Experimental Methods in Magnetism

I. GENERATION AND COMPUTATION OF MAGNETIC FIELDS

BY

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PREFACE

Only too often in the literature on magnetic phenomena little or no information is given on the measuring methods used. In particular it is not always clear to what extent the result is influenced by the methods of preparation and measurement of the sample. Literature on measuring apparatus is scattered in a great number of scientific periodicals and fluctuates considerably as regards what is considered as known and what is explained.

This book presents a treatment of the principles of a number of widely-used methods for measuring magnetic quantities. It is hoped that the book will be useful to those engaged in magnetic experiments, whether in scientific research or in routine measurements. The reader is presumed to be familiar with the basic concepts of magnetism and with elementary vector analysis. The latter, however, is not necessary if the proofs are taken for granted and only the results are used.

To increase the practical value of this book much attention has been given to those details of a method where errors are liable to be introduced and how this can be prevented.

The book is certainly not meant to give a complete survey of the literature. The examples are mainly chosen from the immediate neighbourhood of the author; this does not of course imply that there may not be better or earlier ones. However, several chapters are provided with a bibliography of books and survey articles consulted by the author; these contain fairly complete lists of the literature.

For technical reasons the book is split into two parts. The first part contains two chapters on the theory of the magnetic potential needed for the understanding (though not for the application) of several subjects discussed in the subsequent chapters of the book. It further contains a chapter on the generation of magnetic fields by ironless solenoids. This chapter is rather detailed as it is considered that the experimenter is often faced with the design and construction of coils of all sorts when building his apparatus. On the other hand a chapter on iron-core magnets is kept very elementary as these magnets are commonly bought and thus only a limited understanding of their differences is required.

Part 2 of the book deals with the measurement of magnetic quantities. For the discussion of the various methods use is often made of the results obtained in Part 1. Therefore the two parts should be considered as one whole. Two important fields are not treated, namely neutron diffraction and domain techniques. This is because the experimental techniques used in the former field are covered in detail in a book by BACON [1955] and two books on the latter subject have appeared recently (CRAIK and TEBBLE [1965] and CAREY and ISAAC [1966]). The chapter on resonance methods (Ch. 6) is kept very elementary because here too several books are available, mentioned at the end of that chapter.

As to the remaining subjects it is hoped that the book presents a useful supplement to the already existing literature. If so, this is in no small measure due to the generous help I received from my colleagues K. Compaan, P. Cornelius, U. Enz, W.P.J. Fontein, N.J. Freedman, P.R. Locher, A.L. Luiten, G.W. Van Oosterhout, R.P. Van Stapele, D.L.A. Tjaden, J.S. Van Wieringen, D. Wilkinson and from the Editor of this series E.P. Wohlfarth. Their assistance is gratefully acknowledged here.

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I also wish to express my appreciation to the management of the Philips Research Laboratories for the facilities granted, to E. Deimel, J. Geel, R. Gersdorf, J.P. Morel and N.F. Verster for permission to mention their unpublished results, to the publishers of Philips Technical Review for permission to reproduce Figs. 3.10 and 3.40, and to Mr. P. Vissers for making the drawings.

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LIST OF MOST IMPORTANT SYMBOLS

The symbols may have other meanings, incidentally, than those mentioned below. The list applies to both Parts 1 and 2.

A	Area
$\mathbf{B}$, B	Magnetic induction (flux density)
C	Specific heat
C	Capacitance
C	Curie constant
E	Potential energy
$\mathbf{E}$	Electric fieldstrength
e	Electromotive force
$\mathbf{F}$, F	Force
f	Electric fieldstrength
f	Mössbauer fraction
f	Frequency
G	Shear modulus
G	Galvanometer constant
$\mathbf{H}$, H	Magnetic field
$\mathbf{I}$, I	Magnetization
i	Electric current
J_l	Bessel function of order l
J	Polar moment of inertia
$\mathbf{J}$, J	Angular momentum
K	Bulk modulus (modulus of compression)
K	Anisotropy constant
k	Torsion constant

k	Thermal conductivity
L	Inductance
l	Length
M	Mutual inductance
M	Molar mass
M	Mass
M	Torque
$\mathbf{m}$	Magnetic moment
N	Number of turns
N	Number of photons
$\|N\|$	Demagnetization tensor
N_f	Fluxmetric or ballistic demagnetization factor
N_m	Magnetometric demagnetization factor
n	Density of turns
P_l	Legendre polynomial
P_l^m	Associated Legendre polynomial
p	Pressure
$\mathbf{p}$	Dipole density
$\mathbf{p}$	Volume force density
Q	Factor of merit
Q_0	Magnetic charge
Q_l	Strength of magnetic 2^l-pole
q	Charge density
R	Resistance
R_H	Hall constant
r	Reduced resistivity
$\mathbf{r}$	Radius vector
S	Stress
s	Stress density
$\mathbf{s}$	Angular momentum
T	Temperature
T	Volume
T	Torque
t	Time

