Shiing-shen Chern

Selected Papers

Volume III

陈省身论文集 第3卷 [英]

Springer-Verlag

World Publishing Corp

Shiing-shen Chern

Selected Papers

Volume III

Springer-Verlag

World Publishing Corp

Shiing-shen Chern
Mathematical Sciences Research Institute
Berkeley, CA 94720
USA

Mathematics Subject Classification (1980): 01A75

Library of Congress Cataloging-in-Publication Data
(Revised for Volumes II, III, and IV)

Chern, Shiing-Shen, 1911–
Selected papers.

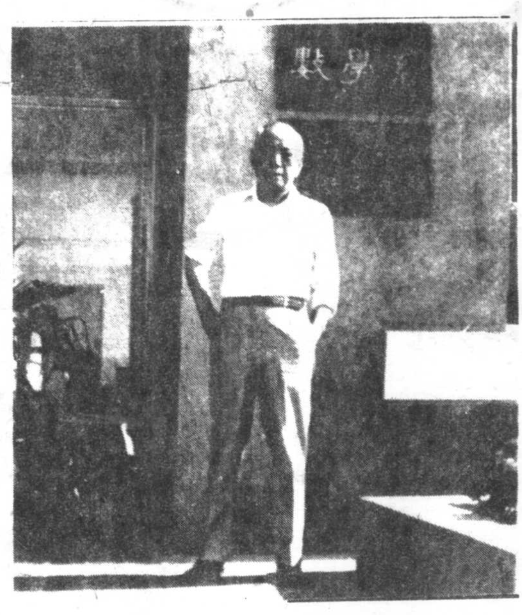
Published in conjunction with the International Symposium in Global Analysis and Global Geometry to be held in Berkeley, Calif., June 1979.
"Bibliography of the publications of S.S. Chern": p.
1. Chern, Shiing-Shen, 1911– . 2. Geometry, Differential. 3. Global analysis (Mathematics)
I. Title.
QA641.C47 514'.7 78-22093
ISBN 0-387-90339-9 (v. 1)

Reprinted by World Publishing Corporation, Beijing, 1990
for distribution and sale in The People's Republic of China only
ISBN 7-5062-0775-3

ISBN 0-387-96816-4 Springer-Verlag New York Berlin Heidelberg
ISBN 3-540-96816-4 Springer-Verlag Berlin Heidelberg New York

陳省身数学
論文選集

With Father, 1917



At Nankai Institute of Mathematics, 1986

Bibliography of the Publications of S.S. Chern

Note: **Boldface** numbers at the end of each entry denote the volume in which the entry appears.

I. *Books and Monographs*

1. *Topics in Differential Geometry* (mimeographed), Institute for Advanced Study, Princeton (1951), 106 pp. **(IV)**
2. *Differentiable Manifolds* (mimeographed). University of Chicago, Chicago (1953), 166 pp.
3. *Complex Manifolds*
   a. University of Chicago, Chicago (1956), 195 pp.
   b. University of Recife, Recife, Brazil (1959), 181 pp.
   c. Russian translation, Moscow (1961), 239 pp.
4. *Studies in Global Geometry and Analysis* (Editor), Mathematical Association of America (1967), 200 pp.
5. *Complex Manifolds without Potential Theory*, van Nostrand (1968), 92 pp. Second edition, revised, Springer-Verlag (1979) 152 pp.
6. *Minimal Submanifolds in a Riemannian Manifold* (mimeographed), University of Kansas Lawrence (1968), 55 pp. **(IV)**
7. (with Wei-huan Chen) *Differential Geometry Notes*, in Chinese, Beijing University Press (1983), 321 pp.
8. *Studies in Global Differential Geometry* (Editor), Mathematical Association of America (1988), 350 pp.

II. *Papers*

1932

[1] Pairs of plane curves with points in one-to-one correspondence. *Science Reports Nat. Tsing Hua Univ.* **1** (1932) 145–153. **(II)**

1935

*[2] Triads of rectilinear congruences with generators in correspondence. *Tohoku Math. J.* **40** (1935) 179–188.

