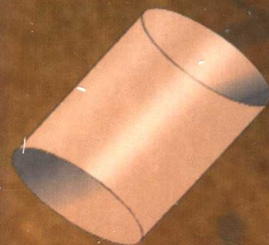
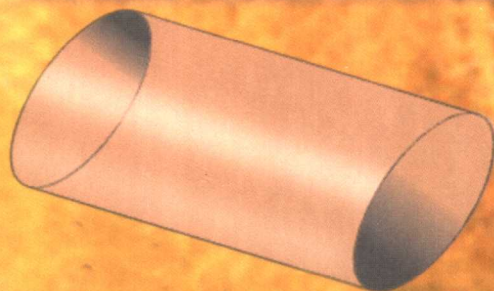


# PREFACULUS

## FUNCTIONS AND GRAPHS

FOURTH EDITION

$$f(x) = 13x + 41 + 1$$



**Raymond A. Barnett**  
**Michael R. Ziegler**  
**Karl E. Byleen**

**FOURTH EDITION**

# **PRECALCULUS: FUNCTIONS AND GRAPHS**

**Raymond A. Barnett**

*Merritt College*

**Michael R. Ziegler**

*Marquette University*

**Karl E. Byleen**

*Marquette University*



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---

# PREFACE

*Precalculus: Functions and Graphs* is one of three books in the authors' precalculus series (see page ii for a brief comparison of all three books). Improvements in this edition evolved out of the generous response from a large number of users of the last and previous editions as well as survey results from instructors, mathematics departments, course outlines, and college catalogs. Fundamental to a book's growth and effectiveness is classroom use and feedback. Now in its fourth edition, *Precalculus: Functions and Graphs* has had the benefit of having a substantial amount of both.

## Emphasis and Style

The text is written for student comprehension. Great care has been taken to write a book that is mathematically correct and accessible to students. Emphasis is on computational skills, ideas, and problem-solving rather than mathematical theory. Most derivations and proofs are omitted except where their inclusion adds significant insight into a particular concept. General concepts and results are usually presented only after particular cases have been discussed.

## Examples and Matched Problems

Over 375 completely worked examples are used to introduce concepts and to demonstrate problem-solving techniques. Each example is followed by a similar matched problem for the student to work while reading the material. This actively involves the student in the learning process. The answers to these matched problems are included at the end of each section for easy reference.

## Exploration, Discussion, and Group Activities

Every section contains Explore-Discuss boxes interspersed at appropriate places to encourage a student to think about a relationship or process before a result is stated or to investigate additional consequences of a development in the text. Verbalization of mathematical concepts, results, and processes is encouraged in these Explore-Discuss boxes, as well as in some matched problems, and in particular problems in almost every exercise set. The Explore-Discuss material can also be used as an in-class or out-of-class group activity. In addition, at the end of every chapter, before the chapter review, is a special chapter group activity that involves a number of the concepts discussed in the chapter. All of these special activities are highlighted to emphasize their importance.

## Exercise Sets


The book contains over 5,600 problems. Each exercise set is designed so that an average or below-average student will experience success and a very capable student will




be challenged. Exercise sets are mostly divided into A (routine, easy mechanics), B (more difficult mechanics), C (difficult mechanics and some theory) level, and Applications. The most difficult application problems are marked with two stars (\*\*), the moderately difficult application problems with one star (\*), and the easier application problems are not marked. Please note that some exercises ask the student to work the exercises by hand, then check their work on a graphing utility. The use of a graphing utility on such exercises is strictly optional.

## Applications

One of the primary objectives of this book is to give the student substantial experience in modeling and solving real-world problems. Enough applications are included to convince even the most skeptical student that mathematics is really useful. An Applications Index is included to help locate particular applications. Most of the applications are simplified versions of actual real-world problems taken from professional journals and professional books. No specialized experience is required to solve any of the applications.

Because many students use this book to prepare for calculus, examples and exercises that are especially pertinent to calculus are marked with the icon .

## Technology

The generic term “graphing utility” is used to refer to any of the various graphing calculators or computer software packages that might be available to a student using this book. Although use of a graphing utility is optional, it is likely that many students and instructors will want to make use of one of these devices. To assist these students and instructors, optional graphing utility activities are included throughout the book beginning in Chapter 3. These include brief discussions in the text, examples or portions of examples solved on a graphing utility, and problems for the student to solve. All of the optional graphing utility material is clearly identified by the following symbol  and can be omitted without loss of continuity, if desired.

