ENCYCLOPEDIA OF COMPUTER SCIENCE AND TECHNOLOGY

EXECUTIVE EDITORS

VOLUME 3
Ball to Box

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EXECUTIVE EDITORS

Jack Belzer *Albert G. Holzman* *Allen Kent*

UNIVERSITY OF PITTSBURGH
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BALLISTICS CALCULATIONS

INTRODUCTION

Ballistics has been defined [1] as "The science or art of hurling missile weapons by the use of an engine; the science of the motion of projectiles." Without amplification, this definition may create an impression of narrowly exclusive concern with the dynamics of rigid bodies. In modern practice "ballistics" has actually been much more broadly construed to deal with almost all aspects of conventional weaponry as well as the effects of nuclear weapons, extending from research for development of the technological base for design to evaluation of predicted or observed performance, product improvement, doctrine for use, and quality control for manufacturing tolerances or for monitoring of deterioration in storage. The action involved in the definition is intended to destroy, defeat, damage, or even merely to threaten a remote target. Targets may include various kinds of structures (e.g., buildings, dams, bridges, docks), vehicles (tanks, trucks, trains, ships, aircraft), equipment, industrial plants, storage or supply dumps, transportation facilities, communications systems, information-gathering ability, personnel, or morale. The damage-producing mechanism may be impact of the projectile on the target; blast wave loading from the detonation of chemical explosives; impact of fragments of shell casings, bombs, mines, or grenades; penetration by high-speed metal jets; fire produced by impact with pyrophoric materials; smoke for concealment; illumination for exposure; noise; or even the psychological effect of propaganda leaflets.

Delivery of the warhead to the target can be accomplished by use of rockets, which are self-propelled; by firing nonself-propelled projectiles from guns, howitzers, or mortars; by dropping bombs from aircraft or depth charges from ships; by launching torpedoes or missiles from submarines; by soldiers throwing grenades; or mobile targets may pass unintentionally over land or sea mines. If we disregard grenades, bombs, mines, depth charges, and torpedoes, the motion is generated by the direct interaction with the projectiles of gases formed by the combustion of chemical propellants. After launching there follows a period of flight subject to gravity, aerodynamic forces generated by the interaction of the shell with the atmosphere, and possibly continued thrust for rockets or tracer ammunition. Eventually, by the use of a fuse actuated, e.g., by impact or by a timer, the warhead will be triggered close to the target if the target's location relative to the launching site is accurately known; if the weapon is accurate; if it has been correctly calibrated, aimed, or adjusted; and if it has performed properly. The damage inflicted is a complicated function of the nature of the target and of the forces applied thereto. Since damage is the ultimate purpose of all of the preceding actions, estimation of the vulnerability of targets becomes one of the principal objectives of ballistics, from the point of view of the attackers. Conversely, determination of measures for reduction of vulnerability is a primary concern of the defenders.

Intended to deal systematically with a great variety of targets, possible forms of damage, and agents to accomplish this, ballistics must clearly be based on a tremendous interdisciplinary body of engineering science and technology. No individual can hope to master all aspects of the subject in total relevant detail. Thus, as detailed development and expansion of the field has occurred, ballistics has been split into progressively narrower subdivisions, but with correspondingly increased well-grounded specialized content. During the past 20 years, a generally acceptable productive subdivision into major technical specialties would be:

Interior ballistics
Exterior ballistics
Terminal ballistics
Vulnerability/vulnerability reduction
Weapons systems analysis

The problems to be considered have to be solved by a mixture of intuition, empiricism, tests or experiments, and theory.

The theories originate in physics (largely but not exclusively mechanics), geophysics, and chemistry. Generally they are phrased in mathematical or statistical terms. The equations and constraints are usually nonlinear. To solve problems, at least approximately, or to gain insight into the general behavior of their solutions, three approaches are generally attempted:

Seek special solutions, particularly similarity solutions and principles.
Linearize, especially if this leads to classical equations with extensive accumulations of theory, of simple representations of solutions, or of transformations to reformulate problems in more tractable forms.
Resort to numerical methods.

In general, if nonlinear problems include realistic details, the only general widely effective method available to determine approximate solutions is computation. Until the emergence of the electronic computer the principal deterrent to the computational approach was the prodigious amount of calculation required, of course. Nevertheless, for some of the simpler kinds of nonlinear problems, of which we shall discuss two types, the need very long ago for results was desperate enough to warrant the enormous effort required to compute with inadequate equipment. Accordingly, hand computations had to be undertaken for them with whatever resources were available.