[3] Associate quadratic complexes of a rectilinear congruence. *Tohoku Math. J.* **40** (1935) 293–316. **(II)**

[4] Abzählungen für Gewebe. *Abh. Math. Sem. Univ. Hamburg* **11** (1935) 163–170. **(I)**

1936

[5] Eine Invariantentheorie der Dreigewebe aus r-dimensionalen Mannigfaltigkeiten im R_{2r}. *Abh. Math. Sem. Univ. Hamburg* **11** (1936) 333–358. **(I)**

---

* Does not appear in these volumes.

BIBLIOGRAPHY OF THE PUBLICATIONS OF S.S. CHERN

1937

[6] Sur la géométrie d'une équation différentielle du troisième ordre. *C. R. Acad. Sci. Paris* **204** (1937) 1227–1229. **(II)**

[7] Sur la possibilité de plonger un espace à connexion projective donné dans un espace projectif. *Bull. Sci. Math.* **61** (1937) 234–243. **(I)**

1938

[8] On projective normal coördinates. *Ann. of Math.* **39** (1938) 165–171. **(II)**

[9] On two affine connections. *J. Univ. Yunnan* **1** (1938) 1–18. **(II)**

1939

[10] Sur la géométrie d'un système d'équations différentielles du second ordre. *Bull. Sci. Math* **63** (1939) 206–212. **(II)**

1940

*[11] The geometry of higher path-spaces. *J. Chin. Math. Soc.* **2** (1940) 247–276.

[12] Sur les invariants intégraux en géométrie. *Science Reports Nat. Tsing Hua Univ.* **4** (1940) 85–95. **(II)**

[13] The geometry of the differential equation $y''' = F(x, y', y'')$. *Science Reports Nat. Tsing Hua Univ.* **4** (1940) 97–111. **(I)**

[14] Sur une généralisation d'une formule de Crofton. *C.R. Acad. Sci. Paris* **210** (1940) 757–758. **(II)**

[15] (with C.T. Yen) Sulla formula principale cinematica dello spazio ad *n* dimensioni. *Boll. Un. Mat. Ital.* **2** (1940) 434–437. **(II)**

*[16] Generalization of a formula of Crofton. *Wuhan Univ. J. Sci.* **7** (1940) 1–16.

1941

[17] Sur les invariants de contact en géométrie projective différentielle. *Acta Pontif. Acad. Sci.* **5** (1941) 123–140. **(II)**

1942

[18] On integral geometry in Klein spaces. *Ann. of Math.* **43** (1942) 178–189. **(I)**

[19] On the invariants of contact of curves in a projective space of *N* dimensions and their geometrical interpretation. *Acad. Sinica Sci. Record* **1** (1942) 11–15. **(I)**

[20] The geometry of isotropic surfaces. *Ann. of Math.* **43** (1942) 545–559. **(II)**

[21] On a Weyl geometry defined from an $(n-1)$-parameter family of hypersurfaces in a space of *n* dimensions. *Acad. Sinica Sci. Record* **1** (1942) 7–10. **(I)**

1943

[22] On the Euclidean connections in a Finsler space. *Proc. Nat. Acad. Sci. USA*, **29** (1943) 33–37. **(II)**

[23] A generalization of the projective geometry of linear spaces. *Proc. Nat. Acad. Sci. USA*, **29** (1943) 38–43. **(I)**

1944

[24] Laplace transforms of a class of higher dimensional varieties in a projective space of *n* dimensions. *Proc. Nat. Acad. Sci. USA*, **30** (1944) 95–97. **(II)**

[25] A simple intrinsic proof of the Gauss–Bonnet formula for closed Riemannian manifolds. *Ann of Math.* **45** (1944) 747–752. **(I)**

[26] Integral formulas for the characteristic classes of sphere bundles. *Proc. Nat. Acad. Sci. USA* **30** (1944) 269–273. **(II)**

*[27] On a theorem of algebra and its geometrical application. *J. Indian Math. Soc.* **8** (1944) 29–36.

BIBLIOGRAPHY OF THE PUBLICATIONS OF S.S. CHERN

1945

*[28] On Grassmann and differential rings and their relations to the theory of multiple integrals. *Sankhya* **7** (1945) 2–8.

