Strength of Materials

Part II Advanced

Third Edition

S Timoshenko

STRENGTH OF MATERIALS

PART II

Advanced Theory and Problems

BY

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THIRD EDITION

VNR

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PREFACE TO THE THIRD EDITION

In preparing the latest edition of this book, a considerable amount of new material has been added. Throughout the text, the latest references have been inserted, as well as new problems for solution and additional figures. The major changes in text material occur in the chapters on torsion, plastic deformation and mechanical properties of materials.

With regard to torsion, the problem of the twist of tubular members with intermediate cells is considered, as well as the torsional buckling of thin-walled members of open cross section. Each of these topics is important in the design of thin-walled structures such as the structural components of airplanes. In the chapter on plastic deformation the fundamental principles of limit design are discussed. Several examples of the application of the method to structural analysis are presented.

Major additions were made to the chapter on mechanical properties of materials, so that this single chapter now contains over 160 pages. The purpose of this expanded chapter is to focus attention on the recent developments in the field of experimental studies of the properties of structural materials. Some of the topics discussed are (1) the influence of imperfections on the ultimate strength of brittle materials and the "size effect"; (2) comparison of test results for single-crystal and polycrystal specimens; (3) the testing of materials under two- and three-dimensional stress conditions and various strength theories; (4) the strength of materials under impact; (5) fatigue of metals under various stress conditions and methods for improving the fatigue resistance of machine parts; and (6) strength of materials at high temperature, creep phenomenon and the use of creep test data in design. For the reader who desires to expand his knowledge of these topics further, the numerous references to the recent literature will be helpful. Finally, in the concluding article of the book, information for the proper selection of working stresses is presented in considerable detail.

It is the author's hope that with these additions, the book will be more complete for the teaching of graduate courses in mechanics of materials and also more useful for designers and research engineers in mechanical and structural engineering.

In conclusion the author wishes to thank Professor James M. Gere of Stanford University for his assistance and numerous suggestions in revising the book and in reading the proofs.

S. TIMOSHENKO

STANFORD UNIVERSITY
February 10, 1956

PREFACE TO THE SECOND EDITION

In the preparation of the new edition of this volume, the general character of the book has remained unchanged; the only effort being to make it more complete and up-to-date by including new theoretical and experimental material representing recent developments in the fields of stress analysis and experimental investigation of mechanical properties of structural materials.

The most important additions to the first edition include:

1. A more complete discussion of problems dealing with bending, compression, and torsion of slender and thin-walled structures. This kind of structure finds at present a wide application in airplane constructions, and it was considered desirable to include in the new edition more problems from that field.

2. A chapter on plastic deformations dealing with bending and torsion of beams and shafts beyond the elastic limit and also with plastic flow of material in thick-walled cylinders subjected to high internal pressures.

3. A considerable amount of new material of an experimental character pertaining to the behavior of structural materials at high temperatures and to the fatigue of metals under reversal of stresses, especially in those cases where fatigue is combined with high stress concentration.

4. Important additions to be found in the portion of the book dealing with beams on elastic foundations; in the chapters on the theory of curved bars and theory of plates and shells; and in the chapter on stress concentration, in which some recent results of photoelastic tests have been included.

Since the appearance of the first edition of this book, the author's three volumes of a more advanced character, "Theory of Elasticity," "Theory of Elastic Stability," and "Theory of Plates and Shells" have been published. Reference to these books are made in various places in this volume, especially in those cases where only final results are given without a complete mathematical derivation.

It is hoped that with the additions mentioned above the book will give an up-to-date presentation of the subject of strength of materials which may be useful both to graduate students interested in engineering mechanics and to design engineers dealing with complicated problems of stress analysis.

STEPHEN P. TIMOSHENKO

PALO ALTO, CALIFORNIA
June 12, 1941

PREFACE TO THE FIRST EDITION

The second volume of THE STRENGTH OF MATERIALS is written principally for advanced students, research engineers, and designers. The writer has endeavored to prepare a book which contains the new developments that are of practical importance in the fields of strength of materials and theory of elasticity. Complete derivations of problems of practical interest are given in most cases. In only a comparatively few cases of the more complicated problems, for which solutions cannot be derived without going beyond the limit of the usual standard in engineering mathematics, the final results only are given. In such cases, the practical applications of the results are discussed, and, at the same time, references are given to the literature in which the complete derivation of the solution can be found.

In the first chapter, more complicated problems of bending of prismatical bars are considered. The important problems of bending of bars on an elastic foundation are discussed in detail and applications of the theory in investigating stresses in rails and stresses in tubes are given. The application of trigonometric series in investigating problems of bending is also discussed, and important approximate formulas for combined direct and transverse loading are derived.

In the second chapter, the theory of curved bars is developed in detail. The application of this theory to machine design is illustrated by an analysis of the stresses, for instance, in hooks, fly wheels, links of chains, piston rings, and curved pipes.

