

Robust Technology with Analysis of Interference in Signal Processing

TELMAN ALIEV



Robust Technology with Analysis of Interference in Signal Processing

Telman Aliev

*Academy of Sciences of Azerbaijan
Baku, Azerbaijan*

Kluwer Academic / Plenum Publishers
New York, Boston, Dordrecht, London, Moscow

Library of Congress Cataloging-in-Publication Data

Aliev, T. A.

Robust technology with analysis of interference in signal processing/Telman Aliev.
p. cm.

Includes bibliographical references and index.

ISBN 0-306-47479-4

1. Signal processing—Mathematics. 2. Signal processing—Quality control. 3.
Electromagnetic interference. I. Title.

TK5102.9 .A445 2003

621.382'2—dc21

2002040701

ISBN 0-306-47479-4

©2003 Kluwer Academic / Plenum Publishers, New York
233 Spring Street, New York, N.Y. 10013

<http://www.wkap.com>

10 9 8 7 6 5 4 3 2 1

A C.I.P. record for this book is available from the Library of Congress

All rights reserved

No part of this book may be reproduced, stored in a retrieval system, or transmitted in any form or by any means, electronic, mechanical, photocopying, microfilming, recording, or otherwise, without written permission from the Publisher, with the exception of any material supplied specifically for the purpose of being entered and executed on a computer system, for exclusive use by the purchaser of the work

Printed in the United States of America

Robust Technology with Analysis of Interference in Signal Processing

Acknowledgments

I am very obliged to Professor Genshiro Kitagawa, Professor Dr. Otto Moeschlin, and Ph.D. researcher Elena Kozlovskaya for positive comments on my works.

I am grateful to Academic Publishers Kluwer and particularly to editor Ana Bozicevic for her interest in this monograph.

I must note the great help rendered by my colleagues A.V. Handzel, T.A. Alizadeh, N.G. Golichenko, and U.M. Mamedova during the preparation of this manuscript.

Baku, July 2002

Telman Aliev

Preface

In this book the well-known methods of analysis of noisy signals are revised for the first time. A computer technology of analysis of noise as an information carrier is developed. This technology allows us to determine the estimate of the noise variance, its correlation function, its distribution law, as well as the coefficient of correlation and cross correlation function between the noise and the signal. For the case when classical conditions are not fulfilled the theoretical fundamentals of robust statistical analysis are developed. That makes it possible to improve the estimate of correlation and spectral characteristics, correlation matrix stipulation, and identification adequacy considerably. All of this allows us to realize the solution of a large number of the most important problems, which was impossible in the framework of classical algorithms. The results of computer experiments, examples of solving various problems of forecasting, diagnostics, identification in fields of seismology, petroleum prospecting, oil-gas production, and oil chemistry are given.

This book is intended for specialists who work in various fields of science and technology on applied problems on personal computers.

Contents

CHAPTER 1. NEEDS IN DEVELOPMENT OF STATISTICAL ANALYSIS TECHNOLOGY	1
1.1 Opportunities of Robust Statistical Analysis Technology.....	1
1.2 Features of Improving Correlation Matrix Stipulation.....	3
1.3 Analysis of Factors of Poor Stipulation of Correlation Matrices	8
1.4 Analysis of Methods of Improving Correlation Matrix Stipulation.	13
CHAPTER 2. ROBUST METHODS FOR DETERMINING ESTIMATES OF STATISTICAL CHARACTERISTICS.....	21
2.1 Methods for Determining Noise Variance	21
2.2 Robust Method for Improving Estimates of Autocorrelation Functions	31
2.3 Robust Method for Improving Estimates of Cross Correlation Functions	39

CHAPTER 3. ROBUST TECHNOLOGY OF CORRELATION ANALYSIS.....	47
3.1 Algorithms for Determining Estimates of Noise Variance	47
3.2 Algorithms for Determining Magnitude Providing Robustness of Estimates of Auto- and Cross Correlation Functions.....	52
3.3 Robust Algorithms for Improving Estimates of Auto- and Cross Correlation Functions.....	59
CHAPTER 4. ROBUST TECHNOLOGY FOR IMPROVING CORRELATION MATRIX STIPULATION AND IDENTIFICATION ADEQUACY.....	67
4.1 Robust Technology for Improving Correlation Matrix Stipulation .	67
4.2 Analysis of the Efficiency of Robust Technology for Improving Correlation Matrix Stipulation	70
4.3 Robust Technology for Improving Identification Adequacy of Statics of Technological Processes	79
4.4 Algorithms for Providing Adequacy of Mathematical Models at Expense of Compensation of Errors of Diagonal Elements of Correlation Matrices by Means of Estimates of Noise Variance	86
4.5 Robust Technology of Choice of Regularization Parameters by Statistical Identification.....	88
4.6 Determination of <i>a Priori</i> Values of Model Coefficients in Using Regularization Methods	93
CHAPTER 5. ROBUST TECHNOLOGY OF HARMONIC ANALYSIS.....	99
5.1 Methods of Spectral Analysis of Experimental Information.....	99
5.2 Practical Harmonic Analysis.....	103
5.3 Analysis of Features of Spectral Analysis Algorithms	106