[29] Some new characterizations of the Euclidean sphere. *Duke Math. J.* **12** (1945) 279–290. **(II)**

[30] On the curvature integra in a Riemannian manifold. *Ann. of Math.* **46** (1945) 674–684. **(I)**

*[31] On Riemannian manifolds of four dimensions. *Bull. Amer. Math. Soc.* **51** (1945) 964–971.

1946

[32] Some new viewpoints in the differential geometry in the large. *Bull. Amer. Math. Soc.* **52** (1946) 1–30. **(II)**

[33] Characteristic classes of Hermitian manifolds. *Ann. of Math.* **47** (1946) 85–121. **(I)**

1947

[34] (with H.C. Wang). Differential geometry in symplectic space I. *Science Report Nat. Tsing Hua Univ.* **4** (1947) 453–477. **(II)**

[35] Sur une classe remarquable de variétés dans l'espace projectif à *N* dimensions. *Science Reports Nat. Tsing Hua Univ.* **4** (1947) 328–336. **(I)**

*[36] On the characteristic classes of Riemannian manifolds. *Proc. Nat. Acad. Sci USA.* **33** (1947) 78–82.

*[37] Note of affinely connected manifolds. *Bull. Amer. Math. Soc.* **53** (1947) 820–823; correction ibid **54** (1948) 985–986.

*[38] On the characteristic ring of a differentiable manifold. *Acad. Sinica. Sci. Record* **2** (1947) 1–5.

1948

[39] On the multiplication in the characteristic ring of a sphere bundle. *Ann. of Math.* **49** (1948) 362–372. **(I)**

[40] Note on projective differential line geometry. *Acad. Sinica Sci. Record* **2** (1948) 137–139. **(II)**

*[41] (with Y.L. Jou) On the orientability of differentiable manifolds. *Science Reports Nat. Tsing Hua Univ.* **5** (1948) 13–17.

[42] Local equivalence and Euclidean connections in Finsler spaces. *Science Reports Nat. Tsing Hua Univ.* **5** (1948) 95–121. **(II)**

1949

[43] (with Y.F. Sun). The imbedding theorem for fibre bundles. *Trans. Amer. Math. Soc* **67** (1949) 286–303. **(II)**

*[44] (with S.T. Hu) Parallelisability of principal fibre bundles. *Trans. Amer. Math. Soc,* **67** (1949) 304–309.

1950

[45] (with E. Spanier). The homology structure of sphere bundles. *Proc. Nat. Acad. Sci. USA,* **36** (1950) 248–255. **(II)**

[46] Differential geometry of fiber bundles. *Proc. Int. Congr. Math.* (1950) **II** 397–411. **(II)**

1951

[47] (with E. Spanier). A theorem on orientable surfaces in four-dimensional space. *Comm. Math. Helv.* **25** (1951) 205–209. **(I)**

1952

[48] On the kinematic formula in the Euclidean space of N dimensions. *Amer. J. Math* **74** (1952) 227–236. **(II)**

[49] (with C. Chevalley). Elie Cartan and his mathematical work. *Bull. Amer. Math. Soc.* **58** (1952) 217–250. **(II)**

[50] (with N.H. Kuiper) Some theorems on the isometric imbedding of compact Riemann manifolds in Euclidean space. *Ann. of Math.* **56** (1952) 422–430. **(II)**

1953

[51] On the characteristic classes of complex sphere bundles and algebraic varieties. *Amer. J. of Math.*, **75** (1953) 565–597. **(I)**

*[52] Some formulas in the theory of surfaces. *Boletin de la Sociedad Matematica Mexicana*, **10** (1953) 30–40.

[53] Relations between Riemannian and Hermitian geometries. *Duke Math. J.*, **20** (1953) 575–587. **(II)**

1954

[54] Pseudo-groupes continus infinis *Colloque de Geom. Diff.* Strasbourg (1954) 119–136. **(I)**

[55] (with P. Hartman and A. Wintner) On isothermic coordinates. *Comm. Math. Helv.* **28** (1954) 301–309. **(I)**

1955

[56] La géometrie des sous-variétés d'un espace euclidien à plusieurs dimensions. *l'Ens. Math.*, **40** (1955) 26–46. **(II)**

[57] An elementary proof of the existence of isothermal parameters on a surface. *Proc. Amer. Math. Soc.*, **6** (1955) 771–782. **(II)**

[58] On special W-surfaces. *Proc. Amer. Math. Soc.*, **6** (1955) 783–786. **(II)**

[59] On curvature and characteristic classes of a Riemann manifold. *Abh. Math. Sem. Univ. Hamburg* **20** (1955) 117–126. **(II)**

1956

*[60] Topology and differential geometry of complex manifolds. *Bull. Amer. Math. Soc.*, **62** (1956) 102–117.

