Robust Technology with Analysis of Interference in Signal Processing

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Baku, July 2002 *Telman Aliev*

Preface

In this book the well-known methods of analysis of noisy signals are revised for the first time. A computer technology of analysis of noise as an information carrier is developed. This technology allows us to determine the estimate of the noise variance, its correlation function, its distribution law, as well as the coefficient of correlation and cross correlation function between the noise and the signal. For the case when classical conditions are not fulfilled the theoretical fundamentals of robust statistical analysis are developed. That makes it possible to improve the estimate of correlation and spectral characteristics, correlation matrix stipulation, and identification adequacy considerably. All of this allows us to realize the solution of a large number of the most important problems, which was impossible in the framework of classical algorithms. The results of computer experiments, examples of solving various problems of forecasting, diagnostics, identification in fields of seismology, petroleum prospecting, oil-gas production, and oil chemistry are given.

This book is intended for specialists who work in various fields of science and technology on applied problems on personal computers.

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Chapter 1

NEEDS IN DEVELOPMENT OF STATISTICAL ANALYSIS TECHNOLOGY

1.1 Opportunities of Robust Statistical Analysis Technology

The mathematical fundamentals of the modern methods of experimental data analysis were developed by A.N. Kolmogorov, N. Wiener, and A.Ya. Khinchin. The most widely used algorithms for processing of measuring information are created on its basis. It is recommended to use them in the case when the classical conditions are realized, i.e., the analyzed signals are stationary, they obey the normal distribution law, the correlation between the noise and useful signal is equal to zero, and the noise is white noise. For example, while using the spectral correlation and regression analysis, the theory of random processes, and the theory of pattern recognition, we assume that the classical conditions hold true. For the cases when these assumptions are acceptable one can get satisfactory results. But there is a wide class of problems where these assumptions are not acceptable and the results of solving these problems are unsatisfactory. In this case in solving a large number of important problems one gets false results, which lead to disastrous consequences that severely influence economic and social life. It is explained by the following fact. In creating traditional algorithms the specific features of forming real signals are not sufficiently taken into consideration. For this reason the application of these algorithms for solving the most important problems does not give desired results. Therefore it is a problem of great theoretical and practical importance to develop algorithms providing the robustness of calculating appropriate estimates so that their properties remain satisfactory even if the classical conditions are violated considerably. All of this requires the revision of traditional technologies of statistical analysis of experimental data. For this purpose for a long time we carried out the researches the results of which are published in the following journals:

- *Automatic Control and Computer Sciences*: 1998 N2, 5; 1999 N5; 2000 N1, 4, 5; 2001 N1, 2, 4; 2002 N1;
- *Optoelectronics, Instrumentation and Data Processing*: 1995 N4; 1997 N3; 1998 N2; 2001 N5.

Some works are published in journals of the Russian Academy of Sciences:

- *Journal of Computer and Systems Sciences International*: 1995 N3; 1997 N3; 1998 N1;
- *Automation and Remote Control*: 1998 N5, 6;
- *Oil Industry*: 1998 N9, 10; 2000 N1; 2001 N3; 2002 N3.

The present monograph, *Robust Technology with Analysis of Interference in Signal Processing*, is written on the basis of these works. We shall use the terms "noise" and "interference" as synonyms. For the first time anywhere, the theoretical fundamentals of analysis of noise as a carrier of useful information are given and for the case when it is not required to fulfill the classical conditions the technology of statistical signal processing is created. This eliminates the difficulties with the application of the theory of spectral, correlation, and regression analysis, the theory of identification, and the theory of pattern recognition during solving a large number of the most important problems in various fields.

There are many books[1, 2, 3, 4] in which the authors set out the methods and algorithms for analyzing experimental data. For the first time, we revise the well-known technologies of statistical analysis and suggest robust algorithms providing the independence of the estimates of statistical and spectral characteristics of a total signal from the influence of outside factors. We find that if the estimates of the useful signal do not change in time, the estimates of the total signal, which is formed during changing outside factors, are equal to the estimates of the useful signal.

