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Preface

This book is intended as an introduction to plasma physics at a level suitable for advanced undergraduates or beginning postgraduate students in physics, applied mathematics or astrophysics. The main prerequisite is a knowledge of electromagnetism and of the associated mathematics of vector calculus. SI units are used throughout. There is still a tendency amongst some plasma physics researchers to cling to c.g.s. units, but it is the author's view that universal adoption of SI units, which have been the internationally agreed standard since 1960, is to be encouraged.

After a short introductory chapter, the basic properties of a plasma concerning particle orbits, fluid theory, Coulomb collisions and waves are set out in Chapters 2–5, with illustrations drawn from problems in nuclear fusion research and space physics. The emphasis is on the essential physics involved and the theoretical and mathematical approach has been kept as simple and intuitive as possible. An attempt has been made to draw attention to areas of current research and to present plasma physics as a developing subject with many areas of uncertainty, and not as something to be set forth on 'tablets of stone'.

Chapter 6 deals with the theory of nonlinear problems, discussing analytical methods and also the fundamentals of computational techniques, using both fluid and particle codes. In Chapter 7 the principles of some of the diagnostic methods used to obtain information about laboratory and space plasmas are discussed. Finally, Chapter 8 considers some applications of plasma physics to nuclear fusion research, using both magnetic and inertial confinement, and to some problems in space physics.

Most of the chapters end with a few problems, intended to illustrate and test the student's knowledge of the material in them. In an introductory text I have not felt it appropriate to give a comprehensive list of references, but a list of further reading is provided to guide the interested reader either to more advanced texts or to recent review articles.

I should like to extend thanks to many friends, colleagues, teachers and students, particularly at Glasgow University, St Andrews University and Culham Laboratory and amongst the users of the Central Laser Facility at the Rutherford Appleton Laboratory, from whom I have learnt much about plasma physics. Finally, thanks are due to my wife for her help in preparing the manuscript and in eliminating some of the more convoluted examples of English prose from the original draft.

R.A.C.

Contents

1 Introduction

1.1 Nature and occurrence of plasmas

A plasma is essentially a gas consisting of charged particles, electrons and ions, rather than neutral atoms or molecules. In general the plasma is electrically neutral overall, but the existence of charged particles means that it can support an electric current and react to electric and magnetic fields. It cannot, however, be treated simply as an ordinary gas which is electrically conducting. There is a very fundamental difference between a neutral gas and a plasma, resulting from the very different nature of the inter-particle forces in the two cases. In the former the forces are very strong, but of short range, so the dynamics of a gas is dominated by two-body, billiard-ball-like collisions. In a plasma the forces are Coulomb forces, which are comparatively weak and of long range. This makes possible a variety of collective effects in a plasma, involving the interaction of a large number of particles, and makes plasma physics a rich and complicated subject.

It is a commonplace to point out in introductions to plasma physics that most of the matter in the universe is in the plasma state. Since, however, we live in one of the small regions of the universe where matter is predominantly solid, liquid or gas, concern with the properties of plasma is quite recent and has mainly been stimulated by its importance in space physics and in the development of controlled nuclear fusion.

Since most matter outside the lower layers of the Earth's atmosphere is ionized, its relevance to the first of these is obvious. In interplanetary space and in the upper layers of the ionosphere ionization is mainly produced by ultra-violet radiation, and in the diffuse plasmas which result the rate of recombination of electrons and ions is low.

Nuclear fusion, on which we shall elaborate in the next section, is the process by which stars generate their energy but which so far has only been exploited by man in an uncontrolled fashion. The main obstacle in the way of harnessing this energy source is the fact that the reactions will take place at a useful rate only if the temperature of the reacting material is of the order of 10^8 K. Material at this temperature is ionized, since the thermal energy is well above that required to strip electrons from atoms, and research has centred on using magnetic fields to confine and control the hot plasma. Early attempts to do this soon revealed that a plasma was a much more subtle and complicated system than had been thought, and triggered off a programme of theoretical and experimental research into its properties which still continues. In space physics, satellites are being used to obtain ever more detailed data about the plasma in the vicinity of the Earth, while our knowledge of more distant regions is also being extended by improved observational techniques. Such observations reveal that space plasmas show just as rich a variety of phenomena as laboratory plasmas.

The aim of this book is to acquaint the reader with some of the most important properties of a plasma and in the course of doing so to point out ways in which these properties are relevant to laboratory or naturally occurring plasmas. Many of the illustrations will be drawn from applications connected with the controlled fusion programme, since this undoubtedly has been and remains the driving force behind much of the research on plasmas.