5.4 Causes of Appearance of Difference between Positive and Negative Errors Caused by Noise	109
5.5 Algorithms for Providing Robustness of Estimates a_n , b_n	111
5.6 Robust Technology of Determining Coefficients of Fourier Series	115
5.7 Analysis of Results of Computer Experiment	119
CHAPTER 6. POSITION-BINARY TECHNOLOGY OF STATISTICAL ANALYSIS OF CYCLIC PROCESSES AND NOISES.....	121
6.1 Special Features of Cyclic Processes	121
6.2 Position-Binary Technology of Analysis of Cyclic Signals.....	123
6.3 Application of Position-Binary Technology for Noisy Signal Analysis.....	127
6.4 Position-Selective Discretization of Analog Signals.....	130
6.5 Examples of Application of Position-Binary Technology for Analyzing Random Processes	135
6.6 Position-Selective Filtering of Noise	136
6.7 Position-Binary Technology of Analysis of Noise.....	140
CHAPTER 7. ALGORITHMS AND TECHNOLOGY OF ANALYSIS OF INTERFERENCE AS INFORMATION CARRIER	143
7.1 Problem of the Analysis of Interference as an Information Carrier	143
7.2 Algorithm for Determining the Mean-Root-Square Deviation of Interference	145
7.3 Algorithm for Determining the Relative Mean-Root-Square Error of Samples Caused by Interference.....	147

7.4 Decomposition of Noisy Signal to Useful Signal and Interference	149
7.5 Algorithms of Correlation Analysis of Interference.....	152
7.6 Algorithms of Spectral Analysis of Interference.....	156
7.7 Informative Technology of Analysis of Interference	158
CHAPTER 8. APPLICATION OF ROBUST TECHNOLOGY FOR SOLVING PROBLEMS OF CONTROL, DIAGNOSTICS, AND FORECASTING.....	163
8.1 Alternatives of Using Robust Technology in Information Systems	163
8.2 Robust Technology of System Analysis of Seismic Signals.....	168
8.3 Hybrid Robust System of Control and Diagnostics of Sea Platform State.....	173
8.4 Robust Technology of Controlling Seismostability of Objects of Sea Oil-and-Gas Deposits	178
8.5 Robust Information System for Early Detection and Diagnostics of Failures in Drilling Oil Wells	181
8.6 Diagnostics of Oil Well State.....	184
8.7 Robust Technology of Diagnostics of State of Objects of Petrochemical Productions.....	187
References	195
Index.....	197

Chapter 1

NEEDS IN DEVELOPMENT OF STATISTICAL ANALYSIS TECHNOLOGY

1.1 Opportunities of Robust Statistical Analysis Technology

The mathematical fundamentals of the modern methods of experimental data analysis were developed by A.N. Kolmogorov, N. Wiener, and A.Ya. Khinchin. The most widely used algorithms for processing of measuring information are created on its basis. It is recommended to use them in the case when the classical conditions are realized, i.e., the analyzed signals are stationary, they obey the normal distribution law, the correlation between the noise and useful signal is equal to zero, and the noise is white noise. For example, while using the spectral correlation and regression analysis, the theory of random processes, and the theory of pattern recognition, we assume that the classical conditions hold true. For the cases when these assumptions are acceptable one can get satisfactory results. But there is a wide class of problems where these assumptions are not acceptable and the results of solving these problems are unsatisfactory. In this case in solving a large number of important problems one gets false results, which lead to disastrous consequences that severely influence economic and social life. It is explained by the following fact. In creating traditional algorithms the specific features of forming real signals are not sufficiently taken into consideration. For this reason the application of these algorithms for solving the most important problems does not give desired results. Therefore it is a problem of great theoretical and practical importance to develop algorithms providing the robustness of calculating appropriate estimates so that their properties remain satisfactory even if the classical conditions are violated considerably. All of this requires the revision of traditional technologies of

statistical analysis of experimental data. For this purpose for a long time we carried out the researches the results of which are published in the following journals:

- *Automatic Control and Computer Sciences*: 1998 N2, 5; 1999 N5; 2000 N1, 4, 5; 2001 N1, 2, 4; 2002 N1;
- *Optoelectronics, Instrumentation and Data Processing*: 1995 N4; 1997 N3; 1998 N2; 2001 N5.