The possibility of providing the robustness of the required estimates irrespectively of fulfilling corresponding conditions is of great applied importance. Thanks to providing robustness of the mentioned estimates it is possible to improve the results of the correlation and spectral analysis, the stipulation of the correlation matrices, the adequacy of the identification, and it is possible to eliminate serious difficulties in applying the theory of random processes, the theory of pattern recognition, methods of identification and control, and so on in solving many widespread practical problems.

The position-binary technology of statistical analysis of cyclic processes allowing us to obtain estimates with an accuracy that is comparable with the accuracy of measuring devices is also suggested in this work.

Based on the above there is an opportunity for timely recognition and forecasting of the microchanges in the states of monitored and controlled objects.

1.2 Features of Improving Correlation Matrix Stipulation

It should be particularly noted where in solving matrix correlation equations numerically under conditions where the correlation matrix elements involve errors and their values are known approximately, the solution involves inaccuracies, too.

Theoretical solution of the system

$$\vec{R}_{\overset{\circ}{x}\overset{\circ}{x}}(0)\cdot\vec{B}=\vec{R}_{\overset{\circ}{x}\overset{\circ}{y}}(0) \tag{1.1}$$

is given by the formula

$$\vec{B}=\vec{R}^{-1}_{\overset{\circ}{x}\overset{\circ}{x}}(0)\cdot\vec{R}_{\overset{\circ}{x}\overset{\circ}{y}}(0), \tag{1.2}$$

where $\vec{R}_{\overset{\circ}{x}\overset{\circ}{x}}(0)=\left\|R_{\overset{\circ}{x}_i\overset{\circ}{x}_j}(0)\right\|$ is an estimate matrix of auto- and cross correlation functions $R_{\overset{\circ}{x}_i\overset{\circ}{x}_i}(0)$, $R_{\overset{\circ}{x}_i\overset{\circ}{x}_j}(0)$ with time shift $\tau=0$ of centered input signals $\overset{\circ}{x}_i(t)=x_i(t)$ - m_{x_i}; $\vec{R}_{\overset{\circ}{x}\overset{\circ}{y}}(0)=\left\|R_{\overset{\circ}{x}_i\overset{\circ}{y}}(0)\right\|$ is a column matrix of estimates of cross correlation functions $R_{\overset{\circ}{x}_i\overset{\circ}{y}}(0)$ with time shift $\tau=0$ between centered input $\overset{\circ}{x}_i(t)$ and output signals $\overset{\circ}{y}(t)=y(t)-m_y$, $\vec{B}=\|b_i\|$ is the column matrix of coefficients of regression equation; $\vec{R}^{-1}_{\overset{\circ}{x}\overset{\circ}{x}}(0)$ is the matrix inverse with respect to $\vec{R}_{\overset{\circ}{x}\overset{\circ}{x}}(0)$; m_{x_i}, m_{x_j}, m_y are mathematical expectations of signals $x_i(t)$, $x_j(t)$, $y(t)$, respectively.

As known, $\vec{R}^{-1}_{\overset{\circ}{x}\overset{\circ}{x}}(0)$ exists only in the case where the determinant of matrix $\vec{R}_{\overset{\circ}{x}\overset{\circ}{x}}(0)$ is not equal to zero, that is, $\left|\vec{R}_{\overset{\circ}{x}\overset{\circ}{x}}(0)\right| \neq 0$. However, it may happen that in calculating the determinant accurately, proceeding from the approximate values of correlation matrix elements instead of accurate ones, the determinant turns out to differ from zero, but the change of the correlation matrix elements within the given precision can yield a correlation matrix whose determinant is equal to zero.

The inverse matrix $\vec{R}^{-1}_{\overset{\circ}{x}\overset{\circ}{x}}(0)$ is called stable if small changes of elements $R_{\overset{\circ}{x}_i \overset{\circ}{x}_j}(0)$ of the matrix $\vec{R}_{\overset{\circ}{x}\overset{\circ}{x}}(0)$ involve small change of elements of the inverse matrix. There is no doubt that for the stability of the inverse correlation matrix $\vec{R}^{-1}_{\overset{\circ}{x}\overset{\circ}{x}}(0)$, it is necessary that the determinant of the matrix not be too small.