1957

[61] On a generalization of Kähler geometry. *Lefschetz jubilee volume*. Princeton Univ. Press (1957) 103–121. **(I)**

[62] (with R. Lashof) On the total curvature of immersed manifolds. *Amer. J. of Math.* **79** (1957) 306–318. **(I)**

[63] (with F. Hirzebruch and J-P. Serre) On the index of a fibered manifold. *Proc. Amer. Math. Soc.*, **8** (1957) 587–596. **(I)**

[64] A proof of the uniqueness of Minkowski's problem for convex surfaces. *Amer. J. of Math.*, **79** (1957) 949–950. **(II)**

1958

*[65] Geometry of submanifolds in complex projective space. *Symposium International de Topologia Algebraica* (1958) 87–96.

[66] (with R.K. Lashof) On the total curvature of immersed manifolds, II. *Michigan Math. J.* **5** (1958) 5–12. **(II)**

[67] Differential geometry and integral geometry. *Proc. Int. Congr. Math. Edinburgh* (1958) 441–449. **(II)**

1959

[68] Integral formulas for hypersurfaces in Euclidean space and their applications to uniqueness theorems. *J. of Math. and Mech.* **8** (1959) 947–956. **(I)**

1960

[69] (with J. Hano and C.C. Hsiung) A uniqueness theorem on closed convex hypersurfaces in Euclidean space. *J. of Math. and Mech.* **9** (1960) 85–88. **(I)**

[70] Complex analytic mappings of Riemann surfaces I. *Amer. J. of Math.* **82** (1960) 323–337. **(I)**

[71] The integrated form of the first main theorem for complex analytic mappings in several complex variables. *Ann. of Math.* **71** (1960) 536–551. **(I)**

*[72] Geometrical structures on manifolds. *Amer. Math. Soc. Pub.* (1960) 1–31.

*[73] La géométrie des hypersurfaces dans l'espace euclidean. *Seminaire Bourbaki*, **193** (1959–1960).

*[74] Sur les métriques Riemanniens compatibles avec une reduction du groupe structural. *Séminaire Ehresmann*, January 1960.

1961

[75] Holomorphic mappings of complex manifolds. *L'Ens. Math.* **7** (1961) 179–187. **(I)**

1962

*[76] Geometry of quadratic differential form. *J. of SIAM* **10** (1962) 751–755.

1963

[77] (with C.C. Hsiung) On the isometry of compact submanifolds in Euclidean space. *Math. Annalen* **149** (1963) 278–285. **(II)**

[78] Pseudo-Riemannian geometry and Gauss–Bonnet formula. *Academia Brasileira de Ciencias* **35** (1963) 17–26. **(I)**

1965

[79] Minimal surfaces in an Euclidean space of N dimensions. *Differential and Combinatorial Topology*, Princeton Univ. Press (1965) 187–198. **(I)**

[80] (with R. Bott) Hermitian vector bundles and the equidistribution of the zeroes of their holomorphic sections. *Acta. Math.* **114** (1965) 71–112. **(II)**

[81] On the curvatures of a piece of hypersurface in Euclidean space. *Abh. Math. Sem. Univ. Hamburg* **29** (1965) 77–91. **(III)**

[82] On the differential geometry of a piece of submanifold in Euclidean space. *Proc. of U.S.–Japan Seminar in Diff. Geom.* (1965) 17–21. **(III)**

1966

[83] Geometry of G-structures. *Bull. Amer. Math. Soc.* **72** (1966) 167–219. **(III)**

[84] On the kinematic formula in integral geometry. *J. of Math. and Mech.* **16** (1966) 101–118. **(III)**

*[85] Geometrical structures on manifolds and submanifolds. *Some Recent Advances in Basic Sciences*, Yeshiva Univ. Press (1966) 127–135.