## Graphs and Illustrations

All graphs and illustrations in this edition of the text are new. All of the graphs are computer generated to insure mathematical accuracy. Graphing utility screens displayed in the text are actual output from a graphing calculator.

## Important Student Aids

**Annotation** of examples and developments, in small color type, is found throughout the text to help students through critical stages. **Think boxes** (dashed boxes) are used to enclose steps that are usually performed mentally. **Screened boxes** are used to highlight important definitions, theorems, results, and step-by-step processes. **Caution boxes** appear throughout the text where student errors often occur (see Section 1-7). **Functional use of four colors** improves the clarity of many illustrations, graphs, and developments, and guides students through certain critical steps. **Boldface type** is used to introduce new terms and highlight important comments. **Chapter review sections**

include a review of all important terms and symbols and a comprehensive review exercise. **Cumulative review exercises** after every second or third chapter provide additional practice for students. **Answers to review exercises**, keyed to appropriate sections, are included in the back of the book. **Answers to all other odd-numbered problems** are also in the back of the book. **Summaries of formulas and symbols** (keyed to the sections in which they are introduced) are found inside the front and back covers of the text for convenient reference.

## Principal Changes from the Third Edition

As mentioned earlier, exploration and discussion activities have been distributed uniformly throughout the book. These new elements include Explore-Discuss questions in the text and exercise sets, and chapter group activities. The optional material on graphing utilities is also more uniformly distributed.

In Chapter 1, linear equations and their applications are now covered in one section, and a new section on systems of linear equations and their applications has been added.

In Chapter 2, the sections on aids to graphing functions and operations on functions have been combined to present these related topics in a more concise fashion. The section on rational functions has been moved to Chapter 3.

The material in Chapter 3 has been extensively revised and reorganized, due in no small part to the effect that graphing utilities have had on some of these topics. Section 3-2 now deals with techniques for finding exact zeros, including all rational zeros, and Section 3-3 covers approximating real zeros.

In Chapter 6, techniques for solving trigonometric equations have been condensed into a single section, again reflecting some of the impact that technology has had on equation solving. The treatment of polar coordinates and polar graphs has also been condensed into a single section in Chapter 7.

Since solving two linear equations in two variables is now covered in Chapter 1, the first section of Chapter 8 now concentrates on graphical methods and matrix methods. In Chapter 9, matrix addition and multiplication are now covered in a single section.

Counting techniques have been moved to Chapter 10 and the remaining probability material has been deleted.

## Supplements

A comprehensive set of ancillary materials for both the student and the instructor is available for use with this text.

**Instructor's Edition:** This ancillary contains all of the material from the Student Edition of the text, in addition to answers to all of the exercises in the textbook. (The Student Edition contains answers to selected exercises.)

**Student Solutions Manual:** This supplement, written by Fred Saifer of the City College of San Francisco, is available for sale to the student, and includes detailed solutions to all odd-numbered problems and most review exercises.


**Instructor's Solutions Manual:** This manual, written by John R. Martin of Tarrant County Junior College, provides solutions to even-numbered problems and answers to all problems in the text.

**Instructor's Resource Manual:** This supplement provides transparency masters and

sample tests, prepared by Mark Serebransky of Camden County College, for each chapter in the text.

**Print and Computerized Testbanks:** A Computerized Testbank, prepared by ESA, Inc., with assistance from Thomas Roe of South Dakota State University, is available that provides a variety of formats to allow the instructor to create tests using both algorithmically generated test questions and those from a static testbank. This testing system enables the instructor to choose questions either manually or randomly by section, question types, difficulty level, and other criteria. This testing software is available for PC and Macintosh computers. A softcover print version of the testbank, prepared by Mark Stevenson of Oakland Community College, provides most questions found in the computerized version.

**Barnett/Ziegler/Byleen Video Series:** Course videotapes, created new for this edition, provide students with additional reinforcement of the topics presented in the book. These videos are keyed specifically to the text and feature an effective combination of learning techniques, including personal instruction, state-of-the-art graphics, and real-world applications.

