VIA VECTOR TO TENSOR

W. G. BICKLEY
and
R. E. GIBSON

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An Introduction to the Concepts and Techniques of the Vector and Tensor Calculus

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To

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in gratitude for the intellectual stimulus and personal friendship which we both enjoyed and greatly value.

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EDITORS' FOREWORD

The object of this series is to provide texts in applied mathematics which will cover school and university requirements and extend to the post-graduate field. The texts will be convenient for use by pure and applied mathematicians and also by scientists and engineers wishing to acquire mathematical techniques to improve their knowledge of their own subjects. Applied mathematics is expanding at a great rate, and every year mathematical techniques are being applied in new fields of physical, biological and economic sciences. The series will aim at keeping abreast of these developments. Wherever new applications of mathematics arise it is hoped to persuade a leader in the field to describe them briefly.

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PREFACE

This book is based on courses of lectures given by one of us (W. G. B.) to post-graduate students of engineering and science.

In recent years engineers in particular, and scientists and technologists in general, have been making ever-increasing use of the concepts and techniques of higher mathematics. Matrices, integral transforms, numerical analysis, vector and tensor analysis, . . . , appear more and more freely and frequently in technological treatises and text-books, and in the technical press. It therefore behoves those whose business it is to teach the postgraduate students and others who will need or desire to read these books or periodicals, to put them in the way of being able to do so profitably. This is what the lectures, upon which our book is based, set out to do.

The audience to which these lectures were addressed—and the wider one to which this book is dedicated—consists of individuals with widely differing mathematical backgrounds, but, in general, no specialist mathematical training. Their temperament and motivation differ from those of the mathematical specialist, but their ability to understand and to absorb, and ultimately to use advanced mathematics—when properly presented—is far from negligible and, indeed, in many cases, is on a par with that of those who undergo the discipline of an honours course in mathematics.

For such students a heuristic and inductive treatment is desirable. The fashionable axiomatic, postulational and deductive approach is apt to repel and discourage all but the ablest and most courageous of them.

Here, in this book, our primary aim is *understanding*: rigour and manipulative facility are subsidiary—or subsequent. The first need of these students is to be able to *read with understanding* the mathematical formulae and arguments in the books and articles which they will have to consult. We believe that if this understanding is achieved the step to manipulation and constructive use will be relatively easy—and many of our students will not need to make this further step.

Successful exploration of new mathematical territory can be undertaken only from a 'base camp' adequately supplied with the relevant mathematical facts and ideas. In the case of the tensor calculus—the main target of this book—one of these necessary preliminaries is vector analysis. Many engineering students today are familiar with vectors and vector algebra, but the same is much less true of vector analysis. The course on vectors which forms the first four chapters is to be regarded in the first instance as a refresher course.

Opportunity has been taken here and there to present the material in what is believed to be a somewhat unusual manner, and, more especially, to introduce embryonic ideas to be more fully developed in the later tensor chapters. In particular we may mention the emphasis on the distinction between components and resolved parts, and the complementary properties of moment and angular velocity in polar co-ordinates analogous to that of force and linear velocity, which combine to facilitate the acceptance and understanding of the distinction between covariant and contravariant components.

Also, we have used Gauss' and Stokes' "theorems" in a manner that amounts to definitions of divergence and curl, since this enables these quantities to be defined in a way equally applicable to any co-ordinate system. An attempt has been made—at least in outline—to show how the elements common to diverse situations in applied mathematics and mathematical physics lead, by abstraction, to the definitions of such quantities as scalar and vector products, gradient, divergence and curl. By such means it is hoped to enable the student to read vector language with the understanding which comes of the ability to construct a corresponding mental image.

This last statement may seem rather paradoxical, for surely one of the virtues of the tensor calculus is that it replaces the necessity to visualise the complicated geometry of co-ordinate systems by routine manipulation of formulae. In our opinion, however—at least for most of our audience—this manipulative facility is reached with greater speed and certainty when its relevance to simple well-known or readily assimilated examples is clearly demonstrated.

In view of the needs of our audience the use of the tensor calculus in mathematical physics and applied mathematics is more important than the elaboration of differential geometry. Albeit, in view of the fundamental role of co-ordinate systems, the chapter on 'Some Geometry' was inevitable—and, for example, civil engineers designing shell roofs need to know more than a smattering of the geometry of curves and surfaces. It was also inevitable that this chapter should be the longest in the book, and although the book claims to be no more than an introduction to the tensor calculus, it is difficult to see what could have been left out of this chapter without loss. It may also seem strange that the last chapter contains no specific reference to the theory of relativity. To have included this would have demanded much space and reference to what would have been unfamiliar background to most of our students. At the same time we feel that there is very little in the book that anyone wishing to make a study of the theory of general relativity could well do without.

