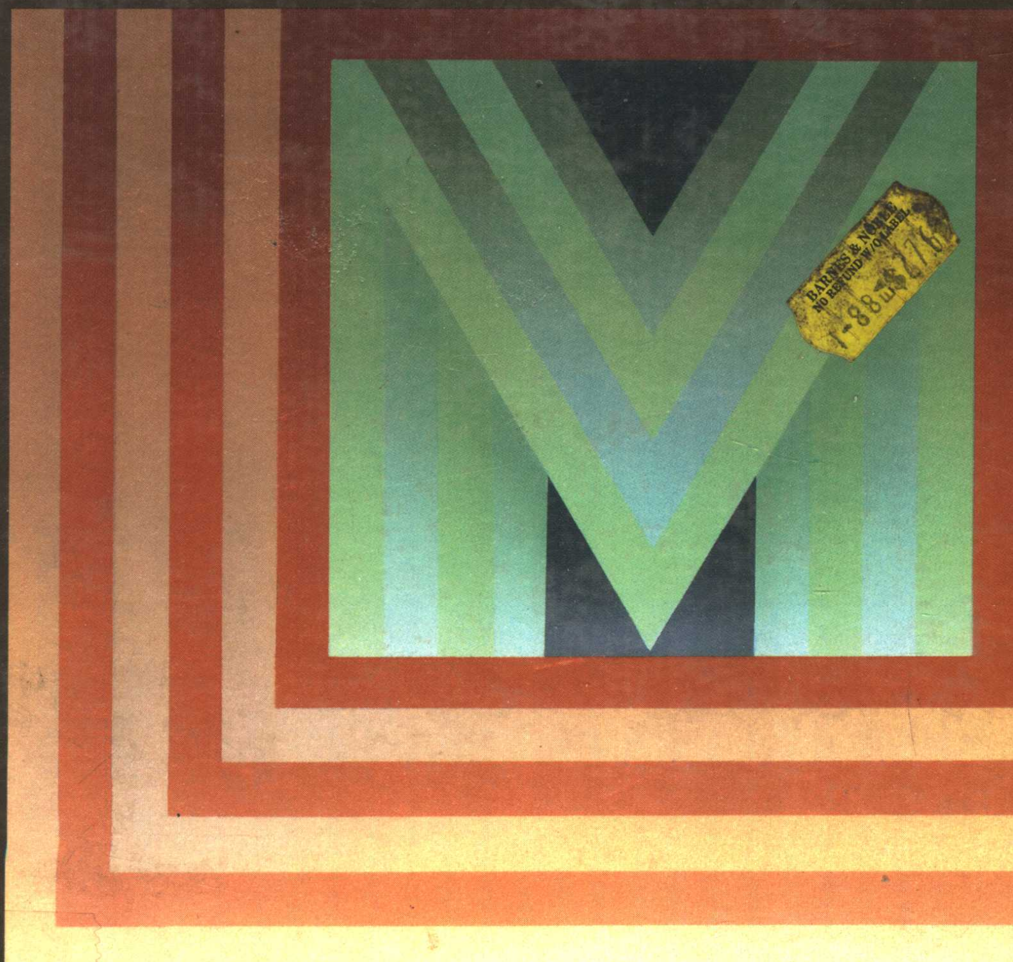
Algebra and Trigonometry

Fourth Edition



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Fourth Edition

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Scott, Foresman and Company • Glenview, Illinois • London, England

To the Student

A ***Solutions and Study Guide*** *to accompany this textbook is available from your college bookstore. This book contains* **additional worked-out examples** *beyond what is in the textbook, detailed* ***step-by-step solutions*** *to approximately half of the odd-numbered exercises in the textbook, and a* ***self-test for each chapter.*** *These features can help you study and understand the course material.*

Library of Congress Cataloging-in-Publication Data

Lial, Margaret L.
Algebra and trigonometry.

Includes index.
1. Algebra. 2. Trigonometry. I. Miller, Charles David, 1942– . II. Title.
QA154.2.L49 1986 512'.13 85-25098
ISBN 0-673-18296-7

Printed in the United States of America.
123456-RRC-90 89 88 87 86 85

Preface

The fourth edition of *Algebra and Trigonometry* is designed to give a mathematically sound approach to the topics in algebra and trigonometry needed for success in later courses. It is intended to prepare students for calculus by providing them with thorough explanations that are precise, yet written for student comprehension.