The first type included calculation of orbits in astronomy; the preparation of nautical almanacs and ephemerides; and later on the closely related calculation of trajectories to construct firing tables for artillery. Much of this work involves the numerical solution of small systems of ordinary differential equations. In the context of construction of firing tables, the equations of motion contain aerodynamic forces and moments as well as physical parameters that depend on the particular shell. The initial conditions depend on the angle of elevation of the gun barrel and on the powder charge (which determines the initial velocity). A common practical problem is to determine combinations of elevations and charges that produce sets of equally spaced horizontal ranges, between which the gunner can easily interpolate, if necessary. If table preparation is to be attacked numerically without any kind of computer more elaborate than an abacus, the most likely approach would be to calculate a few trajectories for each charge and then to fill in the desired table by a mixture of direct and inverse interpolation, probably with some reservations or misgivings about the accuracy of the results. In slightly less austere circumstances, with an electromechanical desk calculator of vintage 1940, it took an average of about 20 hours to compute a trajectory. To obtain smooth enough results for accurate interpolation, as many as 50 elevations and seven charges might have had to be used. At this preelectronic speed it would have taken about 2.8 man-years to produce the table for a particular shell. It should also be mentioned that there may be several types of shell for each of a great variety of weapons. Furthermore, recomputations may be required if dynamically significant changes are made in existing types of shell. All of this implies a large computing load that obviously calls for much faster and more efficient means for computation.

As a second kind of early exploration of numerical methods important for ballistics even prior to the appearance of electronic computers, various calculations for problems in continuum mechanics should be mentioned. In general, these would require the approximate solution of nonlinear partial differential equations. However, the numerical treatment of even linear equations was quickly found to require great volumes of computation. To make it clear that awareness of such ideas and difficulties was widespread, it should suffice to recall some of the early publications on this subject. Theoretical foundations for the validity of various algorithms for the numerical solution of typical classical linear partial differential equations were established by Phillips and Wiener [2], and Courant, Friedrichs, and Lewy [3]. Extensive applications which were heuristically plausible methods at the time and computational short cuts to specific problems were made, e.g., by Southwell [4] and Shortley [5] and their associates. Preliminary studies of the numerical solution of nonlinear partial differential equations were made at about the same time by Friedrichs and Lewy [6] and Frankl and Aleksjeva [7]. It may even be worth mentioning, as an indication of its possible origin in a problem in ballistics, that the latter paper refers to earlier publications on the same problem in Soviet technical military journals.

We have briefly suggested the need in ballistics to be able to perform, rapidly and easily, massive amounts of calculation. Obviously this was just a reflection of the even more extensively perceived and acknowledged needs of all kinds of engineering and of the physical sciences, as a whole, for computation on a grand scale. Perusal of the early volumes of one of the first exclusively applied mathematics journals, the *Zeitschrift fuer angewandte*

Mathematik und Mechanik, reveals that astonishing numbers of papers on numerical methods were published in the 1920s and 1930s. The times cried out for efficient automatic aids to computation. The first responses to this growing need were various kinds of electromechanical desk calculators, more elaborate commercial accounting machines, differential analyzers, and relay computers. Eventually these were followed by an all-electronic computer, its appearance hastened by military needs. An accident of history brought about very productive close professional associations of talented ballisticians, engineers, chemists, physicists, mathematicians, and statisticians at the military and civilian research laboratories of the major warring nations during World War II. One such group at the Ballistic Research Laboratories (BRL) at Aberdeen Proving Ground, Maryland used a Vannevar Bush differential analyzer for part of their mission, the preparation of firing tables. There was a life and death urgency for preparing many new firing tables for new weapons or new ammunition more rapidly than ever. In this connection, the Moore School of Electrical Engineering of the University of Pennsylvania, comparatively nearby in Philadelphia, also had a Bush differential analyzer. As an obvious measure to double the BRL's computing facilities, an additional group of computers was established to prepare firing tables on the Moore School's machine. Association with or awareness of this important task inspired some of the faculty of the Moore School to speculate about designing, with modern high-speed electronic counting techniques, a digital computer that would perform arithmetic at millisecond rates. Eventually support of their proposal by the U.S. Army Ordinance Department to produce a device that would radically speed up the preparation of firing tables led to the construction of the world's first all-electronic digital computer, the ENIAC. In operation it actually achieved computing speeds in the desired range, viz. 0.2 msec for addition and 3.4 msec for multiplication. Detailed accounts of the development and technical background of this machine are contained in Goldstine's recently published authoritative history of digital computers [8].