The known estimate by Hadamard[2] for the determinant, when the matrix can be considered to be stable, has the form

$$\det \vec{R}_{\overset{\circ}{x}\overset{\circ}{x}}(0) \leq \sqrt{\prod_{i=1}^{n} \sum_{j=1}^{n} \left| R_{\overset{\circ}{x}_i \overset{\circ}{x}_j}(0) \right|^2}$$

and means that the determinant module can reach up to the value $\sqrt{\prod_{i=1}^{n} \sum_{j=1}^{n} \left| R_{\overset{\circ}{x}_i \overset{\circ}{x}_j}(0) \right|^2}$ with the same sums of squares of modules of line elements.

In a general form a change of each element $dR^{-1}_{\overset{\circ}{x}_k \overset{\circ}{x}_l}(0)$ of the inverse correlation matrix, caused by a change of element $R_{\overset{\circ}{x}_i \overset{\circ}{x}_j}(0)$, is equal to this change $dR_{\overset{\circ}{x}_i \overset{\circ}{x}_j}(0)$ multiplied by production of two elements of the inverse matrix:

$$dR^{-1}_{\overset{\circ}{x}_k \overset{\circ}{x}_l}(0) = -\sum_{i,j} R^{-1}_{\overset{\circ}{x}_k \overset{\circ}{x}_i}(0) R^{-1}_{\overset{\circ}{x}_j \overset{\circ}{x}_l}(0) dR_{\overset{\circ}{x}_i \overset{\circ}{x}_j}(0).$$

The correlation matrix is considered to be poorly stipulated if its corresponding inverse matrix is unstable. It is natural that a linear correlation equation system with poorly stipulated matrix is less stable, that is, its solution strongly changes with small changes of elements $R_{\overset{\circ}{x}_i\overset{\circ}{x}_i}(0)$, $R_{\overset{\circ}{x}_i\overset{\circ}{x}_j}(0)$ of correlation matrix $\vec{R}_{\overset{\circ}{x}\overset{\circ}{x}}(0)$ of auto- and cross correlation functions of input signals and elements $R_{\overset{\circ}{x}_i\overset{\circ}{y}}(0)$ of correlation matrix $\vec{R}_{\overset{\circ}{x}\overset{\circ}{y}}(0)$ of cross correlation functions between input and output signals.

It is natural that a system of linear correlation equations with poorly stipulated matrix is less stable, i.e., its solution strongly changes with small changes of the values of correlation functions $R_{\overset{\circ}{x}_i\overset{\circ}{x}_i}(0)$, $R_{\overset{\circ}{x}_i\overset{\circ}{x}_j}(0)$, $R_{\overset{\circ}{x}_i\overset{\circ}{y}}(0)$. Thus, "trifle" of the determinant is the reason for poor stipulation of the correlation matrix. However, only the value of the determinant cannot characterize the matrix stipulation.

For matrix characteristic with reference to its stipulation different characteristics are proposed which are called numbers of stipulation. These two Turing numbers are

$$N\text{-number} = \frac{1}{n} N\left(\vec{R}_{\overset{\circ}{x}\overset{\circ}{x}}(0)\right) N\left(\vec{R}^{-1}_{\overset{\circ}{x}\overset{\circ}{x}}(0)\right) = v\left(\vec{R}_{\overset{\circ}{x}\overset{\circ}{x}}(0)\right), \tag{1.3}$$

$$M\text{ - number} = \frac{1}{n} N\left(\vec{R}_{\overset{\circ}{x}\overset{\circ}{x}}(0)\right) N\left(\vec{R}^{-1}_{\overset{\circ}{x}\overset{\circ}{x}}(0)\right) = v\left(\vec{R}_{\overset{\circ}{x}\overset{\circ}{x}}(0)\right), \tag{1.4}$$

where $N\left(\vec{R}_{\overset{\circ}{x}\overset{\circ}{x}}(0)\right) = \sqrt{Sp\vec{R}^{T}_{\overset{\circ}{x}\overset{\circ}{x}}(0) \cdot \vec{R}_{\overset{\circ}{x}\overset{\circ}{x}}(0)}$ or