**Interactive Diagram CD-ROM:** This software package is available for sale to the student. This CD contains 45 Interactive Diagrams that are designed for use with this textbook. Each Interactive Diagram (ID) is a separate Java Applet that contains an illustration that can be manipulated by the user for further conceptual understanding of the topic presented. For each section of the text where an ID has been created, an icon  has been placed in the margin.

**Multimedia Tutorial:** This multimedia supplement is a self-paced tutorial specifically linked to the text and reinforces topics through unlimited opportunities to review concepts and practice problem solving.

In addition to the supplements listed above, a number of other technology and Web-based ancillaries are under development; they will support the ever-changing technology needs in college algebra and precalculus. For further information about these or any supplements, please contact your local WCB/McGraw-Hill sales representative.

## Accuracy

Because of the careful checking and proofing by a number of mathematics instructors (acting independently), the authors and publisher believe this book to be substantially error-free. For any errors remaining, the authors would be grateful if they were sent to: Michael R. Ziegler, 509 West Dean Court, Fox Point, WI 53217; or, by email, to [michaelziegler@execpc.com](mailto:michaelziegler@execpc.com).

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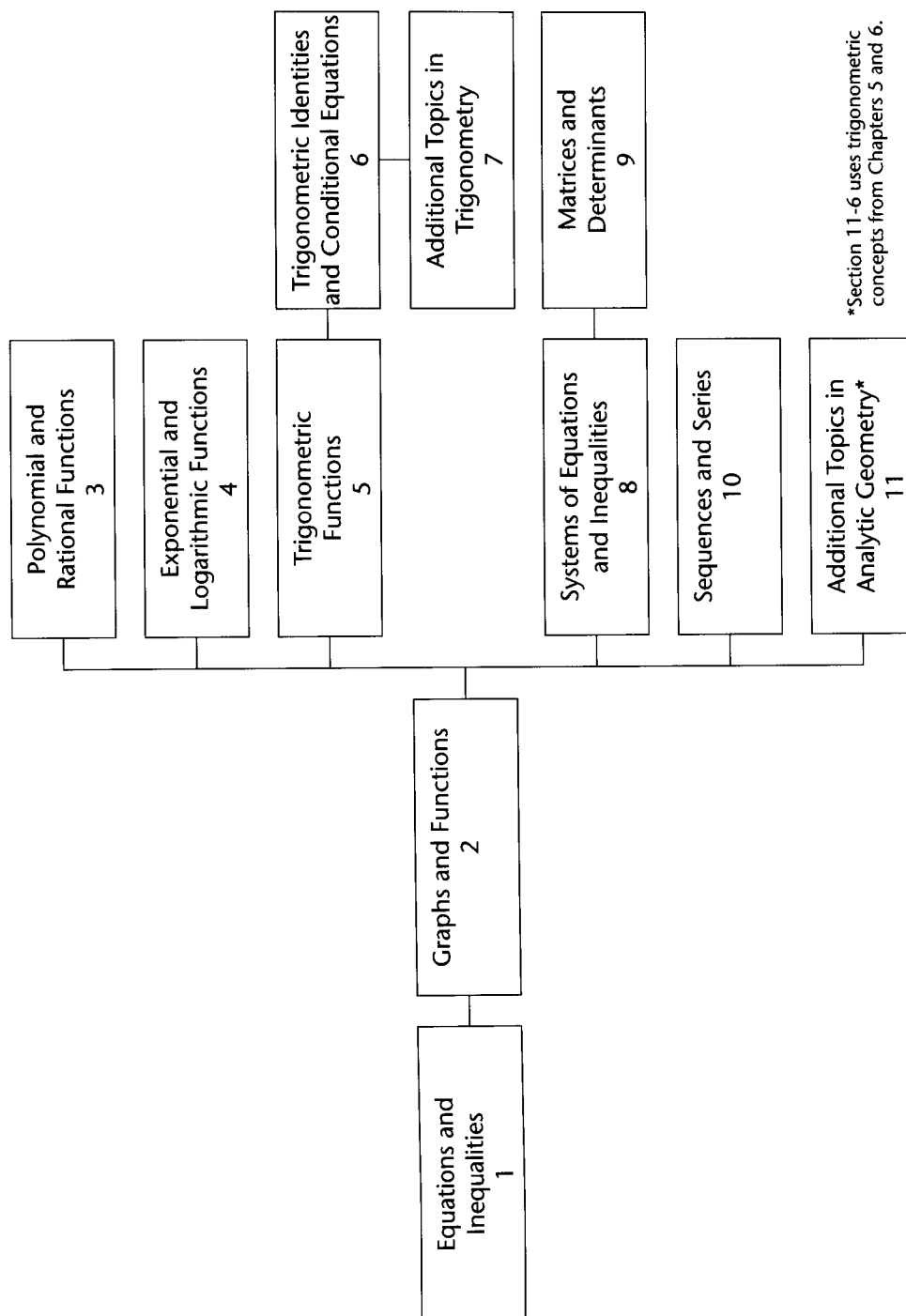
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Producing this new edition with the help of all these extremely competent people has been a most satisfying experience.