1.2 Controlled nuclear fusion

Because the most strongly bound nuclei are those in the middle of the periodic table, nuclear energy can be released in two ways. These are fusion of light nuclei into heavier ones, and fission of heavy nuclei, the latter being the process which fuels the current generation of nuclear power stations. To obtain energy in a controlled way from fusion, the most suitable reactions involve fusion of hydrogen isotopes to helium. That which requires the lowest temperature involves deuterium and tritium reacting to form helium and a neutron

$$_1D^2 + {_1T^3} \rightarrow {_2He^4} + n + 17.56\,\text{MeV},$$

and if fusion is eventually demonstrated to be feasible it will probably be with this system. Since tritium is an unstable radioactive gas, deuterium–deuterium fusion

$$_1D^2 + {_1D^2} \rightarrow {_2He^4}$$

may be more desirable in the long run, but it requires a higher temperature.

In order to understand the problems which stand in the way of controlled nuclear fusion it is necessary to consider the conditions under which a reaction yielding a net power output can take place. For fusion to occur, the reacting nuclei must have enough energy to overcome their Coulomb repulsion and to approach sufficiently closely for there to be a reasonable probability of fusion. For the D–T reaction the cross-section, measuring the probability of the reaction's taking place, has a maximum when the particle energy, in the rest frame of the centre of mass of the two particles, is of the order of 10 keV. As a result, an effective power output requires that the particle thermal energies be of this order. Generally temperatures are measured in eV, the energy equivalent of the temperature, κT, with κ Boltzmann's constant, being expressed in these units. Since 1 eV is equivalent to about 10^4 K, an effective fusion reactor requires a temperature of about 10^8 K to be sustained.

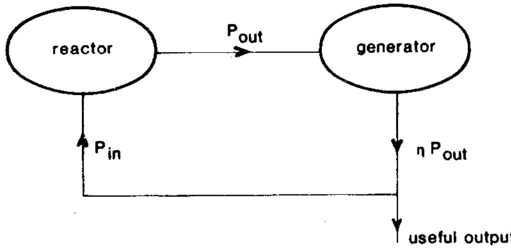

Figure 1.1 Schematic reactor system with energy flow.

Supposing that this can be done we may obtain a further condition to be satisfied if we consider the energy flow in the schematic reactor system shown in Figure 1.1.

In the reactor let us suppose that n is the ion density, there being an equal density of electrons, so that at a temperature T the thermal energy density is $3n\kappa T$. No confinement system is perfect and there is always loss of particles and energy across the magnetic field lines to the walls.

If the energy loss time associated with this is τ, then the rate of loss of thermal energy per unit volume is $3n\kappa T/\tau$. There is also a substantial loss by radiation, which we denote by P_{rad} per unit volume. Plasmas under fusion conditions are almost transparent to radiation, so the radiation loss is a volume rather than a surface effect. Finally we shall suppose that nuclear reactions produce a power P_{nuc} per unit volume.

In order to maintain the temperature, the input power must balance the radiation and diffusion losses, so that, per unit volume

$$P_{in} = 3n\kappa T/\tau + P_{rad}.$$

As well as the nuclear power, the radiated power and that lost by transport across the containing field contribute to the output of the reactor, which is thus

$$P_{out} = 3n\kappa T/\tau + P_{rad} + P_{nuc}.$$

This output, in the form of heat, has to be converted to electricity by a conventional turbine system, with an efficiency η which is typically about 30%, so that the output from this stage is ηP_{out}. Part of this power, P_{in}, must be recirculated within the system to maintain the temperature of the reacting material, so the final useful power output is

$$\begin{aligned} P &= \eta P_{out} - P_{in} \\ &= \eta P_{nuc} - (1-\eta)P_{rad} - (1-\eta)\frac{3n\kappa T}{\tau} \end{aligned} \tag{1.1}$$

If the system is to generate any useful power this must be positive. Now, both P_{nuc} and P_{rad} are proportional to the square of the density, but at temperatures in the range below about 10 keV P_{nuc} increases much more rapidly with temperature than P_{rad} which is proportional to $T^{1/2}$. If the temperature can be kept high enough, of the order of 10 keV, then the combination of the first two terms in (1.1) is positive. Given that this temperature is maintained, the first two terms, which we recall are proportional to n^2, can be made to outweigh the last one if $n\tau$ is big enough. Thus we arrive at the other condition which must be satisfied in addition to the requirement of a high enough temperature. This condition, known as the Lawson criterion, sets a minimum value on the product of density and energy containment time. Its numerical value is around $10^{20}\,\mathrm{m}^{-3}\,\mathrm{s}$.