The prerequisite is a high school course in algebra or intermediate algebra at the college level. For students whose algebra background is not as solid as it might be, we have included a review of algebra in Chapter 1. Chapter 2 on equations also may be review for students.

Content Highlights

The **thorough review of algebra** in Chapter 1 discusses operations on the real numbers, absolute value, exponents, polynomials, factoring, rational expressions, radicals, and complex numbers.

Functions, an essential concept in this text, are discussed beginning in Chapter 3 and continuing throughout the book. **Functions and their graphs** are closely related, so the graphs of functions are discussed again and again. The general techniques of graphing also are presented in Chapter 3; later, new graphs are obtained from familiar ones by investigating symmetry and translations.

Exponential and logarithmic functions, presented in Chapter 5, are introduced in such a way that students see the functional aspects, with the numerical aspects downplayed.

Trigonometry is introduced via the unit circle. It is given a thorough treatment in order to prepare students for what is traditionally a troublesome area in their study of calculus. For those instructors who wish to emphasize the analytic aspects of the subject, the numerical topics have been placed in a separate chapter (Chapter 8).

The book features a great **many applications problems,** from the fields of business, engineering, physics, chemistry, biology, astronomy, navigation, demography, and political science. These applications are designed to be optional, so instructors may decide which ones to assign.

Key Features The **format** of the book has been carefully designed to facilitate learning. **Definitions and rules are set off in boxes with marginal headings,** to help students with review and study, and also to make it easy for instructors to use the text. **Second color has been used pedagogically** to clarify explanations of techniques. **Numerous figures and graphs** illustrate examples and exercises.

The textbook features an **abundance of examples,** which present major points through detailed steps and explanations. The symbol ■ makes it clear when an example ends and the discussion continues. Sections are designed to be covered in one class period.

Exercises **Extensive exercise sets** have been a strong feature of this textbook. This edition contains nearly 6800 exercises. These problems include a lengthy set of exercises for each section plus review exercises at the end of each chapter (about 1250 review problems in all). Answers to odd-numbered exercises are located at the back of the book.

A special set of cumulative review exercises at the end of Chapter 2 can be used to decide which portions of the algebra review in Chapter 1 or the discussion of equations in Chapter 2 need be covered.

Algebra problems similar to those in calculus are included in many exercise sets, and most of the review exercise sets contain exercises from standard calculus textbooks. These exercises help students see the importance of algebra and trigonometry for their future work in calculus.

Supplements The **Solutions and Study Guide,** by Eldon L. Miller, University of Mississippi, features additional examples for each section in the textbook, as well as solutions to approximately half of the odd-numbered exercises and a self-test for each chapter. This book can help students study and understand the course material by providing additional worked-out examples.

The **Instructor's Guide** contains a diagnostic pretest, a sample minimum assignment for the course, answers to even-numbered exercises, and an extensive test bank that can be used to prepare examinations or to provide additional problems for students to work.

Acknowledgments A great many people helped with this revision through their suggestions and comments. In particular, we would like to thank those who reviewed the manuscript: Charles Applebaum, Bowling Green State University; Lee H. Armstrong, University of Central Florida; Alan A. Bishop, Western Illinois University; August J. Garver, University of Missouri, Rolla; James Hodge, College of Lake County; John Hornsby, University of New Orleans; Janice McFatter, Gulf Coast Community College; Brenda Marshall, Parkland College; and Eldon L. Miller, University of Mississippi. We also would like to thank the following people who helped us check the answers: James Arnold, University of Wisconsin–Milwaukee; Lewis Blake III, Duke University; Louis F. Hoelzle, Bucks County Community College; Kathleen Pirtle; Marjorie Seachrist; and Priscilla Ware, Columbus Technical College.

We have received a great deal of help from two outstanding people at Scott, Foresman and Company: both Barbara Maring and Linda Youngman helped make the writing of this text a great pleasure.

Margaret L. Lial
Charles D. Miller

Contents

1

Fundamentals of Algebra

Today, algebra is required in a great many fields, ranging from accounting to zoology. The reason that algebra is required in all these fields is that algebra is *useful*. Equations, graphs, and functions occur again and again in many different areas. For example, in this text algebra is used to predict population growth, to determine the path of objects orbiting in space, and to investigate the costs versus the benefits of removing pollutants from a substance. To prepare for this study, the book begins with a review of the basics of algebra.