V	Magnetic potential
V	Voltage
V	Volume
W	Power (energy per unit time)
Y	Young's modulus of elasticity
Y_l^m	Spherical harmonic
Z	Impedance
α	Dimensional ratio
α	Direction cosine
β	Dimensional ratio
β	Direction cosine
γ	Dimensional ratio
γ	Gyromagnetic ratio
Δ	Laplace operator
δ	Difference operator
δ	Packing density (volume fraction occupied by matter)
ε	Strain
ε	Absorption coefficient
ζ	Dimensional ratio
ζ	Heat transfer coefficient
η	Electrical resistivity
θ	Polar angle
θ	Debye temperature
ϑ	Reduced temperature
κ	Efficiency
λ	Packing density (volume fraction occupied by matter)
λ	Magnetostriction constant
μ	Absolute permeability
μ_0	Permeability of vacuum
μ_r	Relative permeability (vacuum = 1)
ξ	Surfacial current density
ξ	Poisson's ratio of contraction

ξ	Lagrange's undetermined multiplier
ρ	Radius
ρ_V	Verdet's constant
σ	Area
σ	Absorbing cross-section
τ	Area
τ	Time constant (relaxation time)
τ	Current density
Φ	Magnetic flux
φ	Azimuthal angle
χ	Magnetic susceptibility
ψ	Electric potential
ω	Angular frequency
∇	Gradient operator

CONTENTS

CHAPTER 1

THE MAGNETIC POTENTIAL

§ 1. INTRODUCTION

Many methods used in the study of magnetic phenomena in solids are based on the observation of the effect of an external magnetic field on the material under investigation. Therefore a great deal of attention will be paid to the generation of magnetic fields and the potential theory connected with it. We shall use the concept of magnetic charges and charge densities and relate these to electrical currents. This approach is made for convenience of calculation and does not imply any judgement on the physical reality of magnetic monopoles.

The quantitative treatment of the theory involves the choice of a unit system. In this book the mksA- or Giorgi-system will be used. However, for convenience of those who are used to other systems a comparison between this and the current cgs-systems is made in the next section.

§ 2. COMPARISON OF UNIT SYSTEMS

For giving a numerical value to a magnetic quantity the electromagnetic centimeter-gram-second system (emcgs-system) is widely used. Its principal feature is that the magnetic permeability μ_0 of the vacuum is put equal to one. Since

$$\mu_0 \varepsilon_0 c^2 = 1$$

the dielectric constant ε_0 of the vacuum is consequently equal to c^{-2}, c being the velocity of light in vacuum.

In the cgs-system of Gauss the permeability and the dielectric constant of the vacuum are both equal to one. The consequence is that a factor c^{-1} appears in the basic formulae. This system is also widely used in magnetism.

The meter-kilogram-second-Ampère system of Giorgi (mksA-system) is very convenient if both electric and magnetic quantities are involved in the calculation. It is rationalized, which means that the factor 4π that occurs in the relation between electric field-strength and polarization and also between magnetic fieldstrength and magnetization in the cgs-systems, is no longer present in the corresponding relations in the mksA-system. This has been obtained by putting the permeability of the vacuum equal to

$$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{H\,m^{-1}}.$$

The dielectric constant then is

$$\varepsilon_0 = 1/\mu_0 c^2 = 8.855 \times 10^{-12}\ \mathrm{F\,m^{-1}}.$$

A unit charge emits a unit flux in this system rather than a flux 4π as it does in the cgs-systems.

The mksA-system has become widespread during the last decade, mainly because of its practical possibilities. It is adopted in this book since the measurement of magnetic quantities is very often done by methods based on the interaction between magnetic fields and electric currents.

A comparison between a few basic formulae as they occur in the above mentioned systems, together with a conversion table is given in Appendix 1. A detailed survey on the use of these systems may be found in the book by CORNELIUS [1961].

§ 3. POTENTIAL DUE TO MAGNETIC CHARGE SYSTEMS

§ 3.1. *Single magnetic charge*

Experimenting with long magnetized wires thus simulating free magnetic charges Coulomb showed that the force **F** between two magnetic charges Q_0 and Q_0' is given by

$$\boldsymbol{F} \propto \frac{Q_0 Q_0'}{r^3} \boldsymbol{r}, \tag{1.1}$$

where $\boldsymbol{r}$ is the vector connecting the two charges. From eq. (1.1) it follows directly that along a closed path

$$\oint \boldsymbol{F} \cdot \mathrm{d}\boldsymbol{r} = 0, \tag{1.2}$$

which means that the line integral of $\boldsymbol{F}$ from one point to another is irrespective of the path chosen.