1967

[86] (with R. Osserman) Complete minimal surfaces in Euclidean n-space. *J. de l'Analyse Math.* **19** (1967) 15–34. **(III)**

[87] Einstein hypersurfaces in a Kählerian manifold of constant holomorphic curvature. *J. Diff. Geom.* **1** (1967) 21–31. **(III)**

1968

[88] On holomorphic mappings of Hermitian manifolds of the same dimension. *Proc. Symp. Pure Math.* **11**. Entire Functions and Related Parts of Analysis (1968) 157-170. **(I)**

1969

[89] Simple proofs of two theorems on minimal surfaces. *L'Ens. Math.* **15** (1969) 53-61. **(I)**

1970

[90] (with H. Levine and L. Nirenberg) Intrinsic norms on a complex manifold. *Global analysis*, Princeton Univ. Press (1970) 119-139. **(I)**

[91] (with M. do Carmo and S. Kobayashi) Minimal submanifolds of a sphere with second fundamental form of constant length. *Functional Analysis and Related Fields*, Springer-Verlag (1970) 59-75. **(I)**

[92] (with R. Bott) Some formulas related to complex transgression. *Essays on Topology and Related Topics*, Springer-Verlag, (1970) 48-57. **(I)**

*[93] Holomorphic curves and minimal surfaces. *Carolina Conference Proceedings* (1970) 28 pp.

[94] On minimal spheres in the four sphere. Studies and Essays Presented to Y. W. Chen, Taiwan, (1970) 137-150. **(I)**

[95] Differential geometry: Its past and its future. *Actes Congrès Intern. Math.* (1970) **1**, 41-53. **(III)**

[96] On the minimal immersions of the two-sphere in a space of constant curvature. *Problems in Analysis*, Princeton Univ. Press, (1970) 27-40. **(III)**

1971

[97] Brief survey of minimal submanifolds. *Differentialgeometrie im Grossen*, W. Klingenberg (ed.), **4** (1971) 43-60. **(III)**

*[98] (with J. Simons) Some cohomology classes in principal fibre bundles and their application to Riemannian geometry. *Proc. Nat. Acad. Sci. USA*, **68** (1971) 791-794.

1972

[99] Holomorphic curves in the plane. *Diff. Geom., in honor of K. Yano*, (1972) 73-94. **(III)**

*[100] Geometry of characteristic classes. *Proc. 13th Biennial Sem. Canadian Math. Congress*, (1972) 1-40. Also pub. in Russian translation.

1973

[101] Meromorphic vector fields and characteristic numbers. *Scripta Math.* **29** (1973) 243-251. **(I)**

*[102] The mathematical works of Wilhelm Blaschke. *Abh. Math. Sem. Univ. Hamburg* **39** (1973) 1-9.

1974

[103] (with. J. Simons) Characteristic forms and geometrical invariants. *Ann. of Math.* **99** (1974) 48-69. **(I)**

[104] (with M. Cowen, A. Vitter III) Frenet frames along holomorphic curves. *Proc. of Conf. on Value Distribution Theory*, Tulane Univ. (1974) 191-203. **(III)**

[105] (with J. Moser) Real hypersurfaces in complex manifolds. *Acta. Math.* **133** (1974) 219-271. **(III)**

1975

[106] (with S.I. Goldberg) On the volume decreasing property of a class of real harmonic mappings. *Amer. J. of Math.* **97** (1975) 133-147. **(III)**

[107] On the projective structure of a real hypersurface in C_{n+1}. *Math. Scand.* **36** (1975) 74–82. **(III)**

1976

[108] (with J. White) Duality properties of characteristic forms. *Inv. Math.* **35** (1976) 285–297. **(III)**

1977

*[109] Circle bundles. *Geometry and topology. III.* Latin Amer. School of Math. Lecture Notes in Math. Springer-Verlag. **597** (1977) 114–131.

*[110] (with P.A. Griffiths) Linearization of webs of codimension one and maximum rank. *Proc. Int. Symp. on Algebraic Geometry, Kyoto* (1977) 85–91.

