CALCULUS

WITH APPLICATIONS

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Preface

This book is designed for beginning calculus students in business, economics, social sciences, and biological and health sciences. The text can be used in either a one-semester course or a more detailed two-semester course. A review of algebra in an appendix provides the option of supplementing the precalculus material found in Chapter 1. Suggested outlines for various courses are shown on page xiv.

APPLICATIONS

The applications appearing as examples or as exercises are realistic yet not too complicated. Unfortunately, true applications involve many symbols and calculations. Although such situations have been simplified, their authenticity has been carefully preserved. Special care is taken with units, so that meaningful results are obtained.

ORGANIZATION

Each section follows the basic sequence of motivation, development, and then application. New topics are motivated by introducing a realistic problem that leads to the development of new techniques necessary to solve the problem. General properties associated with the new concepts are accompanied by intuitive justification and by examples that demonstrate their importance. Finally, applications are presented to show how typical problems can be solved with these new methods. In the more difficult procedures, a step-by-step process is outlined for the student to follow.

EXERCISES

Each set of exercises begins with easier drill problems. Then the student is led into applications that relate the material to realistic problems. The more difficult problems are decomposed into parts that carefully lead the student to an understanding of the problem. The numerous exercises give the instructor freedom in choosing from a wide range of applications. Furthermore, some exploratory and challenging problems are included for more advanced students. *A star denotes the more difficult exercises*. Answers, including figures, for odd-numbered problems are found in Appendix D.

CALCULATOR AND COMPUTER

The use of a calculator is assumed throughout the book. Since most students rely on calculators early in their approach to a problem, answers to problems usually appear in both algebraic and calculated forms. Each new topic includes brief instructions for the use of a calculator in computing numerical results.

The use of a calculator is encouraged to give the student better understanding of concepts (such as limits) and to appreciate the results obtained. However, care must be taken to warn students of round-off errors and truncation errors when using a calculator.

The appendix includes a short computer program in BASIC that can be used to demonstrate Riemann sums and their convergence to areas in the discussion of integration. The student is encouraged to write variations of this program in a section on numerical methods of integration.

SPECIAL FEATURES

The development of limits is more complete than in many other applied calculus books. The student is led (with the help of figures and a calculator) to understand the concept of a limit through the natural approach with one-sided limits. The relationship of limits with graphs and the importance of infinite limits are emphasized in applications.

Memory devices, step-by-step outlines, graphs, and abundant examples all aid the student in understanding and applying the concepts of calculus. A generous number of figures are found in the examples and applications, emphasizing the value of a good graph and enhancing the analysis of the problem at hand.

A direct method of integration is introduced along with integration by parts as an alternative approach to integrating products of functions. This direct method has proved helpful in the actual computation of integrals as well as in an understanding of the concept of antidifferentiation.

The chapter on several variables includes a helpful discussion on contour curves as an aid to understanding functions and visualizing their graphs. The emphasis on applications involves both partial differentiation and multiple integration.

SUPPLEMENTS

An Instructor's Manual, available upon request, includes all answers, solutions to selected problems, and suggestions for problem assignments and classroom presentations.

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Coreen L. Mott
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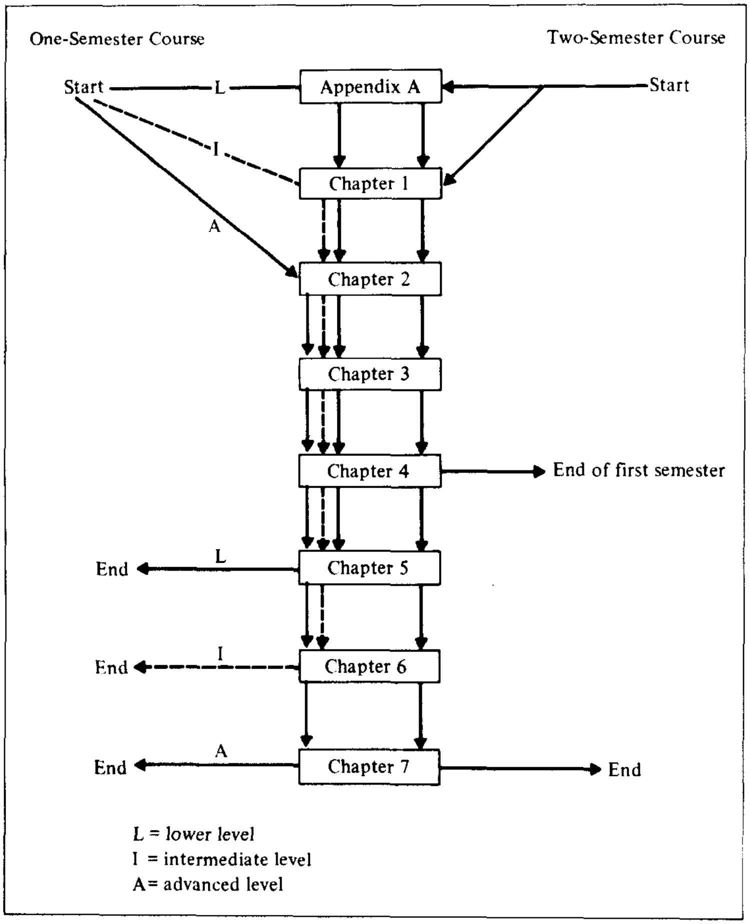