1.1 The Real Numbers

Numbers are the foundation of mathematics. The most common numbers in mathematics are the **real numbers,** the numbers that can be written as a decimal, either repeating, such as

$$\frac{1}{3} = .33333\ldots, \quad \frac{3}{4} = .75000\ldots, \quad \text{or} \quad 2\frac{4}{7} = 2.571428571428\ldots,$$

or nonrepeating, such as

$$\sqrt{2} = 1.4142135\ldots \quad \text{or} \quad \pi = 3.14159\ldots.$$

There are four familiar operations on real numbers: addition, subtraction, multiplication, and division. Addition is indicated with the symbol $+$. Subtraction is written with the symbol $-$, as in $8 - 2 = 6$. Multiplication is written in a variety of ways. All the symbols 2×8, $2 \cdot 8$, $2(8)$, and $(2)(8)$ represent the product of 2 and 8, or 16. When writing products involving **variables** (letters used to represent numbers), no operation symbols may be necessary: $2x$ represents the product of 2 and x, while xy indicates the product of x and y. Division of the real numbers a and b is written $a \div b$, or more commonly a/b.

The set of real numbers, together with the relation of equality and the basic operations of addition and multiplication, form the **real number system.** (Informally, a **set** is a collection of objects.) The key properties of the real number system are given below, where a, b, and c are letters used to represent any real number.

Properties of the Real Numbers

Closure properties	$a + b$ is a real number ab is a real number
Commutative properties	$a + b = b + a$ $ab = ba$
Associative properties	$(a + b) + c = a + (b + c)$ $(ab)c = a(bc)$
Identity properties	There exists a unique real number 0 such that $a + 0 = a$ and $0 + a = a$. There exists a unique real number 1 such that $a \cdot 1 = a$ and $1 \cdot a = a$.
Inverse properties	There exists a unique real number $-a$ such that $a + (-a) = 0$ and $(-a) + a = 0$. If $a \neq 0$, there exists a unique real number $1/a$ such that $a \cdot \frac{1}{a} = 1$ and $\frac{1}{a} \cdot a = 1$.
Distributive property	$a(b + c) = ab + ac$

Example 1 By the closure properties, the sum or product of two real numbers is a real number. For example,

(a) $4 + 5$ is a real number, 9.
(b) $-9(-4)$ is a real number, 36.
(c) $8 \cdot 0$ is a real number, 0.
(d) $\sqrt{5} + \sqrt{3}$ is a real number. ■

Example 2 The following statements are examples of the commutative property. Notice how the order of the numbers changes from one side of the equals sign to the other.

(a) $6 + x = x + 6$
(b) $(6 + x) + 9 = (x + 6) + 9$
(c) $5 \cdot (9 \cdot 8) = (9 \cdot 8) \cdot 5$
(d) $5 \cdot (9 \cdot 8) = 5 \cdot (8 \cdot 9)$ ■

The associative properties are used to add or multiply three or more numbers. For example, the associative property for addition says that the sum $a + b + c$ of the real numbers a, b, and c can be found either by first adding a and b, and then adding c to the result, indicated by the association

$$(a + b) + c,$$

or by first adding b and c, and then adding a to the result, indicated by the association

$$a + (b + c),$$

since, by the associative property, either method gives the same result.

Example 3 As the following statements illustrate, with the associative properties the order of the numbers does not change, but the placement of the parentheses does change.

(a) $4 + (9 + 8) = (4 + 9) + 8$ **(b)** $3(9x) = (3 \cdot 9)x$

(c) $(\sqrt{3} + \sqrt{7}) + 2\sqrt{6} = \sqrt{3} + (\sqrt{7} + 2\sqrt{6})$ ■

The identity properties show that 0 and 1 are special numbers: the sum of 0 and any real number a is the number a, so that 0 preserves the identity of a real number under addition. For this reason, 0 is the **identity element for addition.** In the same way, 1 preserves the identity of a real number under multiplication, making 1 the **identity element for multiplication.**

Example 4 By the identity properties,

(a) $-8 + 0 = -8$ **(b)** $(-9)1 = -9.$ ■

According to the addition inverse property, for any real number a there is a real number, written $-a$, such that the sum of a and $-a$ is 0, or $a + (-a) = 0$. The number $-a$ is called the **addition inverse, opposite,** or **negative** of a. The addition inverse property also says that this number $-a$ is *unique;* that is, a given number has only one addition inverse.