For histories of ballistics the reader should consult the works of Charbonnier [9], Mandryka [10], Szabo [11], and Barnes [12]. The published lectures of Cranz [13, 14], also available in English translations, provide definitive texts that have exerted international influence on the development of ballistics. A testimonial to their widely acknowledged importance is the compilation of modern advances published in the Cranz Centennial volume, edited by Nelson [15]. Specialized accounts of exterior ballistics, some of which contain more modern material, are presented in the books of Bliss [16]; Garnier [17]; McShane, Kelley, and Reno [18]; lectures by Green [19]; and reports by Murphy [20a, 20b]. References for interior ballistics are books by Corner [21] and Tranter [22]; a more recent (popular) account has been written by Lowry [23]. The reader cannot have failed to notice the preponderance of exterior ballistics in these references. This can be explained partly by the fact that as a subject ultimately based on particle or rigid body dynamics, exterior ballistics had its theoretical beginnings in late seventeenth century science, partly because of the apparently much greater complexity of the subject matter of the other subdivisions of ballistics. By contrast with the relative antiquity of exterior ballistics, Corner has remarked that, with one exception, modern interior ballistics began in about 1840. The remaining subdivisions of modern ballistics, listed earlier, have been established since World War II. To the extent that their subject matter is of widespread importance outside of ballistics, general reference works can easily be found in large technical libraries. Specialized accounts of terminal ballistics and vulnerability estimation tend to have rather limited circulation.

A SHORT SURVEY OF PROBLEMS IN BALLISTICS

As a preliminary to discussing selected ballistic computations in the next section, we mention some general types of scientific and engineering problems of ballistics. To be sure, many of these problems also occur in nonmilitary contexts and have been studied at great length. If they have been treated numerically, the computational algorithms and computed results have very likely been published in technical journals or in series of laboratory reports that are readily accessible to the general professional public. Many references to such material can be found by looking under index headings such as blast waves, boundary layers, detonation, explosions, impact, jets, nozzle flow, reacting flow, shells (structural), shock waves, and systems identification in the author's bibliography for the numerical solution of partial differential equations [24]. Hence there is no need for extended consideration of such universal calculations (see the section entitled Selected Examples of Computation in Ballistics for a discussion of computations that have some features peculiar to ballistics).

We shall organize our survey in accordance with the specialized subdivision presented in the Introduction.

Interior Ballistics is dedicated primarily to the events that culminate in the launching of a projectile or a rocket. First, there are problems in chemistry. Combustion [25, 26] of a chemical propellant in a gun produces gas at high pressures which sets the projectile in motion at high speed. In a rocket motor it produces a flux of momentum across the exit plane of the nozzle that thrusts the rocket forward. Searches for new or better propellants and igniters begin from chemistry or chemical engineering considerations. To try to determine the necessary chemical and physical properties of potential or proposed propellants completely by theoretical means leads to problems in quantum mechanics. For reactions that involve more than about five atoms, the calculations are beyond the capabilities of present-day computers. Semiempirical approaches to more complicated reactions have greater prospects for success. For the most immediate response to the needs of interior ballistics technology, chemical reaction rates have to be determined experimentally. Interpretation of the data comes down to a problem of parameter identification. The reaction rates in the systems of ordinary differential equations of chemical kinetics must be adjusted to fit observed results. Unfortunately, such systems are the archetypal *stiff differential equations*, notoriously difficult to handle numerically.