1978

[111] On projective connections and projective relativity. *Science of Matter*, dedicated to Ta-you Wu. (1978) 225–232. **(III)**

[112] (with P.A. Griffiths) Abel's theorem and webs. *Jber. d. Dt. Math. Verein.* **80** (1978) 13–110. **(III)**

[113] (with P.A. Griffiths) An inequality for the rank of a web and webs of maximum rank. *Annali Sc. Norm. Super. Pisa, Serie IV*, **5** (1978) 539–557. **(III)**

[114] Affine minimal hypersurfaces. *Minimal Submanifolds and Geodesics*. Kaigai Publications. Ltd. (1978) 1–14. **(III)**

1979

*[115] Herglotz's work on geometry. *Ges. Schriften Gustav Herglotz*. Göttingen (1979) xx–xxi.

[116] (with C.L. Terng) An analogue of Bäcklund's theorem in affine geometry. *Rocky Mountain J. Math.* **10** (1979) 105–124. **(III)**

[117] From triangles to manifolds. *Amer. Math. Monthly* **86** (1979) 339–349. **(III)**

[118] (with C.K. Peng) Lie groups and KdV equations. *Manuscripta Math.* **28** (1979) 207–217. **(III)**

1980

[119] General relativity and differential geometry. *Some Strangeness in the Proportion: A Centennial Symp. to Celebrate the Achievements of Albert Einstein*. *Harry Woolf* (ed.), Addison-Wesley Publ. (1980) 271–287. **(III)**

[120] (with W.M. Boothby and S.P. Wang) The mathematical work of H.C. Wang. *Bull. Inst. of Math*, **8** (1980) xiii–xxiv. **(III)**

*[121] Geometry and physics. *Math. Medley*, Singapore, **8** (1980) 1–6.

*[122] (with R. Bryant and P.A. Griffiths) Exterior differential systems. *Proc. of 1980 Beijing DD-Symposium*, (1980) 219–338.

1981

[123] Geometrical interpretation of the sinh-Gordon equation. *Annales Polonici Mathematici* **39** (1981) 63–69. **(IV)**

[124] (with P.A. Griffiths) Corrections and addenda to our paper: "Abel's theorem and webs." *Jber. d. Dt. Math.-Verein.* **83** (1981) 78–83. **(III)**

[125] (with R. Osserman) Remarks on the Riemannian metric of a minimal submanifold. *Geometry Symposium Utrecht 1980*, Springer Lecture Notes **894** (1981) 49–90. **(IV)**

[126] (with J. Wolfson) A simple proof of Frobenius theorem. *Manifolds and Lie Groups, Papers in Honor of Y. Matsushima*. Birkhäuser (1981) 67–69. **(IV)**

[127] (with K. Tenenblat) Foliations on a surface of constant curvature and modified Korteweg–de Vries equations. *J. Diff. Geom.* **16** (1981) 347–349. **(IV)**

[128] (with C.K. Peng) On the Bäcklund transformations of KdV equations and modified KdV equations. *J. of China Univ. of Sci. and Tech.*, **11** (1981) 1–6. **(IV)**

1982

[129] Web geometry. *Proc. Symp. in Pure Math.* **39** (1983) 3–10. **(IV)**

[130] Projective geometry, contact transformations, and *CR*-structures. *Archiv der Math.* **38** (1982) 1–5. **(IV)**

1983

[131] (with J. Wolfson) Minimal surfaces by moving frames. *Amer. J. Math.* **105** (1983) 59–83. **(IV)**

[132] On surfaces of constant mean curvature in a three-dimensional space of constant curvature. *Geometric Dynamics*, Springer Lecture Notes **1007** (1983) 104–108. **(IV)**

1984

[133] Deformation of surfaces preserving principal curvatures, *Differential Geometry and Complex Analysis*, Volume in Memory of H. Rauch, Springer-Verlag (1984) 155–163. **(IV)**

1985

[134] (with R. Hamilton) On Riemannian metrics adapted to three-dimensional contact manifolds. *Arbeitstagung Bonn 1984* Springer Lecture Notes **1111** (1985) 279–308. **(IV)**

[135] (with J. Wolfson) Harmonic maps of S^2 into a complex Grassmann manifold. *Proc. Nat. Acad. Sci.* USA **82** (1985) 2217–2219. **(IV)**

[136] Moving frames, *Soc. Math. de France*, Astérisque, (1985) 67–77. **(IV)**

*[137] Wilhelm Blaschke and web geometry, Wilhelm Blaschke—Gesammelte Werke. **5**, Thales Verlag, (1985) 25–27.