Some works are published in journals of the Russian Academy of Sciences:

- *Journal of Computer and Systems Sciences International*: 1995 N3; 1997 N3; 1998 N1;
- *Automation and Remote Control*: 1998 N5, 6;
- *Oil Industry*: 1998 N9, 10; 2000 N1; 2001 N3; 2002 N3.

The present monograph, *Robust Technology with Analysis of Interference in Signal Processing*, is written on the basis of these works. We shall use the terms "noise" and "interference" as synonyms. For the first time anywhere, the theoretical fundamentals of analysis of noise as a carrier of useful information are given and for the case when it is not required to fulfill the classical conditions the technology of statistical signal processing is created. This eliminates the difficulties with the application of the theory of spectral, correlation, and regression analysis, the theory of identification, and the theory of pattern recognition during solving a large number of the most important problems in various fields.

There are many books^{1, 2, 3, 4} in which the authors set out the methods and algorithms for analyzing experimental data. For the first time, we revise the well-known technologies of statistical analysis and suggest robust algorithms providing the independence of the estimates of statistical and spectral characteristics of a total signal from the influence of outside factors. We find that if the estimates of the useful signal do not change in time, the estimates of the total signal, which is formed during changing outside factors, are equal to the estimates of the useful signal.

The possibility of providing the robustness of the required estimates irrespectively of fulfilling corresponding conditions is of great applied importance. Thanks to providing robustness of the mentioned estimates it is possible to improve the results of the correlation and spectral analysis, the stipulation of the correlation matrices, the adequacy of the identification, and it is possible to eliminate serious difficulties in applying the theory of random processes, the theory of pattern recognition, methods of identification and control, and so on in solving many widespread practical problems.

The position-binary technology of statistical analysis of cyclic processes allowing us to obtain estimates with an accuracy that is comparable with the accuracy of measuring devices is also suggested in this work.

Based on the above there is an opportunity for timely recognition and forecasting of the microchanges in the states of monitored and controlled objects.

1.2 Features of Improving Correlation Matrix Stipulation

It should be particularly noted where in solving matrix correlation equations numerically under conditions where the correlation matrix elements involve errors and their values are known approximately, the solution involves inaccuracies, too.

Theoretical solution of the system

$$\bar{R}_{xx}(0) \cdot \bar{B} = \bar{R}_{xy}(0) \quad (1.1)$$

is given by the formula

$$\bar{B} = \bar{R}_{xx}^{-1}(0) \cdot \bar{R}_{xy}(0), \quad (1.2)$$

where $\bar{R}_{xx}(0) = \left\| R_{x_i x_j}(0) \right\|$ is an estimate matrix of auto- and cross correlation functions $R_{x_i x_i}(0)$, $R_{x_i x_j}(0)$ with time shift $\tau = 0$ of centered input signals $\dot{x}_i(t) = x_i(t) - m_{x_i}$; $\bar{R}_{xy}(0) = \left\| R_{x_i y}(0) \right\|$ is a column matrix of estimates of cross correlation functions $R_{x_i y}(0)$ with time shift $\tau = 0$ between centered input $\dot{x}_i(t)$ and output signals $\dot{y}(t) = y(t) - m_y$, $\bar{B} = \|b_i\|$ is the column matrix of coefficients of regression equation; $\bar{R}_{xx}^{-1}(0)$ is the matrix inverse with respect to $\bar{R}_{xx}(0)$; m_{x_i} , m_{x_j} , m_y are mathematical expectations of signals $x_i(t)$, $x_j(t)$, $y(t)$, respectively.

As known, $\bar{R}_{xx}^{-1}(0)$ exists only in the case where the determinant of matrix $\bar{R}_{xx}(0)$ is not equal to zero, that is, $\left| \bar{R}_{xx}(0) \right| \neq 0$. However, it may happen that in calculating the determinant accurately, proceeding from the approximate values of correlation matrix elements instead of accurate ones, the determinant turns out to differ from zero, but the change of the correlation matrix elements within the given precision can yield a correlation matrix whose determinant is equal to zero.