Second, there are problems in fluid mechanics of various degrees of complexity associated with the behavior of the propellant gases. In the pre-computer era a similarity solution was often used to approximate the gas velocity, pressure, etc., but experience has shown this to be inadequate. Consequently, more elaborate computations of the development of the gas flow-field have been undertaken. In this connection, one of the simplest classical problems is a one-dimensional nonsteady idealization of the motion of a projectile in a gun barrel, the so-called problem of Lagrange [21, 27–29]. Another commonly encountered simple example is the approximation of a steady plane or a symmetric flow through a nozzle. Both problems can be characterized mathematically by hyperbolic systems of partial differential equations, subject to appropriate boundary and initial conditions. One of the principal problems in undertaking computations for such problems is general uncertainty about the choice of initial conditions. Success of the calculations may depend on the fact that the flow is so extensive and of such long duration that the influence of the initial conditions decays comparatively rapidly, giving the boundary conditions the predominant effect in determining the solution. Speculation about these matters could lead to the following inverse form of Lagrange's problem. Supplement the gasdynamics equations by experimental observations of the motion of the projectile in the gun tube to try to determine the initial conditions for the propellant gas. Variants of all of these fluid dynamics problems result from adding more realistic features, e.g., chemical reactions, more elaborate geometry, asymmetry, which will increase the number of independent variables; additional physical properties of the gases or fluids, such as viscosity and heat conduction; variations in the cross-sectional area of the bore; and bore friction.

Third, there are problems of rigid body dynamics. One arises in the in-bore motion of projectiles. In general, it would simplify the analysis if the axis of symmetry of the shell moved along the axis of symmetry of the barrel. In reality, the paths of points of the projectile's axis will resemble spirals. This can happen because there can be some "play" in the way the projectile's driving band engages and its bourrelet rides on the rifling. Another source of problems is the functioning of mechanical fuses which may depend on the constrained rigid body motion of some of their components.

Fourth, there are many kinds of mechanical engineering problems. Continuum mechanics problems will appear in the design of gun barrels to resist the high-pressure loading produced by the propellant gases. For most kinds of gun mounts it will be necessary to provide spring or hydraulic cylinder recoil mechanisms. Various problems of vibration or wave motion are associated with automatic weapons. Tracking and aiming at moving targets involve problems in the dynamics of combinations of rigid and elastic bodies, to say nothing of the associated control or optimal control problems. For both artillery shell and rockets the period when the projectile begins free flight involves phenomena, such as muzzle blast or launcher interaction, that limit the accuracy of the presumed initial conditions of the trajectory and hence the accuracy of the location of the predicted impact point. Also associated with this phase of projectile and gun dynamics are problems of muzzle brakes, flash suppressors, and silencers. In addition to everything else that has been mentioned, there are numerous mechanical problems involved in the design of shell and fuses to withstand high g-loadings inside the gun barrel.

Fifth, there obviously must be many problems associated with the analysis of test and experimental data or the correction of malfunctions. These will involve probability theory, statistics, design of experiments, reliability studies, and many matters related to quality control. As a curiosity we mention, in this connection, that the manufacturing lot to lot variations of propellant behavior have to be taken into account in preparing and using firing tables.

Exterior Ballistics deals with the phenomenon of flight from launch site to burst or impact point. In this realm, first there are problems in the dynamics of rigid bodies related to trajectory calculation or flight performance. These are associated with the equations of motion, a system of six second-order differential equations which contain parameters and functions that have to be specified to deal with a particular projectile. The parameters include the mass and principal moments of inertia. The functions are components of forces and moments. In ballistics these are (1) the force due to gravity, and (2) the aerodynamic force and moment expressed in terms of the shape, location, orientation, and state of motion of the projectile. These can be approximated by appeal to a mixture of theory and experiment, to be discussed later. If the motor of a rocket operates during flight, then the mass will vary with time, and the resultant thrust will have to be added somehow to the forces acting on the projectile. To complete the specification of a particular trajectory for shell or rockets, in the generality we have been contemplating, requires initial data for the projectile: (1) location of its center of mass, (2) its orientation, (3) its velocity, and (4) its angular velocity. In addition, generalized meteorological information concerning atmospheric density and wind distribution will be required, since these affect the forces and ultimately the burst or impact point.

An example of motion of several rigid bodies arises, e.g., when it has been found desirable to use subcaliber ammunition. To support the projectile in the gun and to seal in the propellant gases, a sabot is attached to or fitted around the projectile. By this means a finned projectile can be launched from a rifled gun. After emergence, centrifugal force can be used to cause a well-designed sabot to separate and fly away from the projectile. Concern for the safety of gun crews and friendly troops dictates the need to analyze such motion thoroughly.