Section 3.3 (related rates), Section 5.4, and Theorem 3 in Section 5.5 can be omitted without affecting subsequent material. Any or all of Chapter 6 can be deleted, but Sections 6.4, 6.5, and 6.6 should be treated as a package. Sections 7.1, 7.4, and any parts of Section 7.6 can be omitted.

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Curves and Functions

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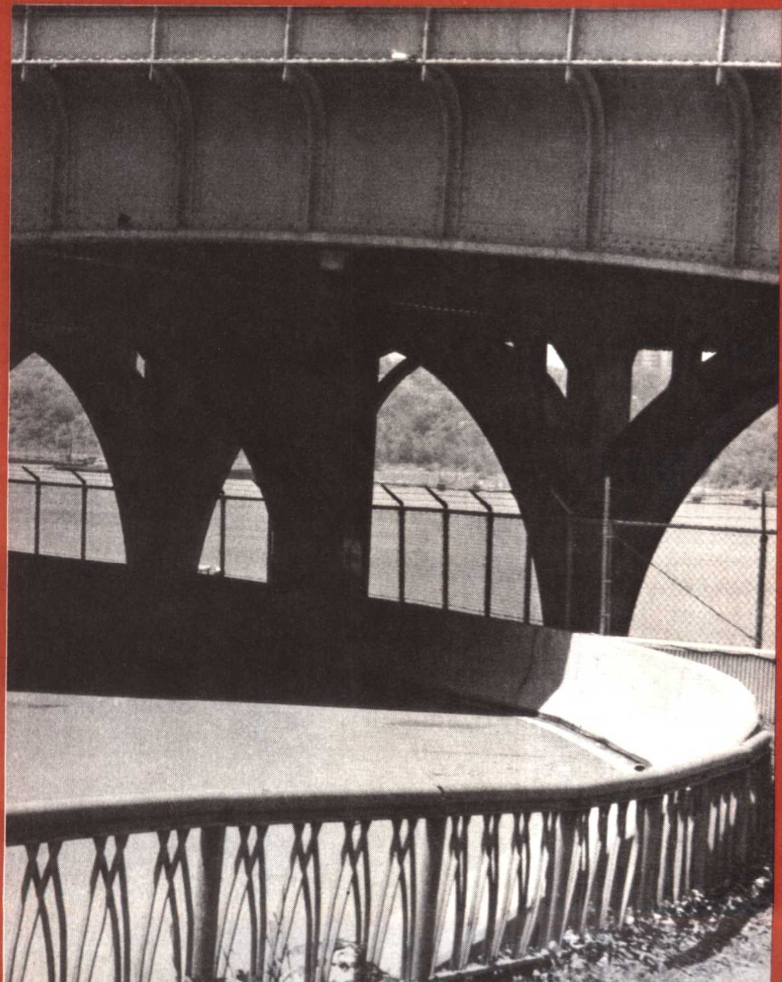

1.1 FUNCTIONS

Functions are everywhere in our lives. Certain physical dependencies and relationships can be mathematically modeled by functions. For example, our monthly water and electric bills depend on the quantity of water or electricity we use. In other words, some bills are a function of (or depend on) the quantity of service used. When a package is mailed, it is first weighed at the post office to determine the postage required. In mathematical terminology, the postage is ''a function of'' the weight of a package. Similarly, crop yield on a farm is a function of (is related to) the amount of rainfall. Generally, a function denotes a dependency of one quantity on another or a relationship between quantities. We shall be concerned with analyzing the properties of such functions, and in so doing we shall consider the following questions. Can we express a mathematical formula for the dependency of the quantities? Can we display this relationship graphically? What kinds of relationships and predictions can be extracted from a formula or a graph? We begin with a precise definition.

DEFINITION 1 Let D be a set of real numbers. A **function on D** is a rule f that assigns to each number x in D a unique real number, denoted by $f(x)$. The set D is called the **domain of the function,** and $f(x)$ is usually called the **value of f at x** (Fig. 1.1).

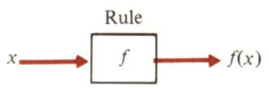

FIG. 1.1

Notation If f is the symbol that denotes the rule or function defined on D, we often write the domain of f as D_f.

EXAMPLE 1

Consider the following table showing the weights and corresponding postage costs for seven packages.