*[138] The mathematical works of Wilhelm Blaschke—an update. Thales Verlag, (1985), 21–23.

1986

[139] (with K. Tenenblat) Pseudospherical surfaces and evolution equations. *Studies in Applied Math.* MIT **74** (1986) 55–83. **(IV)**

[140] On a conformal invariant of three-dimensional manifolds. *Aspects of Mathematics and Its Applications* Elsevier Science Publishers B.V. (1986) 245–252. **(IV)**

*[141] (with P.A. Griffiths) Pfaffian systems in involution. *Proceedings of 1982 Changchun Symposium on Differential Geometry and Differential Equations*, Science Press, China, (1986) 233–256.

1987

[142] (with J. Wolfson) Harmonic maps of the two-sphere into a complex Grassmann manifold II. *Ann. of Math.* **125** (1987) 301–335. **(IV)**

[143] (with T. Cecil) Tautness and Lie Sphere geometry *Math. Annalen*, Volume Dedicated to F. Hirzebruch **278** (1987) 381–399. **(IV)**

1988

[144] Vector bundles with a connection. *Studies in Global Differential Geometry*, MAA, no. 27 (1988), 1–26.

1989

[145] (with T. Cecil) Dupin submanifolds in Lie sphere geometry, to appear in *Differential Geometry and Topology*, Springer Lecture Notes 1989.

[146] Historical remarks on Gauss–Bonnet, to appear in Moser Volume, Academic Press.

[147] An introduction to Dupin submanifolds, to appear in Do Carmo Volume.

Contents

Bibliography of the Publications of S.S. Chern vii

[81]* On the Curvatures of a Piece of Hypersurface in Euclidean Space ... 1
[82] On the Differential Geometry of a Piece of Submanifold in Euclidean Space 17
[83] The Geometry of G-Structures 23
[84] On the Kinematic Formula in Integral Geometry 77
[86] Complete Minimal Surfaces in Euclidean n-Space (with Robert Osserman) 95
[87] Einstein Hypersurfaces in a Kählerian Manifold of Constant Holomorphic Curvature 115
[95] Differential Geometry: Its Past and Its Future 127
[96] On the Minimal Immersions of the Two-sphere in a Space of Constant Curvature 141
[97] Brief Survey of Minimal Submanifolds 155
[99] Holomorphic Curves in the Plane 173
[104] Frenet Frames Along Holomorphic Curves (with Michael J. Cowen and Albert L. Vitter III) 195
[105] Real Hypersurfaces in Complex Manifolds (with J.K. Moser) 209
[106] On the Volume Decreasing Property of a Class of Real Harmonic Mappings (with Samuel I. Goldberg) 263
[107] On the Projective Structure of a Real Hypersurface in C_{n+1} 278
[108] Duality Properties of Characteristic Forms (with James White) 287
[111] On Projective Connections and Projective Relativity 301
[112] Abel's Theorem and Webs (with Phillip Griffiths) 309
[124] Corrections and Addenda to our Paper: Abel's Theorem and Webs (with P. Griffiths) 398
[113] An Inequality for the Rank of a Web and Webs of Maximum Rank (with Phillip A. Griffiths) 405
[114] Affine Minimal Hypersurfaces 425
[116] An Analogue of Bäcklund's Theorem in Affine Geometry (with Chuu-Lian Terng) 439

* Numbers in brackets refer to the Bibliography of the Publications of S.S. Chern (see pages vii–xiv).