Second, there are modeling problems associated with simplification of the equations of motion. It should be mentioned that accurate computation of the complete six-degrees-of-freedom equations of motion must be carried out with very small time increments. This implies a very long time to compute a trajectory and a high computing cost per trajectory. But such calculations also yield details about the fine structure of a trajectory which are unnecessary for most applications. For problems that involve great volumes of trajectory calculation, the need to reduce costs or time provides an incentive to try to simplify the equations of motion without unacceptable losses of accuracy. We have already mentioned that the preparation of firing tables is a classical example of this type. Another is the determination of aerodynamic forces and moment functions from photographic records of projectiles in free flight in elaborately instrumented firing ranges. Suppose that the aerodynamic functions have known plausible approximate functional forms that contain several parameters. Then an attempt can be made, e.g., to adjust the parameters to make the corresponding calculated trajectory fit the observed data as well as possible. Simplicity of the equations or approximate solutions is also imperative if small programs stored in a small field computer are to be used as substitutes for firing tables. Speed is an incentive for real time (or faster) tracking and control for automatic weapons, e.g., antiaircraft guns. On-board computers for large missiles also have similar needs for speed, which can be partly achieved by use of simplified models.

Third, there are problems of accuracy and stability. To be employed effectively against a remote target, a weapon with an ordinary high-explosive warhead, which has a limited lethal volume, must be sufficiently accurate. Accuracy depends on the extent to which the weapon's performance is reproducible within tolerances accepted partly on grounds of experience. In general, the solutions of the equations of rigid body motion depend continuously on the projectile parameters, aerodynamic force and moment functions, initial conditions, and meteorological conditions. Loosely speaking, this means that we would expect, with an exception to be discussed later, that small enough variations from standard tabulated conditions should have small enough effects, i.e., the artillerist's problems, insofar as they concern trajectories, should be well posed. To determine what perturbations are actually permissible for a given tolerance is a matter best decided by computation. Many errors originate in manufacturing ammunition, e.g., variations in mass, in location of the center of mass, in moments of inertia, unintentional geometrical asymmetries, or variations of composition and loading of propellants and explosive. All of these perturbations, plus wear or erosion of the gun barrel, produce variations in initial conditions that, in turn, produce changes in range. That there are inevitable variations in meteorological conditions need scarcely be mentioned.

A more serious and difficult problem, with important implications for accuracy, arises as follows. To design projectiles that will fly farther (by virtue of reduced aerodynamic drag) and carry a large payload of explosive, shell shapes that are elongated in the direction of motion and have somewhat streamlined noses must be used. In general, in flight the axis of symmetry of the shell should form a small angle (which in the nature of things cannot be zero) with the tangent to the trajectory of the center of mass. To develop a qualitative description of the motion of the shell, temporarily consider a rather specialized case of motion. Suppose that the center of mass moves on a plane trajectory, and that the instantaneous axis of rotation is normal to this plane. Then a classical theorem in hydrodynamics [30] states that the axis of symmetry will tend to turn to a position athwart the trajectory or to tumble end-over-end. Such a motion would radically increase the aerodynamic drag and disastrously decrease the range. Of course, such behavior would render a projectile useless for close support of friendly troops. If such dangerous behavior is to be prevented, the instantaneous axis of rotation cannot be normal to the plane of the trajectory. A substantial component of angular velocity must be introduced in this plane. This can be accomplished by spinning the projectile by firing it from a rifled gun barrel. Now the angular part of the motion of the shell will become comparable to the well-documented and thoroughly understood oscillatory behavior of a top or gyroscope. Stability criteria involving shell rigid-body parameters and aerodynamic (or ballistic) coefficients, state of motion, and atmospheric properties have been developed [18] but will not be discussed here. For calculations to discuss in-flight observations of misbehavior of unstable projectiles, or to interpret or reduce flight data for satisfactory projectiles, or to determine the effects of design changes on stability, it may be necessary to use the full six-degrees-of-freedom equations of motion, or simplified versions that contain terms not ordinarily used for firing tables calculations.