We have discussed this criterion in terms of a steady-state reactor with a magnetic confinement system. However, similar energy balance considerations lead to the same condition being obtained for pulsed systems. One concept which is currently receiving much attention is that of inertial confinement, in which a small target pellet is heated and compressed by a laser or particle beam. No attempt is made to contain the resulting small dense plasma and the confinement time is just the finite time which it takes to fly apart because of its inertia. While magnetic confinement systems aim for densities of the order $10^{20}\,\mathrm{m}^{-3}$ and confinement times of the order of seconds, inertial confinement aims for densities of about a thousand times that of the solid target material and correspondingly short confinement times. Fusion systems will be discussed in more detail in Chapter 8.

1.3 Fluid and kinetic descriptions of a plasma

In order to gain a theoretical understanding of the behaviour of a plasma we must formulate a set of equations to describe it, then try to solve them with the boundary or other conditions imposed by the problem we have in mind. One of the difficulties of plasma physics is that there is no one set of equations which we can write down and identify as the starting point for the theory. Different levels of description and different sets of basic equations are used depending on the problem to be solved.

The difficulties in describing a plasma may perhaps be best explained if we begin with a brief informal outline of the kinetic theory of a neutral gas. In general the behaviour of a gas is very adequately described by a set of equations giving the evolution of its density, velocity, pressure etc., rather than by any kinetic description which contains information about the motion of individual particles. If, however, a kinetic theory is required then it is usually formulated in terms of a particle velocity distribution function $f(\boldsymbol{r}, \boldsymbol{v}, t)$ giving the density of particles in a six-dimensional space with coordinates $(\boldsymbol{r}, \boldsymbol{v})$ at time t. The usual fluid quantities are velocity moments of this, for instance the density is

$$\rho(\boldsymbol{r}, t) = m \int f(\boldsymbol{r}, \boldsymbol{v}, t)\, \mathrm{d}^3 v,$$

with m the mass of a particle, and the fluid velocity is the average particle velocity, that is

$$u(r,t) = \frac{1}{n}\int v f(r,v,t)\,d^3v.$$

Here n is the number of particles per unit volume, equal to ρ/m. The evolution of this velocity distribution is described by some kinetic equation, which must be derived from a detailed consideration of the particle dynamics of the system, and which for a neutral gas is Boltzmann's equation. A fundamental property of this equation is that any initial spatially uniform distribution function relaxes towards a Maxwellian

$$f(v) = \frac{n}{(2\pi\kappa T/m)^{3/2}} \exp(-m(v-u)^2/2\kappa T), \tag{1.2}$$

where T is the temperature, κ Boltzmann's constant, u the average velocity of the gas and n the density of the particles. The Maxwellian is the velocity distribution function of a system in thermal equilibrium.

The crucial feature of gas dynamics, which allows a fluid rather than a kinetic description to be adequate for most purposes, is that the length scales and time scales over which the system changes are usually much greater than the mean free path and the inter-particle collision time. This means that in the kinetic description the effects of gradients and time variations are small compared with the effects of collisions which push the distribution towards a Maxwellian. In consequence, a systematic perturbation theory can be developed (the Chapman–Enskog method) in which the distribution is, to lowest order, Maxwellian as in (1.2), but with the parameters n, u, T allowed to be slowly varying functions of space and time. Equations for n, u and T which are just the usual equations for a fluid can then be derived. The essential feature is that the dominant role of collisions in gas dynamics means that the distribution function is very close to being locally Maxwellian which, in turn, means that the gas is described by the parameters n, u and T.

In a plasma the inter-particle force is the long-range Coulomb force which is weak compared to the strong forces between colliding neutral gas molecules A particle in a plasma feels the effect of many particles at once and its velocity undergoes a random series of small changes, rather than the sudden changes in velocity produced by neutral gas collisions.

The effect of these Coulomb collisions is again to reduce the velocity distribution to the thermal equilibrium Maxwellian, but an essential feature of plasma physics is that many phenomena of importance take place on time scales much shorter than that associated with the relaxation to thermal equilibrium. For such high-frequency, short-scale phenomena the plasma must be described by a kinetic equation.

Thus, both fluid and kinetic descriptions of a plasma are in regular use, whereas in neutral gas dynamics kinetic theory is only necessary to describe some rather specialized phenomena. Fluid equations are used in plasma physics when large-scale behaviour is being considered, for instance the equilibria of containment systems and their stability against gross motions of the whole plasma. Kinetic equations are used to describe high-frequency wave propagation and the so-called microinstabilities which involve the growth of small-scale perturbations and are driven by the existence of non-Maxwellian velocity distributions. Both types of description will be used in later parts of this book.