Weight (oz)	10	12	18	25	39	42	58
Cost ($)	0.68	0.72	0.85	1.03	1.87	2.08	2.95

If f represents the rule assigning to each weight its corresponding mailing cost, then $f(10) = 0.68$, which is read ''f of 10 is 0.68'' or ''f assigns to the weight 10 (ounces) the cost \$0.68.'' Similarly, $f(42) = 2.08$. Note that for this example the domain of f is

$$D_f = \{10, 12, 18, 25, 39, 42, 58\}$$

EXAMPLE 2

Let R be the set of all real numbers. Let g be the rule that assigns to each number x the number x^2. The arrows in Fig. 1.2 show some of the assignments under this rule g.

In our mathematical notation, we have

$$g(-2) = 4 \qquad g(2) = 4$$
$$g(-4) = 16 \qquad g(7) = 49$$

and D_g is the set of all real numbers.

In this case, we actually have a formula that represents the rule g. Namely,

$$g(x) = x^2$$

where x is any real number. In this formula, x is called an **independent variable** because it is simply an artificial name or symbol representing any real number in the domain.

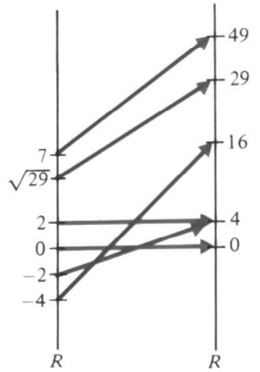

FIG. 1.2

Now that we have seen some examples, let us reflect on the definition. The important words are "rule" and "unique":

Rule A function is a rule. This rule is stated either as a formula [such as $g(x) = x^2$], as a table (as in Example 1), or by describing the assignment (as in Example 2).

Unique A function assigns to each member of its domain only one number.

For a better understanding of this "unique" requirement, let us consider a "rule" that is *not* a function.

EXAMPLE 3

Let D be the set of positive real numbers. Let h be the rule that assigns to each number x in D a number z whose square is x. That is,

$$h(x) = z \qquad \text{such that} \qquad z^2 = x$$

The number z is known as "a square root of x" (see Appendix A, Section A.2). We see, however, that $h(4) = 2$, since $2^2 = 4$; but also $h(4) = -2$, since $(-2)^2 = 4$. Since the rule h as stated does not specify which square root, it does not make a *unique* assignment. Hence rule h is *not* a function.

Of course, rule h could be altered to form a legitimate function. For example, if rule h assigns to each number in D its *positive* square root, then this assignment is unique and hence is a function.

Notation We have seen that the term "square root" is ambiguous without specifying either the positive or negative root. It is conventional to assume that the square root symbol used without sign represents the nonnegative square root. That is,

$$\sqrt{x} = \textbf{nonnegative} \text{ square root of } x$$

Since $\sqrt{x}$ makes *unique* assignments, it is a function for $\{x \mid x \geq 0\}$.* For example,

$$\sqrt{4} = 2 \quad \text{and} \quad \sqrt{9} = 3$$

EXAMPLE 4

Suppose rule f assigns values to numbers x according to the following table:

x	1	7	9	−7	9
$f(x)$	5	11	27	−14	6

Then $f(1) = 5$, and $f(7) = 11$; however, $f(9) = 27$ and $f(9) = 6$. Since this rule does not assign a "unique" number to 9, it does *not* represent a function.

Note

It is important to realize that for functions, assignments must be unique; however, it is permissible to assign the same value to several different numbers in the domain. For example, the function $g(x) = x^2$ defined in Example 2 assigns the value 4 to both $+2$ and -2. That is, $g(2) = 4$ and $g(-2) = 4$. Figure 1.3 summarizes.

A function *may not* assign two different values to the same number x.

A function *may* assign the same value to several different members of the domain.

Not legal — Legal

FIG. 1.3

Remark

A specific description of a domain is often omitted when a function is defined by a formula. In such a case, we assume that the domain is the set of all values for which the rule makes sense. For example, the function g defined by the formula $g(x) = x^2$ has an "implied" domain D_g, which is the set of all real numbers.

We will now examine more functions of real numbers in which the domain is not explicitly specified.

* As noted in Appendix A.1, the notation $\{x \mid x \geq 0\}$ represents the set of all real numbers x such that x is greater than or equal to zero. This set is also frequently denoted by the interval $[0, \infty)$. Refer to Appendix A.1 for a further discussion of set notation and interval notation.

EXAMPLE 5

Consider the function given by the formula

$$F(x) = 2x + 1$$

The "rule" here says take a number x, multiply it by 2, and then add 1. For example,

$$\begin{aligned} F(3) &= 2(3) + 1 = 7 \\ F(a) &= 2a + 1 \\ F(x + 1) &= 2(x + 1) + 1 = 2x + 3 \\ F(x + a) &= 2(x + a) + 1 = 2x + 2a + 1 \end{aligned}$$

It is assumed that the domain D_F is the set of all real numbers, since the rule F makes sense when x is any real number.