Fourth, there are problems in fluid mechanics. These may include matters of universal interest, such as similarity principles, e.g., for transonic flow. More frequently, there are efforts to devise analytical approximations for force and moment coefficients for projectiles. If such formulas can be derived by truncating some kind of series expansions, e.g., in terms of relevant parameters, this would be a fertile field for use of symbol-manipulating programs. To validate such mathematical models, one option would be first to determine the flow field about the projectile by solving numerically, for various levels of physical and geometrical complexity and realism, the fundamental equations of hydro- or gas-dynamics. From such information further straightforward computation would yield some of the forces and moments. Another kind of problem, which introduces an internal flow field, occurs for shells that contain liquids or thixotropic substances liquefied by the tremendous setback pressures created at the instant of firing. Motion of the liquid cargo may alter the projectile's stability characteristics and dynamic behavior radically enough to cause unacceptable deterioration in accuracy. Analysis of the basic viscous flow leads to difficult high-order eigenvalue problems, so far satisfactorily solved for only the simplest cavity shapes.

Fifth, there are numerous requirements for quantitative interpretation of test and experimental data, i.e., systems or parameter identification problems. For each production lot of propellant, full-scale proof tests of production or prototype ammunition from an actual gun are performed to obtain values of ballistic coefficients or empirical correction factors, primarily from observations of dispersions of range and lateral deflection. Rocket or bomb flight data can be recorded by photogrammetric or cinetheodolite methods. Radar tracking or on-board radio transmitters can also be used to acquire appropriate forms of trajectory data. Models can be fired in carefully and elaborately instrumented spark ranges to produce numerous crossed photographs from which the location and orientation of the projectile can be seen at many points of a flat trajectory. Similarly, high-speed motion pictures can be made of unsupported models launched for brief periods of flight upstream in a supersonic wind tunnel's test section. For strut-mounted, possibly spinning models, more straightforward and direct measurements of forces and moments can be made in wind tunnels, although these results require correction to eliminate the effects of strut interference on the flow at the base of the model. To turn to flow data less immediately related to forces and moments, in addition to ordinary spark photography, which reveals discontinuities or rapid changes of second partial derivatives of air density in the flow field, schlieren or interferometric techniques [31], respectively, produce photographic records of first derivatives or of the density itself. In particular, the unscrambling of interferometric records of axisymmetric flows (traversed by an interferometer beam perpendicular to the axis of symmetry) reduces to the numerical solution of an Abel integral equation [32].

Terminal Ballistics is concerned with the basic phenomena in the infliction of damage on a great diversity of targets, matched by great diversity of weapons to inflict the damage. Small arms perforate weak targets, generally by impact of a solid metal slug. Grenades, land mines, and some larger projectiles damage unarmored or lightly armored targets with a spray of metal fragments produced by detonation of an explosive charge. Shaped charges perforate heavy armor with an explosively produced hypervelocity metal jet. Bombs, depth charges, torpedoes, and naval mines produce blast waves in air or water which impulsively overload a target, in order to rupture, permanently deform, or weaken it. If the target contains fuel or other combustible material, an attempt may be made to ignite it. In general, the logical scope of terminal ballistics ranges over explosives, explosions, transmission of their energy through some intermediary (air, water, or fragments) to a target, and the mechanical response of the target to the energy it has absorbed.

With regard to computation, first there are problems in chemistry. Searches for new or improved explosives start from chemical, chemical engineering, or physical considerations. In principle, the properties of energetic materials should be derivable from applications of quantum and statistical mechanical concepts. The difficulties involved in doing this in practice and the need to resort to semiempirical methods have been alluded to already in connection with similar theoretical studies of propellants and igniters in interior ballistics.

Second, there are problems in fluid mechanics associated with the behavior of gaseous explosion products (in which chemical reactions may be occurring) and the transmission of blast waves into the surrounding air or water (or even metal!). Even after radical simplification is achieved by ignoring chemical reactions, viscosity, heat conduction, and temperature dependencies of various physical parameters, so that the type of partial differential equations will be hyperbolic, the problems that arise, at least with realistic geometry, can be discouragingly complex and can consume enormous amounts of computer time. To give some idea of the nature of the problems to be solved, let us start with the simplest example of transmission of energy from one gas to another as it occurs in a shock-tube [27]. Consider a closed rigid tube which contains at one end a uniform stagnant "driver"-gas at high pressure, separated by a plastic film membrane from another uniform stagnant "driven"-gas at lower pressure at the other end. When the membrane is ruptured, the driver-gas expands to compress the driven-gas. This generates a complex but qualitatively easily