1.4 The Debye length

This parameter, which first appeared in the theory of electrolytes, is of fundamental importance in a plasma, and its relation to the average inter-particle spacing plays an important role in determining the strength of Coulomb collisions. If a positive charge, say, is placed in a plasma then electrons are attracted towards it and ions repelled and the effect is to create a screen of negative charge around it. As a result the potential produced by the charge falls off faster than $1/r$ and the Debye length is a characteristic length beyond which the effect of the charge is screened off.

We consider the electrons and suppose that their density is n in the unperturbed system. Then in a potential ϕ the probability of finding an electron with speed v is proportional to $\exp[(-\frac{1}{2}mv^2 + e\phi)/\kappa T]$, this being simply the thermal equilibrium Boltzmann energy distribution. Integrating over velocity we find the electron density is proportional to $\exp(e\phi/\kappa T)$, so that

$$n_e = n_0 e^{e\phi/\kappa T} \approx n_0(1 + e\phi/\kappa T),$$

and in a potential ϕ the charge density perturbation due to the electrons is $-n_0 e^2\phi/\kappa T$, if $|e\phi|$ is assumed to be small compared to κT. Assuming, for simplicity, that the ions just form a uniform background, then Poisson's equation relates the potential to the charge density and gives

$$\nabla^2\phi = \frac{n_0 e^2 \phi}{\varepsilon_0 \kappa T}.$$

If we look for a spherically symmetrical solution to this, then we have

$$\frac{1}{r}\frac{d}{dr}\left(r^2\frac{d\phi}{dr}\right) = \frac{1}{\lambda^2}\phi,$$

the solution of which tending to zero at infinity is

$$\phi = A\frac{e^{-r/\lambda}}{r},$$

with $\lambda^2 = \varepsilon_0\kappa T/(n_0 e^2)$ and A an arbitrary constant. This gives the behaviour of the potential surrounding a charge, which goes as $e^{-r/\lambda}/r$ instead of $1/r$

as it would in a vacuum. The effect of the charge is only felt within a distance of order λ, the Debye length.

In a plasma any given particle attracts towards it a screening charge in the same way, with the result that the effective range of the inter-particle force is of the order of the Debye length. A test particle moving through the plasma interacts at any instant with the particles in the Debye sphere, i.e. sphere of radius λ, surrounding it. Any change in its velocity due to 'collisions' with these particles, is the result of a non-zero resultant force on the test particle from the particles within its Debye sphere. The more particles there are within the Debye sphere the more uniformly will they be distributed around the test particle and the less the likelihood of any imbalance producing a force on the test particle. We can thus arrive in an intuitive way at one of the basic results of more elaborate plasma kinetic theories, namely that the parameter which determines the strength of collisions in a plasma and hence the rate of relaxation to thermal equilibrium is the number of particles in the Debye sphere, $n_0\lambda^3$. For magnetically confined fusion plasmas and most space plasmas this is generally a large number, so that collisional effects are weak. The dynamics of the plasma is then not dominated by inter-particle collisions and it can support a variety of waves and instabilities which must be described by kinetic theory. Sometimes this property is taken as part of the definition of a plasma. Systems with few particles in the Debye sphere are called non-ideal or non-Debye plasmas and are a field of specialized study in their own right. They can occur in the compressed plasmas of interest in inertial confinement systems.

The rich variety of phenomena exhibited in a plasma, from large-scale fluid motions to microscopic high frequency oscillations, makes it a challenging and interesting object of study, both for the experimentalist and theorist. Despite the considerable progress which has been made towards its understanding there are still many problems awaiting solution.

2 Motion of a charged particle

2.1 Introduction

It has been pointed out in the introductory chapter that in a hot plasma inter-particle collisions are relatively weak, so that over time scales of interest a particle may remain close to its orbit in the macroscopic fields in the plasma, without being significantly deflected by the microscopic fields arising from other particles. For this reason it is useful to have a knowledge of the behaviour of single particles in electric and magnetic fields. This is not only an essential step towards a more exact analysis of plasma behaviour, but also by itself allows us to understand the basic ideas underlying some magnetic confinement systems.

The usual procedure in developing this orbit theory, and the one which we shall follow here, is to begin with motion in a steady uniform magnetic field, where the exact particle orbit is easily calculated, then use this as the basis for a perturbation theory which will describe the particle orbit in fields varying over suitable long length and time scales. This basic theory is developed in the first part of the chapter and some applications of it discussed in the latter part.