$$N = \frac{1}{n}\sqrt{\sum_{i,j}^{n}\left|R_{\overset{\circ}{x}_i\overset{\circ}{x}_j}(0)\right|^2}\sqrt{\sum_{i,j}^{n}\left|R^{-1}_{\overset{\circ}{x}_i\overset{\circ}{x}_j}(0)\right|^2} =$$

$$= \frac{1}{n}\sqrt{\sum_{i,j}^{n}\left|R_{\overset{\circ}{x}_i\overset{\circ}{x}_j}(0)\right|^2}\sqrt{\sum_{i,j}^{n}\left|P_{\overset{\circ}{x}_i\overset{\circ}{x}_j}(0)\big/\det\vec{R}_{\overset{\circ}{x}\overset{\circ}{x}}(0)\right|^2},$$

$R^{-1}_{\overset{\circ}{x}_i\overset{\circ}{x}_j}(0)$ are the elements of the matrix $\vec{R}^{-1}_{\overset{\circ}{x}\overset{\circ}{x}}(0)$ inverse with reference to the matrix $\vec{R}_{\overset{\circ}{x}\overset{\circ}{x}}(0)$; $P_{\overset{\circ}{x}_i\overset{\circ}{x}_j}(0)$ is algebraic complement of elements $R_{\overset{\circ}{x}_i\overset{\circ}{x}_j}(0)$;

$\vec{R}^T_{\mathring{x}\mathring{x}}(0)$ is the matrix transposed with reference to $\vec{R}_{\mathring{x}\mathring{x}}(0)$; $M\left(\vec{R}_{\mathring{x}\mathring{x}}(0)\right) = n \cdot \max\limits_{ij}\left|R_{\mathring{x}_i \mathring{x}_j}(0)\right|$ are the norms of the matrices:

$$P\text{ - number} = \frac{\max|\lambda_i|}{\min|\lambda_i|} = \rho\left(\vec{R}_{\mathring{x}\mathring{x}}(0)\right) \tag{1.5}$$

is Tod number, where λ_i are eigenvalues of the matrix $\vec{R}_{\mathring{x}\mathring{x}}(0)$;

$$H\text{ - number} = \left\|\vec{R}_{\mathring{x}\mathring{x}}(0)\right\|\left\|\vec{R}^{-1}_{\mathring{x}\mathring{x}}(0)\right\|, \tag{1.6}$$

where $\left\|\vec{R}_{\mathring{x}\mathring{x}}(0)\right\| = \sqrt{\lambda_1}$ is the norm of the matrix; λ_1 is the eigenvalue of the matrix of $\vec{R}'_{\mathring{x}\mathring{x}}(0) \cdot \vec{R}_{\mathring{x}\mathring{x}}(0)$; $\vec{R}'_{\mathring{x}\mathring{x}}(0)$ is the matrix which is conjugate to $\vec{R}_{\mathring{x}\mathring{x}}(0)$. The conjugate complex matrix $\overline{\vec{R}_{\mathring{x}\mathring{x}}(0)}$ is obtained by replacing matrix elements with conjugate complex numbers. If elements of $\vec{R}_{\mathring{x}\mathring{x}}(0)$ are real, then $\overline{\vec{R}_{\mathring{x}\mathring{x}}(0)} = \vec{R}_{\mathring{x}\mathring{x}}(0)$. The matrix $\vec{R}'_{\mathring{x}\mathring{x}}(0) = \overline{\vec{R}^T_{\mathring{x}\mathring{x}}(0)}$ which is conjugate complex to the transposed one is called the matrix which is conjugate to the matrix $\vec{R}_{\mathring{x}\mathring{x}}(0)$. It is obvious that $\left(\vec{R}'_{\mathring{x}\mathring{x}}(0)\right)' = \vec{R}_{\mathring{x}\mathring{x}}(0)$. If matrix $\vec{R}_{\mathring{x}\mathring{x}}(0)$ is real, then the matrix which is conjugate to it coincides with the transposed one.

