltage is then amplified and used to position the throttle. The throttle actuator could be a reversible electric motor, geared to the throttle arm. When the engine shaft speed is equal to the desired speed (when the difference or *error* is zero), the throttle remains fixed. If a change in load or a change in the engine components should occur in the system, and the actual speed is no longer equal to the desired speed, the error voltage becomes nonzero, causing the throttle setting to change so that the actual speed approaches the desired speed. The controller here consists of the voltage difference amplifier, throttle actuator, and carburetor.

Some of the advantages which feedback control offer to the designer are

1. *Increased accuracy.* The closed-loop system may be designed to drive the error between desired and measured response to zero.
2. *Reduced sensitivity to changes in components.* As in the previous example, the system may be designed to seek zero error despite changes in the plant.
3. *Reduced effects of disturbances.* The effects of disturbances to the system may be greatly attenuated.
4. *Increased speed of response and bandwidth.* Feedback may be used to increase