Exercises which will enable the student to test his understanding of the ideas and command of the technique, and, in some instances, to penetrate a little deeper than does the text, are appended to each chapter.

As has been said, this book was written to cater for the needs of students who have not specialised in mathematics. The fashion today seems to be to try to condition the minds of undergraduate mathematicians to the acceptance of ready-made abstractions in the form of axioms and definitions without reference to the process of abstraction and the array of facts whence the abstractions came. This makes mathematics difficult, even for mathematicians. We believe that the understanding which our exposition is intended to engender will, even for the specialist mathematics student, enlighten the abstract definitions and postulates which appear in the more erudite treatises on the tensor calculus.

W. G. BICKLEY

R. E. GIBSON

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PART I

VECTORS

CHAPTER 1

VECTOR ALGEBRA

What is a vector?

It would be fashionable to answer this question, and commence this book, by a definition and some axioms, but that would not suit our purpose. That method may have certain virtues of logical efficiency and of rigour, but it is not informative. One needs to know a great deal about any entity before one is in a position to think about defining it, and it is our purpose gradually to build up the knowledge upon which definitions can be based, and the necessary axioms formulated. Our purpose is to try to present the ideas and techniques of vector algebra and of vector analysis so that their use in various fields of applied mathematics may be, in the first place, clearly understood, and in the second, competently used.

The very fact that he or she is reading these words implies that the reader knows enough about vectors at least to believe that greater knowledge will be useful or interesting—and we hope, both! It may well be that the reader, like the author, can recall the feeling of almost incredulous surprise that quantities, physically so different as forces and velocities, should miraculously obey *exactly* the same laws. Both need for their specification, not only a magnitude, but also a direction and a sense. Both can be represented by a directed line segment. Both can be compounded by the parallelogram, triangle, or polygon law, in terms of this graphical representation. It is the fact that forces and velocities, and other entities, share these and other abstract properties, which leads the mathematician to classify them together, inventing a name for the class, studying their properties and discovering and elaborating the techniques for dealing with them, examining other candidates for entry into this class, and exploring the unifying influence which they have in the study of diverse branches of applied mathematics.

The first thing of importance, is then, to be able to recognise whether an entity falls within this class of vectors, or not.

Scalars and vectors.

Many physical quantities, such as mass, energy, volume, are specified by their magnitude alone, expressed, of course, in terms of some chosen unit. Such quantities are called **scalars**, because many of them are usually measured by some instrument in which a pointer moves over a scale. Other quantities, such as forces and velocities which have already been mentioned, need for their complete specification in addition to their magnitude, a direction in space, and a sense in which this direction is to be thought of as proceeding. Such quantities are termed **vectors.**

Fundamentally, although a single quantity is not sufficient completely to specify them, vectors are to be considered as single entities, and vector methods and formulae are most powerful when this can be done.

In what follows we shall normally denote scalars by ordinary italic type, but to emphasise their nature, we shall normally denote vector quantities by Clarendon (bold-faced) type.

We recall the more elementary properties which are common to forces and to velocities, and which will be common to all vector quantities:—

Vector quantities need *magnitude*, *direction* and *sense* for their complete specification.

Vector quantities can be represented by *directed line segments.*

In terms of such a representation, vector quantities are compounded by the *polygon*, *parallelogram* or *triangle law.*

Other properties common to all vectors will emerge later, but the above are all *necessary* conditions. The question as to how many of these are *sufficient* will be deferred for the present.

Some other vector quantities.

In addition to force and velocity already mentioned, there are many other types of entities which can be recognised as having the properties of a vector.

First, of course, the fact that vectors can be represented by directed line segments shows that these directed line segments themselves are vectors. Such a directed line segment can also represent a *displacement* and so displacement is thus another kind of quantity which is a vector.

Velocity, already recognised as a vector, is the rate of change of vector displacement, a fact that will be taken up later. Here it leads to the consideration of the rate of change of velocity, or *acceleration.* This, again, has magnitude, direction and sense and is recognised as another type of vector quantity. Since mass is a scalar, mass times acceleration is also a vector, so that the Newtonian law of motion is a *vector law.* This consideration also directs attention to the more general form of the law, that force is rate of change of momentum, and to the realisation that *momentum* is a vector quantity.

In the first instance the ideas of displacement, velocity, acceleration

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