It is easy to see that

$$H\text{ - number} = \sqrt{\frac{\mu_1}{\mu_n}} = \eta\left(\vec{R}_{\mathring{x}\mathring{x}}(0)\right), \tag{1.7}$$

where μ_1 and μ_n are the maximum and minimum eigenvalues of matrix $\vec{R}^T_{\mathring{x}\mathring{x}}(0) \cdot \vec{R}_{\mathring{x}\mathring{x}}(0)$. It is clear that

$$\eta\left(\vec{R}_{\mathring{x}\mathring{x}}(0)\right) = \sqrt{\rho\left(\vec{R}^T_{\mathring{x}\mathring{x}}(0) \cdot \vec{R}_{\mathring{x}\mathring{x}}(0)\right)}. \tag{1.8}$$

For symmetrical matrices the stipulation number-P coincides with number-H .

Note inequalities relating these numbers:

$$\nu\left(\vec{R}_{\overset{\circ}{x}\overset{\circ}{x}}(0)\right)\le\mu\left(\vec{R}_{\overset{\circ}{x}\overset{\circ}{x}}(0)\right)\le n^2\nu\left(\vec{R}_{\overset{\circ}{x}\overset{\circ}{x}}(0)\right), \quad (1.9)$$

$$\nu\left(\vec{R}_{\overset{\circ}{x}\overset{\circ}{x}}(0)\right)\le\eta\left(\vec{R}_{\overset{\circ}{x}\overset{\circ}{x}}(0)\right)\le n\nu\left(\vec{R}_{\overset{\circ}{x}\overset{\circ}{x}}(0)\right), \quad (1.10)$$

$$\rho\left(\vec{R}_{\overset{\circ}{x}\overset{\circ}{x}}(0)\right)\le\eta\left(\vec{R}_{\overset{\circ}{x}\overset{\circ}{x}}(0)\right).$$

For orthogonal matrices:

$$\nu\left(\vec{R}_{\overset{\circ}{x}\overset{\circ}{x}}(0)\right)=\eta\left(\vec{R}_{\overset{\circ}{x}\overset{\circ}{x}}(0)\right)=\rho\left(\vec{R}_{\overset{\circ}{x}\overset{\circ}{x}}(0)\right)=1 .$$

All the stipulation numbers are not less than one. The stipulation numbers $\nu\left(\vec{R}_{\overset{\circ}{x}\overset{\circ}{x}}(0)\right)$ and $\eta\left(\vec{R}_{\overset{\circ}{x}\overset{\circ}{x}}(0)\right)$ have the following probabilistic sense. Consider a system of linear correlation equations $\vec{R}_{\overset{\circ}{g}\overset{\circ}{g}}(0)\cdot\vec{B}=\vec{R}_{\overset{\circ}{x}\overset{\circ}{y}}(0)$, where correlation column-matrix $\vec{R}_{\overset{\circ}{x}\overset{\circ}{y}}(0)$ is given exactly, and the values of elements $R_{\overset{\circ}{g}_i\overset{\circ}{g}_j}(0)$ of correlation matrix $\vec{R}_{\overset{\circ}{g}\overset{\circ}{g}}(0)$ consist of values $R_{\overset{\circ}{x}_i\overset{\circ}{x}_j}(0)$ and error $\Lambda_{\overset{\circ}{x}_i\overset{\circ}{x}_j}(0)$ with identical variance $D\left[\Lambda_{\overset{\circ}{x}_i\overset{\circ}{x}_j}(0)\right]$ whose value is assumed to be very small in comparison with values $R_{\overset{\circ}{x}_i\overset{\circ}{x}_j}(0)$. Then the stipulation number-N shows by how many times the ratio of mean-root-square value of errors of the coefficients $\vec{B}$ to mean-root-square value of errors of coefficients themselves is greater than the ratio of mean-root-square value of errors $\Lambda_{\overset{\circ}{x}_i\overset{\circ}{x}_j}(0)$ of estimates $\vec{R}_{\overset{\circ}{g}_i\overset{\circ}{g}_j}(0)$ to mean-root-square value of $\vec{R}_{\overset{\circ}{x}_i\overset{\circ}{x}_j}(0)$ themselves. Number-H gives the ratio of the maximum semiaxis