In the mksA-system we have

$$\boldsymbol{F} = \frac{1}{4\pi\mu_0} \frac{Q_0 Q_0'}{r^3} \boldsymbol{r} = \boldsymbol{H} Q_0', \tag{1.3}$$

where $\boldsymbol{H}$ is the force per unit charge exerted by Q_0 on the charge located at a distance $\boldsymbol{r}$. This force is called the magnetic fieldstrength at a distance $\boldsymbol{r}$ from the single charge or monopole of strength Q_0. The potential V_A at a point A is defined as the work per unit charge required to bring a positive charge from infinity (where $V = 0$ by definition) to A:

$$V_A = \int_{\infty}^{A} -\boldsymbol{H} \cdot \mathrm{d}\boldsymbol{r} = \frac{1}{4\pi\mu_0} \frac{Q_0}{r}, \tag{1.4}$$

where the fieldstrength $\boldsymbol{H}$ at any point is related to the potential V by

$$\boldsymbol{H} = -\operatorname{grad} V. \tag{1.5}$$

The integral of eq. (1.4) does not depend on the path chosen.

The potential difference between two points A and B is

$$V_A - V_B = \int_{A}^{B} \boldsymbol{H} \cdot \mathrm{d}\boldsymbol{r} \tag{1.6}$$

and if the integration path is closed

$$\oint \boldsymbol{H} \cdot \mathrm{d}\boldsymbol{r} = 0, \tag{1.7}$$

which also follows directly from eq. (1.2).

Equation (1.7) only holds if the path does not enclose an electric current (see § 4 of this chapter).

§ 3.2. *Additivity of fields and potentials*

If more point charges Q_i are present the force exerted on a positive unit charge at a point A is the resultant of the forces due to each charge individually:

$$\boldsymbol{F} = \sum \boldsymbol{F}_i,$$

where $\boldsymbol{F}_i$ is the force due to the charge Q_i. Hence the resultant fieldstrength at any point is

$$\boldsymbol{H} = \sum \boldsymbol{H}_i,$$

where $\boldsymbol{H}_i$ is the fieldstrength due to the charge Q_i.

The total potential at the point A is

$$\begin{aligned} V_A &= -\int_{\infty}^{A} (\sum \boldsymbol{H}_i) \cdot \mathrm{d}\boldsymbol{r} = \\ &= -\sum \int_{\infty}^{A} \boldsymbol{H}_i \cdot \mathrm{d}\boldsymbol{r} = \\ &= \sum V_i, \end{aligned} \tag{1.8}$$

where V_i is the potential at A due to the charge Q_i.

We thus see that the potentials in a point due to several sources separately must be added to give the potential due to the whole system of sources.

§ 3.3. *Laplace's and Poisson's equations*

It can be shown from eq. (1.3) that

$$\oint \boldsymbol{H} \cdot \mathrm{d}\boldsymbol{A} = Q_0/\mu_0\,, \tag{1.9}$$

where dA is a surface element of a closed surface enclosing the point charge Q_0.

If the charge is not concentrated in a point but distributed as a charge density q inside the closed surface the right-hand side of eq. (1.9) can be written as

$$\frac{Q_0}{\mu_0} = \frac{1}{\mu_0}\int q\,\mathrm{d}v\,, \tag{1.10}$$

where dv is an element of the volume enclosed by the surface. Application of Gauss' theorem to the left-hand side of eq. (1.9) gives

$$\oint \boldsymbol{H} \cdot \mathrm{d}\boldsymbol{A} = \int \operatorname{div} \boldsymbol{H}\,\mathrm{d}v\,. \tag{1.11}$$

Since the right-hand sides of eqs. (1.10) and (1.11) are equal for any surface the integrands must be equal which gives

$$\operatorname{div} \boldsymbol{H} = q/\mu_0\,. \tag{1.12}$$

Using eq. (1.5) we have then

$$\operatorname{div}\operatorname{grad} V = \Delta V = -\,q/\mu_0\,, \tag{1.13}$$

which is called Poisson's equation.

In a region where the charge density equals zero

$$\Delta V = 0\,, \tag{1.14}$$

which is called Laplace's equation.

Equation (1.14) will be used in Chapter 2 to calculate the magnetic field due to a given system of magnetic charges. The procedure is as follows: The magnetic charges provide a set of boundary con-