The inverse matrix $\bar{R}_{xx}^{-1}(0)$ is called stable if small changes of elements $R_{x_i x_j}(0)$ of the matrix $\bar{R}_{xx}(0)$ involve small change of elements of the inverse matrix. There is no doubt that for the stability of the inverse correlation matrix $\bar{R}_{xx}^{-1}(0)$, it is necessary that the determinant of the matrix not be too small.

The known estimate by Hadamard² for the determinant, when the matrix can be considered to be stable, has the form

$$\det \bar{R}_{xx}(0) \leq \sqrt{\prod_{i=1}^n \sum_{j=1}^n \left| R_{x_i x_j}(0) \right|^2}$$

and means that the determinant module can reach up to the value

$\sqrt{\prod_{i=1}^n \sum_{j=1}^n \left| R_{x_i x_j}(0) \right|^2}$ with the same sums of squares of modules of line elements.

In a general form a change of each element $dR_{x_k x_l}^{-1}(0)$ of the inverse correlation matrix, caused by a change of element $R_{x_i x_j}(0)$, is equal to this change $dR_{x_i x_j}(0)$ multiplied by production of two elements of the inverse matrix:

$$dR_{x_k x_l}^{-1}(0) = - \sum_{i,j} R_{x_k x_i}^{-1}(0) R_{x_j x_l}^{-1}(0) dR_{x_i x_j}(0).$$

The correlation matrix is considered to be poorly stipulated if its corresponding inverse matrix is unstable. It is natural that a linear correlation equation system with poorly stipulated matrix is less stable, that is, its

solution strongly changes with small changes of elements $R_{x_i x_i}^{\circ}(0)$, $R_{x_i x_j}^{\circ}(0)$ of correlation matrix $\vec{R}_{xx}^{\circ}(0)$ of auto- and cross correlation functions of input signals and elements $R_{x_i y}^{\circ}(0)$ of correlation matrix $\vec{R}_{xy}^{\circ}(0)$ of cross correlation functions between input and output signals.

It is natural that a system of linear correlation equations with poorly stipulated matrix is less stable, i.e., its solution strongly changes with small changes of the values of correlation functions $R_{x_i x_i}^{\circ}(0)$, $R_{x_i x_j}^{\circ}(0)$, $R_{x_i y}^{\circ}(0)$.

Thus, "trifle" of the determinant is the reason for poor stipulation of the correlation matrix. However, only the value of the determinant cannot characterize the matrix stipulation.

For matrix characteristic with reference to its stipulation different characteristics are proposed which are called numbers of stipulation. These two Turing numbers are

$$N\text{-number} = \frac{1}{n} N\left(\vec{R}_{xx}^{\circ}(0)\right) N\left(\vec{R}_{xx}^{-1}(0)\right) = v\left(\vec{R}_{xx}^{\circ}(0)\right), \quad (1.3)$$

$$M\text{-number} = \frac{1}{n} N\left(\vec{R}_{xx}^{\circ}(0)\right) N\left(\vec{R}_{xx}^{-1}(0)\right) = v\left(\vec{R}_{xx}^{\circ}(0)\right), \quad (1.4)$$

where $N\left(\vec{R}_{xx}^{\circ}(0)\right) = \sqrt{\text{Sp} \vec{R}_{xx}^T(0) \cdot \vec{R}_{xx}^{\circ}(0)}$ or

$$\begin{aligned} N &= \frac{1}{n} \sqrt{\sum_{i,j}^n \left| R_{x_i x_j}^{\circ}(0) \right|^2} \sqrt{\sum_{i,j}^n \left| R_{x_i x_j}^{-1}(0) \right|^2} = \\ &= \frac{1}{n} \sqrt{\sum_{i,j}^n \left| R_{x_i x_j}^{\circ}(0) \right|^2} \sqrt{\sum_{i,j}^n \left| P_{x_i x_j}^{\circ}(0) / \det \vec{R}_{xx}^{\circ}(0) \right|^2}, \end{aligned}$$

$R_{x_i x_j}^{-1}(0)$ are the elements of the matrix $\vec{R}_{xx}^{-1}(0)$ inverse with reference to the matrix $\vec{R}_{xx}^{\circ}(0)$; $P_{x_i x_j}^{\circ}(0)$ is algebraic complement of elements $R_{x_i x_j}^{\circ}(0)$;

$\bar{R}_{xx}^T(0)$ is the matrix transposed with reference to $\bar{R}_{xx}(0)$;