R. A. Barnett  
M. R. Ziegler  
K. E. Byleen

# CHAPTER DEPENDENCIES



\*Section 11-6 uses trigonometric concepts from Chapters 5 and 6.


# TO THE STUDENT

The following suggestions are made to help you get the most out of this book and your efforts.

As you study the text we suggest a five step-process. For each section:

- |  |   |  |
|--|---|--|
| <ol style="list-style-type: none"><li>1. Read a mathematical development.</li><li>2. Work through the illustrative examples.</li><li>3. Work the matched problem.</li><li>4. Review the main ideas in the section.</li><li>5. Work the assigned exercises at the end of the section.</li></ol> | } | <p>Repeat the 1-2-3 cycle until the section is finished.</p> |
|--|---|--|

All of this should be done with a calculator, plenty of paper, pencils, and a wastebasket at hand. In fact, no mathematics text should be read without pencil and paper in hand; mathematics is not a spectator sport. Just as you cannot learn to swim by watching someone else swim, you cannot learn mathematics by simply reading **worked examples—you must work problems**, lots of them.

If you have a graphing calculator or access to a computer with mathematical software, such as Maple or Mathematica, you should pay particular attention to the remarks, explore-discuss boxes, and exercises marked with . This is optional material that we have included to help you learn to make effective use of technology as part of the problem-solving process. If you do not have one of these devices, please omit material so marked, since it may involve calculations that cannot be done by hand.

If you have difficulty with the course, then, in addition to doing the regular assignments, spend more time on the examples and matched problems and work more A exercises, even if they are not assigned. If you find the course too easy, then work more C exercises and applied problems, even if they are not assigned.

Raymond A. Barnett  
Michael R. Ziegler  
Karl E. Byleen

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# CHAPTER 1

# EQUATIONS AND INEQUALITIES

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- 1-8 Polynomial and Rational Inequalities

Chapter 1 Group Activity:  
Rates of Change

Chapter 1 Review

$$f(x) = |3x + 4| + 1$$



One of the important uses of algebra is the solving of equations and inequalities. In this chapter we look at techniques for solving linear and nonlinear equations and inequalities. In addition, we consider a number of applications that can be solved using these techniques. Additional techniques for solving polynomial equations will be discussed in Chapter 3.

## SECTION 1-1 Linear Equations and Applications



- Equations
- Solving Linear Equations
- A Strategy for Solving Word Problems
- Number and Geometric Problems
- Rate–Time Problems
- Mixture Problems
- Some Final Observations on Linear Equations

- **Equations** An **algebraic equation** is a mathematical statement that relates two algebraic expressions involving at least one variable. Some examples of equations with  $x$  as the variable are

$$\begin{array}{ll} 3x - 2 = 7 & \frac{1}{1+x} = \frac{x}{x-2} \\ 2x^2 - 3x + 5 = 0 & \sqrt{x+4} = x - 1 \end{array}$$

The **replacement set**, or **domain**, for a variable is defined to be the set of numbers that are permitted to replace the variable.

### Assumption

### On Domains of Variables

Unless stated to the contrary, we assume that the domain for a variable is the set of those real numbers for which the algebraic expressions involving the variable are real numbers.

For example, the domain for the variable  $x$  in the expression

$$2x - 4$$

is  $R$ , the set of all real numbers, since  $2x - 4$  represents a real number for all replacements of  $x$  by real numbers. The domain of  $x$  in the equation

$$\frac{1}{x} = \frac{2}{x-3}$$

is the set of all real numbers except 0 and 3. These values are excluded because the left member is not defined for  $x = 0$  and the right member is not defined for  $x = 3$ .