EXAMPLE 6

Consider the function given by

$$h(x) = \sqrt{x + 9}$$

The rule says that we start with a number x, add 9, and then compute the nonnegative square root.

$$\begin{aligned} h(0) &= \sqrt{0 + 9} = \sqrt{9} = 3 \\ h(-9) &= \sqrt{-9 + 9} = \sqrt{0} = 0 \\ h(a) &= \sqrt{a + 9} \qquad \text{provided } a + 9 \geq 0 \\ h(x + 1) &= \sqrt{x + 1 + 9} = \sqrt{x + 10} \qquad \text{provided } x + 10 \geq 0 \\ h(x + a) &= \sqrt{x + a + 9} \qquad \text{provided } x + a + 9 \geq 0 \end{aligned}$$

Since $h(x)$ is defined only when $x + 9 \geq 0$ or $x \geq -9$, the implied domain is $D_h = \{x \mid x \geq -9\}$. (See Appendix A, Section A.1 for a review of working with inequalities.)

EXAMPLE 7

Consider

$$g(x) = \frac{1}{x - 2}$$

The rule g says to start with x, subtract 2, and then invert [divide 1 by the quantity $(x - 2)$].

$$\begin{aligned} g(-10) &= \frac{1}{-10 - 2} = \frac{1}{-12} \\ g(0) &= -\frac{1}{2} \\ g(1) &= -1 \\ g(3) &= 1 \\ g(10) &= \frac{1}{8} \end{aligned}$$

The implied domain is $D_g = \{x \mid x \neq 2\}$, since division by zero is undefined.

Fortunately, we do not often have to worry about specifying the implied domain of a function. Frequently, the domain is already given in the definition of the function. Otherwise, from the preceding examples we have the following guidelines for determining the implied domain of a function:

1 An expression inside a square root symbol must be nonnegative.

2 Division by zero is not allowed.

Exercises 1.1

Consider the rules defined in exercises 1 to 6. Decide whether or not each satisfies the "uniqueness" criterion of the definition of a function.

1. Assign to each student number the age of the corresponding student.

2. Assign to each student number the age of a parent of the corresponding student.

3. Assign to every real number twice its value.

4. $g(x) = 5x$

5. Assign to every student number the student's listed home telephone number.

6. Assign to each student's home telephone number that person's student number. (*Caution:* Consider two members from the same family in the same university.)

Recall that tables can represent functions. Consider the assignments given in the following tables. Decide whether or not each satisfies the "uniqueness" requirement of the definition of a function.

7.

x	1	2	3	4	5	$\sqrt{3}$
$f(x)$	5	8	10	−2	6	5

8.

x	0	−1	−2	−3	−5
$f(x)$	3	$\sqrt{2}$	π	$\sqrt{2}$	0

9.

x	0	1	2	3	0
$f(x)$	−3	4	−11	25	8

10.

x	−1	−2	−3	−4	−5	−6
$f(x)$	3	3	3	3	1	1

11.

x	-2	-1	0	1	2	5
$f(x)$	-2	-1	0	1	2	3

12.

x	0	1	2	3	4	10
$f(x)$	10	4	3	2	1	0

13.

x	1	2	3	4	1
$f(x)$	5	10	15	20	25

14.

x	0	1	2	3	4
$f(x)$	3	2	1	0	0

Complete the following tables where the rule f is given.

15.

x	0	-4	1	6	-2
$f(x) = 3x$					

16.

x	0	1	2	3
$f(x) = \sqrt{x^2 + 1}$				

17.

x	0	$-\frac{1}{2}$	1	3	5
$f(x) = \dfrac{x}{x+1}$					

18.

x	0	1	2	-3	6
$f(x) = 3x - 5$					

19.

x	0	1	-1	$\frac{1}{2}$	-2
$f(x) = \dfrac{1 - x^2}{2}$					

20.

x	-1	1	-2	2	5
$f(x) = 1 + \dfrac{1}{x}$					

Given the functions specified by the formulas in exercises 21 to 27, compute the assignments.

21. Let $f(x) = x/3$ for all real x. Find $f(0)$, $f(3)$, $f(-6)$, and $f(x + 3)$.

22. Let $f(x) = 2x - 3$ for all real x. Find $f(-4)$, $f(1)$, $f(2)$, and $f(x + 1)$.

23. Let $f(x) = 4 - x$ for $-2 \le x \le 2$. Find $f(-1)$, $f(0)$, $f(2)$, and $f(x + 2)$.

24. Let $g(x) = 2x^2 - 3$ for all real x. Find $g(-4)$, $g(0)$, $g(3)$, and $g(x + 1)$.