Don't confuse the *negative of a number* with a *negative number.* Since a is a variable, it can represent a positive or a negative number (as well as zero). The negative of a, written $-a$, can also be either a negative or a positive number (or zero). It is a common mistake to think that $-a$ *must* represent a negative number, although, for example, if a is -3, then $-a$ is $-(-3)$ or 3.

For each real number a except 0, there is a real number $1/a$ such that the product of a and $1/a$ is 1, or

$$a \cdot \frac{1}{a} = 1, \quad a \neq 0.$$

The symbol $1/a$ is often written a^{-1}, so that, by definition,

> For every nonzero real number a,
>
> $$a^{-1} = \frac{1}{a}.$$

The number $1/a$ or a^{-1} is called the **multiplication inverse** or **reciprocal** of the number a. Every real number except 0 has a reciprocal. As with the addition inverse, the multiplication inverse is unique—a given nonzero real number has only one multiplication inverse.

Example 5 By the inverse properties,

(a) $9 + (-9) = 0$

(b) $-15 + 15 = 0$

(c) $6 \cdot \frac{1}{6} = 1 \left(\text{so that } 6^{-1} = \frac{1}{6}\right)$

(d) $-8 \cdot \left(\frac{1}{-8}\right) = 1$

(e) $\frac{1}{\sqrt{5}} \cdot \sqrt{5} = 1.$

(f) There is no real number x such that $0 \cdot x = 1$, so there is no multiplication inverse for 0. ■

One of the most important properties of the real numbers, and the only one that involves both addition and multiplication, is the distributive property. The next example shows how this property is applied.

Example 6 By the distributive property,

(a) $9(6 + 4) = 9 \cdot 6 + 9 \cdot 4$

(b) $3(x + y) = 3x + 3y$

(c) $-\sqrt{5}(m + 2) = -m\sqrt{5} - 2\sqrt{5}$
(The product of $-\sqrt{5}$ and m is often written $-m\sqrt{5}$, since $-\sqrt{5}m$ is too easily confused with $-\sqrt{5m}$.) ■

The distributive property can be extended to include more than two numbers in the sum, as follows.

$$a(b + c + d + e + \cdots + n) = ab + ac + ad + ae + \cdots + an$$

This form is called the **extended distributive property.**

The **substitution property** is another key property.

Substitution Property

> If $a = b$, then a may replace b or b replace a in any expression without affecting the truth or falsity of the statement.

Many further properties of the real numbers can be proven directly from those given above. For example, the following two important properties are used in the next chapter in solving equations.

Addition and Multiplication Properties

> For all real numbers a, b, and c, if $a = b$, then
>
> $$a + c = b + c, \quad \text{and} \quad ac = bc.$$

In words, these properties say that the same number may be added or multiplied on both sides of a statement of equality.

Two special properties of the number 0 are stated below.

Properties of Zero

> For all real numbers a and b,
>
> $$\mathbf{a \cdot 0 = 0}, \quad \text{and} \quad \mathbf{ab = 0} \quad \textbf{if and only if} \quad \mathbf{a = 0 \text{ or } b = 0.}$$

The second property above contains the phrase "if and only if." By definition,

If and Only If

> p if and only if q
>
> means
>
> if p, then q and if q, then p.

Finally, some useful properties of the negatives of numbers follow.

Properties of Negatives

> For all real numbers a and b,
>
> $$\begin{aligned} -(-a) &= a \\ -a(b) &= -(ab) \\ a(-b) &= -(ab) \\ (-a)(-b) &= ab. \end{aligned}$$

The proofs of many of these properties are included in the exercises below. Example 7 shows a general style of proof that can be used for many of these properties.