The left and right members represent real numbers for all other replacements of  $x$  by real numbers.

The **solution set** for an equation is defined to be the set of elements in the domain of the variable that makes the equation true. Each element of the solution set is called a **solution**, or **root**, of the equation. To **solve an equation** is to find the solution set for the equation.

An equation is called an **identity** if the equation is true for all elements from the domain of the variable. An equation is called a **conditional equation** if it is true for certain domain values and false for others. For example,

$$2x - 4 = 2(x - 2) \quad \text{and} \quad \frac{5}{x^2 - 3x} = \frac{5}{x(x - 3)}$$

are identities, since both equations are true for all elements from the respective domains of their variables. On the other hand, the equations

$$3x - 2 = 5 \quad \text{and} \quad \frac{2}{x - 1} = \frac{1}{x}$$

are conditional equations, since, for example, neither equation is true for the domain value 2.

Knowing what we mean by the solution set of an equation is one thing; finding it is another. To this end we introduce the idea of equivalent equations. Two equations are said to be **equivalent** if they both have the same solution set for a given replacement set. A basic technique for solving equations is to perform operations on equations that produce simpler equivalent equations, and to continue the process until an equation is reached whose solution is obvious.

Application of any of the properties of equality given in Theorem 1 will produce equivalent equations.

### Theorem 1

#### Properties of Equality

For  $a$ ,  $b$ , and  $c$  any real numbers:

1. If  $a = b$ , then  $a + c = b + c$ .
2. If  $a = b$ , then  $a - c = b - c$ .
3. If  $a = b$ , then  $ca = cb$ ,  $c \neq 0$ .
4. If  $a = b$ , then  $\frac{a}{c} = \frac{b}{c}$ ,  $c \neq 0$ .
5. If  $a = b$ , then either may replace the other in any statement without changing the truth or falsity of the statement.

**Addition Property**  
**Subtraction Property**  
**Multiplication Property**  
**Division Property**  
**Substitution Property**

### • Solving Linear Equations

We now turn our attention to methods of solving *first-degree*, or *linear*, equations in one variable.

**DEFINITION 1****Linear Equation in One Variable**

Any equation that can be written in the form

$$ax + b = 0 \quad a \neq 0 \quad \text{Standard Form}$$

where  $a$  and  $b$  are real constants and  $x$  is a variable, is called a **linear**, or **first-degree, equation** in one variable.

$5x - 1 = 2(x + 3)$  is a linear equation, since it can be written in the standard form  $3x - 7 = 0$ .

**EXAMPLE 1 Solving a Linear Equation**

Solve  $5x - 9 = 3x + 7$  and check.

**Solution** We use the properties of equality to transform the given equation into an equivalent equation whose solution is obvious.

$$5x - 9 = 3x + 7 \quad \text{Original equation}$$

$$5x - 9 + 9 = 3x + 7 + 9 \quad \text{Add 9 to both sides.}$$

$$5x = 3x + 16 \quad \text{Combine like terms.}$$

$$5x - 3x = 3x + 16 - 3x \quad \text{Subtract } 3x \text{ from both sides.}$$

$$2x = 16 \quad \text{Combine like terms.}$$

$$\frac{2x}{2} = \frac{16}{2} \quad \text{Divide both sides by 2.}$$

$$x = 8 \quad \text{Simplify.}$$

The solution set for this last equation is obvious:

Solution set:  $\{8\}$

And since the equation  $x = 8$  is equivalent to all the preceding equations in our solution,  $\{8\}$  is also the solution set for all these equations, including the original equation. [Note: If an equation has only one element in its solution set, we generally use the last equation (in this case,  $x = 8$ ) rather than set notation to represent the solution.]

**Check**  $5x - 9 = 3x + 7 \quad \text{Original equation}$

$$5(8) - 9 \stackrel{?}{=} 3(8) + 7 \quad \text{Substitute } x = 8.$$

$$40 - 9 \stackrel{?}{=} 24 + 7 \quad \text{Simplify each side.}$$

$$31 \stackrel{\checkmark}{=} 31 \quad \text{A true statement}$$