[117] From Triangles to Manifolds 459
[118] Lie Groups and KdV Equations (with Chia-kuei Peng) 471
[119] General Relativity and Differential Geometry 483
[120] The Mathematical Work of H.C. Wang (with W.M. Boothby and S.P. Wang) 495

Permissions 503

On the Curvatures of a Piece of Hypersurface in Euclidean Space

By SHIING-SHEN CHERN[1])

Dedicated to Professor EMANUEL SPERNER on His Sixtieth Birthday

Let $z = z(x, y)$ be a C^2-surface in euclidean three-space, defined over the disk $x^2 + y^2 < R^2$ in the (x, y)-plane. Let H and K denote respectively its mean and Gaussian curvatures. In 1955 HEINZ ([3], cf. Bibliography at the end) proved the following theorem:

If $|H| \geqq c > 0$, then $R \leqq \frac{1}{c}$. If $K \geqq c > 0$, then $R \leqq \frac{1}{\sqrt{c}}$. If $K \leqq -c < 0$, then $R \leqq e\left(\frac{3}{c}\right)^{\frac{1}{2}}$, ($c$ = const. in all cases).

The purpose of this paper is to extend this theorem to a hypersurface in an euclidean space of dimension $m + 1$. (Cf. Theorems 1, 2, 4 below). Having the global problems in mind, we will consider an immersed hypersurface and establish, in so far as possible, the intermediary results in this general setting. In this sense some of our formulations are more general even in the classical case $m = 2$.

1. Algebraic Preliminaries

Let $x: M \to E$ be an immersion of an oriented manifold M of class two and dimension m into an euclidean space E of dimension $m + 1$. We will consider x as a vector-valued function on M. For $x, y \in E$ we denote by (x, y) their scalar product. Let $x e_1, \ldots, e_{m+1}$ be orthonormal frames, such that $x = x(p)$, $p \in M$, and e_{m+1} is the unit normal vector at $x(p)$. Then we have

$$(1) \qquad \begin{aligned} dx &= \sum_A \omega_A e_A, \\ de_A &= \sum_B \omega_{AB} e_B, \end{aligned}$$

where

$$(2) \qquad \omega_{AB} + \omega_{BA} = 0, \quad \omega_{m+1} = 0,$$

---

[1]) This work is done under partial support by grants from the National Science Foundation and Office of Naval Research, USA.

and the ω structure equations

$$(3)\qquad \begin{aligned} d\omega_A &= \sum_B \omega_B \wedge \omega_{BA},\\ d\omega_{AB} &= \sum_C \omega_{AC} \wedge \omega_{CB}. \end{aligned}$$

(Throughout this paper we will agree on the following ranges of indices:

$$(4)\qquad \begin{aligned} 1 \leqq A, B, C \leqq m+1\\ 1 \leqq i, j, k, h \leqq m.) \end{aligned}$$

As is well-known, we have

$$(5)\qquad \omega_{i,m+1} = \sum_k h_{ik}\omega_k, \quad h_{ik} = h_{ki}.$$

Let

$$(6)\qquad \det(\lambda\delta_{ik} + h_{ik}) = \sum_{0\leq s\leq m} \binom{m}{s} \sigma_s \lambda^{m-s}.$$

Then σ_s is called the sth curvature of $x(M)$. In particular,

$$(7)\qquad \sigma_1 = \frac{1}{m}\sum_i h_{ii}$$

is called the mean curvature and

$$(8)\qquad \sigma_m = \det(h_{ik})$$

is the GAUSS-KRONECKER curvature. For $m = 2$ they were denoted above by H and K respectively.

Let a_A be a fixed orthonormal frame in E. Then (a_A, x) is a scalar function on M, and is in fact the height function in the direction a_A. We put

$$(9)\qquad \begin{aligned} A_{m-h} = \sum \varepsilon_{i_1,\ldots,i_m} d(a_{i_1}, x) \wedge \cdots \wedge d(a_{i_h}, x) \wedge d(a_{i_{h+1}}, e_{m+1})\\ \wedge \cdots \wedge d(a_{i_m}, e_{m+1}), \end{aligned}$$

where $\varepsilon_{i_1,\ldots,i_m}$ is the KRONECKER symbol which is $+1$ or -1, according as $i_1, \ldots, i_m$ form an even or odd permutation of $1, \ldots, m$, and is otherwise zero, and where the summation is over all the indices $i_1, \ldots, i_m$. Then A_h is a multiple of σ_h according to the following formula:

$$(10)\qquad A_h = (-1)^h m!\, t\sigma_h \Phi,$$

where

$$(11)\qquad t = (a_{m+1}, e_{m+1})$$