Example 7 If a is a real number, prove that

$$-(-a) = a.$$

We shall prove that both $-(-a)$ and a are addition inverses of $-a$. Since a real number has only one addition inverse, this will show that $-(-a)$ and a are equal. First,

$$-a + [-(-a)] = 0$$

since $-(-a)$ is the addition inverse of $-a$. Also,

$$-a + a = 0,$$

since a is the addition inverse of $-a$. Since both $-(-a)$ and a are addition inverses of $-a$, and since $-a$ has only one addition inverse,

$$-(-a) = a. \quad \blacksquare$$

The properties of real numbers given above apply to addition or multiplication. The two other common operations for the real numbers, subtraction and division, are defined in terms of the operations of addition and multiplication, respectively.

Subtraction is defined by saying that the difference of the numbers a and b, written $a - b$, is found by adding a and the *negative* of b. See the next page.

Subtraction

For all real numbers a and b,

$$a - b = a + (-b).$$

Example 8

(a) $7 - 2 = 7 + (-2) = 5$

(b) $-8 - (-3) = -8 + (+3) = -5$

(c) $6 - (-15) = 6 + (+15) = 21$ ■

Division of a real number a by a nonzero real number b is defined in terms of multiplication as follows.

Division

For all real numbers a and b, with $b \neq 0$,

$$\frac{a}{b} = a \cdot \frac{1}{b} = ab^{-1}.$$

That is, to divide a by b, multiply a by the reciprocal of b.

Example 9

(a) $\frac{6}{3} = 6 \cdot \frac{1}{3} = 2$

(b) $\frac{-8}{4} = -8 \cdot \frac{1}{4} = -2$

(c) $\frac{-9}{0}$ is meaningless since there is no reciprocal for 0; also, $\frac{0}{0}$ is meaningless. ■

Several useful properties of quotients are listed below.

Properties of Quotients

For all real numbers a, b, c, and d, with all denominators nonzero,

$\frac{a}{b} = \frac{c}{d}$ **if and only if** $ad = bc$

$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$

$\frac{ac}{bc} = \frac{a}{b}$

$\frac{a}{b} - \frac{c}{d} = \frac{ad - bc}{bd}$

$\frac{a}{-b} = \frac{-a}{b} = -\frac{a}{b}$

$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$

$\frac{-a}{-b} = \frac{a}{b}$

$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}.$

To avoid possible ambiguity when working problems, use the following **order of operations,** which has been generally agreed upon. (By the way, this order of operations is used by computers and many calculators.)

Order of Operations

If parentheses or square brackets are present:

1. Work separately above and below any fraction bar.
2. Use the rules below within each set of parentheses or square brackets. Start with the innermost and work outward.

If no parentheses are present:

1. Do any multiplications or divisions in the order in which they occur, working from left to right.
2. Do any additions or subtractions in the order in which they occur, working from left to right.

If exponents or roots are involved, they are dealt with before using the order of operations.

Example 10 Use the order of operations given above to simplify the following.

(a) $(6 \div 3) + 2 \cdot 4 = 2 + 2 \cdot 4 = 2 + 8 = 10$

(b) $\dfrac{-9(-3) + (-5)}{2(-8) - 5(3)} = \dfrac{27 + (-5)}{-16 - 15} = \dfrac{22}{-31} = -\dfrac{22}{31}$

(c) $4 \cdot 2^3 = 4 \cdot 8 = 32$ ■

There are several subsets* of the set of real numbers that come up so often they are given special names, as listed below. Some of the subsets are written with **set-builder notation;** with this notation,

$$\{x|x \text{ has property } P\},$$

read "the set of all elements x such that x has property P" represents the set of all elements having some specified property P.

Subsets of the Real Numbers

Natural numbers	$\{1, 2, 3, 4, \ldots\}$
Whole numbers	$\{0, 1, 2, 3, 4, \ldots\}$
Integers	$\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$
Rational numbers	$\left\{\frac{p}{q}\middle\vert p \text{ and } q \text{ are integers, } q \neq 0\right\}$
Irrational numbers	$\{x\vert x \text{ is a real number that is not rational}\}$

Example 11 Let set $A = \{-8, -6, -3/4, 0, 3/8, 1/2, 1, \sqrt{2}, \sqrt{5}, 6, \sqrt{-1}\}$. List the elements from set A that belong to each of the following sets.

(a) The *natural numbers* in set A are 1 and 6.
(b) The *whole numbers* are 0, 1, and 6.

*Set A is a **subset** of set B if and only if every element of set A is also an element of set B.