2.2 Motion in a uniform magnetic field

The equation of motion of a particle of mass m and charge q in given electric and magnetic fields is

$$\frac{d\boldsymbol{v}}{dt} = \frac{q}{m}(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}), \tag{2.1}$$

where in the most general case $\boldsymbol{E}$ and $\boldsymbol{B}$ could be functions of both position and time. Unfortunately this general case is not amenable to analytic solution, so to obtain a description of the motion, other than by numerical integration of the equation, we have to resort to various approximation techniques. We begin by discussing the case where $\boldsymbol{E} = 0$, while $\boldsymbol{B}$ is constant, then in subsequent sections extend the description to more general cases.

Taking B along the z-axis, the equations of motion are

$$\frac{\mathrm{d}v_x}{\mathrm{d}t} = \Omega v_y$$

$$\frac{\mathrm{d}v_y}{\mathrm{d}t} = -\Omega v_x$$

$$\frac{\mathrm{d}v_z}{\mathrm{d}t} = 0, \tag{2.2}$$

where $\Omega = qB/m$. The general solution is

$$\begin{aligned} v_x &= v_\perp \cos(\Omega t + \theta) \\ v_y &= -v_\perp \sin(\Omega t + \theta) \\ v_z &= v_\parallel , \end{aligned} \tag{2.3}$$

with $v_\perp$, $v_\parallel$, and θ constants. In the x–y plane the solution simply represents uniform motion in a circle with speed $v_\perp$, the perpendicular (to the magnetic field) velocity component. The constant θ just fixes the position of the particle on the circle at $t = 0$. Superimposed on this circular motion is a uniform parallel velocity, $v_\parallel$, so the final result is motion along a helix. The nature of this motion is easily understood if it is noted that the magnetic force is always perpendicular to the particle velocity, so that it does no work on the particle and the magnitude of the velocity is constant. The force is then of constant magnitude and at right angles to the velocity and the field, just what is needed to produce circular motion in the plane perpendicular to the field. The quantity Ω, the angular frequency of the circular motion, is called the cyclotron frequency, while the radius of the orbit in the x–y plane, given by

$$r_{\mathrm{L}} = \frac{v_\perp}{|\Omega|}$$

is called the Larmor radius.

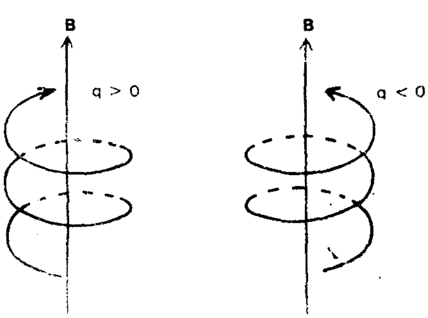

Figure 2.1 Particle orbits in a steady uniform field.

The direction of gyration around the field direction can be deduced from (2.3), or more easily by noting that the magnetic force must be towards the centre of the orbit, the conclusion being that a positively charged particle rotates in a left-handed sense when viewed along the field and a negatively charged particle in the opposite sense. This is illustrated in Figure 2.1.

2.3 Guiding centre drifts

The motion described above can be thought of as gyration about a centre (the guiding centre), which is moving along the direction of the field. In many important applications field gradients or rates of change are such that the particle sees only a small variation during each gyration. Under these circumstances the motion is close to that in a uniform field and can still be thought of as circular motion on which is superimposed a guiding centre drift which may no longer be uniform motion along the field.

The simplest extension is to consider a steady uniform electric field superimposed on the magnetic field of section (2.1). A component along z simply produces a constant acceleration along the z-direction and is of no great interest. Less trivial is a component perpendicular to the magnetic field, let us say in the y-direction, so that our equations of motion are (neglecting the z-component)

$$\frac{\mathrm{d}v_x}{\mathrm{d}t} = \Omega v_y$$

$$\frac{\mathrm{d}v_y}{\mathrm{d}t} = -\Omega v_x + \frac{q}{m}E.$$

These are most easily solved by noting that, putting

$$v_x = \frac{qE}{m\Omega} + V_x$$

transforms them to

$$\frac{\mathrm{d}V_x}{\mathrm{d}t} = \Omega v_y$$

$$\frac{\mathrm{d}v_y}{\mathrm{d}t} = -\Omega V_x,$$

i.e. a set identical to those for a uniform field. Thus we have the motion described before, with a velocity $qE/m\Omega = E/B$ in the x-direction superimposed on it. Writing this velocity in a form independent of the particular coordinate system used here we see that it is

$$\boldsymbol{V}_{\mathrm{E}} = \frac{\boldsymbol{E} \times \boldsymbol{B}}{B^2}. \tag{2.4}$$