$M\left(\bar{R}_{xx}(0)\right) = n \cdot \max_{ij} \left| R_{0 \ 0} \begin{smallmatrix} x_i & x_j \end{smallmatrix} \right|$ are the norms of the matrices:

$$P\text{-number} = \frac{\max |\lambda_i|}{\min |\lambda_i|} = \rho\left(\bar{R}_{xx}(0)\right) \quad (1.5)$$

is Tod number, where λ_i are eigenvalues of the matrix $\bar{R}_{xx}(0)$;

$$H\text{-number} = \left\| \bar{R}_{xx}(0) \right\| \left\| \bar{R}_{xx}^{-1}(0) \right\|, \quad (1.6)$$

where $\left\| \bar{R}_{xx}(0) \right\| = \sqrt{\lambda_1}$ is the norm of the matrix; λ_1 is the eigenvalue of the matrix of $\bar{R}_{xx}'(0) \cdot \bar{R}_{xx}(0)$; $\bar{R}_{xx}'(0)$ is the matrix which is conjugate to $\bar{R}_{xx}(0)$.

The conjugate complex matrix $\overline{\bar{R}_{xx}(0)}$ is obtained by replacing matrix elements with conjugate complex numbers. If elements of $\bar{R}_{xx}(0)$ are real, then $\overline{\bar{R}_{xx}(0)} = \bar{R}_{xx}(0)$. The matrix $\bar{R}_{xx}'(0) = \overline{\bar{R}_{xx}^T(0)}$ which is conjugate complex to the transposed one is called the matrix which is conjugate to the matrix $\bar{R}_{xx}(0)$. It is obvious that $\left(\bar{R}_{xx}'(0) \right)' = \bar{R}_{xx}(0)$. If matrix $\bar{R}_{xx}(0)$ is real, then the matrix which is conjugate to it coincides with the transposed one.

It is easy to see that

$$H\text{-number} = \sqrt{\frac{\mu_1}{\mu_n}} = \eta\left(\bar{R}_{xx}(0)\right), \quad (1.7)$$

where μ_1 and μ_n are the maximum and minimum eigenvalues of matrix $\bar{R}_{xx}^T(0) \cdot \bar{R}_{xx}(0)$. It is clear that

$$\eta\left(\bar{R}_{xx}(0)\right) = \sqrt{\rho\left(\bar{R}_{xx}^T(0) \cdot \bar{R}_{xx}(0)\right)}. \quad (1.8)$$

For symmetrical matrices the stipulation number- P coincides with number- H .

Note inequalities relating these numbers:

$$v\left(\bar{R}_{xx}(0)\right) \leq \mu\left(\bar{R}_{xx}(0)\right) \leq n^2 v\left(\bar{R}_{xx}(0)\right), \quad (1.9)$$

$$v\left(\bar{R}_{xx}(0)\right) \leq \eta\left(\bar{R}_{xx}(0)\right) \leq n v\left(\bar{R}_{xx}(0)\right), \quad (1.10)$$

$$\rho\left(\bar{R}_{xx}(0)\right) \leq \eta\left(\bar{R}_{xx}(0)\right).$$

For orthogonal matrices:

$$v\left(\bar{R}_{xx}(0)\right) = \eta\left(\bar{R}_{xx}(0)\right) = \rho\left(\bar{R}_{xx}(0)\right) = 1.$$

All the stipulation numbers are not less than one. The stipulation numbers $v\left(\bar{R}_{xx}(0)\right)$ and $\eta\left(\bar{R}_{xx}(0)\right)$ have the following probabilistic sense. Consider a system of linear correlation equations $\bar{R}_{xx}(0) \cdot \bar{B} = \bar{R}_{xy}(0)$, where correlation column-matrix $\bar{R}_{xy}(0)$ is given exactly, and the values of elements $R_{gi} (0)$ of correlation matrix $\bar{R}_{xx}(0)$ consist of values $R_{xi xj} (0)$ and error $\Lambda_{xi xj} (0)$ with identical variance $D\left[\Lambda_{xi xj} (0)\right]$ whose value is assumed to be very small in comparison with values $R_{xi xj} (0)$. Then the stipulation number- N shows by how many times the ratio of mean-root-square value of errors of the coefficients \bar{B} to mean-root-square value of errors of coefficients themselves is greater than the ratio of mean-root-square value of errors $\Lambda_{xi xj} (0)$ of estimates $\bar{R}_{gi} (0)$ to mean-root-square value of $\bar{R}_{xi xj} (0)$ themselves. Number- H gives the ratio of the maximum semiaxis