The third chapter contains the theory of bending of plates. The cases of deflection of plates to a cylindrical shape and the symmetrical bending of circular plates are discussed in detail and practical applications are given. Some data regarding the bending of rectangular plates under uniform load are also given.

In the fourth chapter are discussed problems of stress distribution in parts having the form of a generated body and symmetrically loaded. These problems are especially important for designers of vessels submitted to internal pressure and of rotating machinery. Tensile and bending stresses in thin-walled vessels, stresses in thick-walled cylinders, shrink-fit stresses, and also dynamic stresses produced in rotors and rotating discs by inertia forces and the stresses due to non-uniform heating are given attention.

The fifth chapter contains the theory of sidewise buckling of compressed members and thin plates due to elastic instability. These problems are of utmost importance in many modern structures where the cross sectional dimensions are being reduced to a minimum due to the use of stronger materials and the desire to decrease weight. In many cases, failure of an engineering structure is to be attributed to elastic instability and not to lack of strength on the part of the material.

In the sixth chapter, the irregularities in stress distribution produced by sharp variations in cross sections of bars caused by holes and grooves are considered, and the practical significance of stress concentration is discussed. The photoelastic method, which has proved very useful in investigating stress concentration, is also described. The membrane analogy in torsional problems and its application in investigating stress concentration at reentrant corners, as in rolled sections and in tubular sections, is explained. Circular shafts of variable diameter are also discussed, and an electrical analogy is used in explaining local stresses at the fillets in such shafts.

In the last chapter, the mechanical properties of materials are discussed. Attention is directed to the general principles rather than to a description of established, standardized methods of testing materials and manipulating apparatus. The results of modern investigations of the mechanical properties of single crystals and the practical significance of this information are described. Such subjects as the fatigue of metals and the strength of metals at high temperature are of decided practical interest in modern machine design. These problems are treated more particularly with reference to new developments in these fields.

In concluding, various strength theories are considered. The important subject of the relation of the theories to the method of establishing working stresses under various stress conditions is developed.

It was mentioned that the book was written partially for teaching purposes, and that it is intended also to be used for advanced courses. The writer has, in his experience, usually divided the content of the book into three courses as follows: (1) A course embodying chapters 1, 3, and 5 principally for advanced students interested in structural engineering. (2) A course covering chapters 2, 3, 4, and 6 for students whose chief interest is in machine design. (3) A course using chapter 7 as a basis and accompanied by demonstrations in the material testing laboratory. The author feels that such a course, which treats the fundamentals of mechanical properties of materials and which establishes the relation between these properties and the working stresses used under various conditions in design, is of practical importance, and more attention should be given this sort of study in our engineering curricula.

The author takes this opportunity of thanking his friends who have assisted him by suggestions, reading of manuscript and proofs, particularly Messrs. W. M. Coates and L. H. Donnell, teachers of mathematics and mechanics in the Engineering College of the University of Michigan, and Mr. F. L. Everett of the Department of Engineering Research of the University of Michigan. He is indebted also to Mr. F. C. Wilharm for the preparation of drawings, to Mrs. E. D. Webster for the typing of the manuscript, and to the D. Van Nostrand Company for their care in the publication of the book.

S. TIMOSHENKO

ANN ARBOR, MICHIGAN
May 1, 1930

NOTATIONS

α	Angle, coefficient of thermal expansion, numerical coefficient
β	Angle, numerical coefficient
γ	Shearing strain, weight per unit volume
Δ	Unit volume expansion, distance
δ	Total elongation, total deflection, distance
ϵ	Unit strain
ϵ_x, ϵ_y, ϵ_z	Unit strains in *x*, *y*, and *z* directions
θ	Angle, angle of twist per unit length of a shaft
μ	Poisson's ratio
ρ	Distance, radius
σ	Unit normal stress
σ_1, σ_2, σ_3	Principal stresses
σ_x, σ_y, σ_z	Unit normal stresses on planes perpendicular to the *x*, *y*, and *z* axes
σ_E	Unit stress at endurance limit
σ_{ult}	Ultimate stress
σ_{uc}, σ_{ut}	Ultimate stresses in compression and tension
σ_W	Working stress
$\sigma_{Y.P}$	Yield point stress
τ	Unit shear stress
τ_{xy}, τ_{yz}, τ_{zx}	Unit shear stresses on planes perpendicular to the *x*, *y*, and *z* axes, and parallel to the *y*, *z*, and *x* axes
τ_E	Endurance limit in shear
τ_{oct}	Unit shear stress on octahedral plane
τ_{ult}	Ultimate shear stress
τ_W	Working stress in shear
$\tau_{Y.P}$	Yield point stress in shear
φ	Angle, angle of twist of shaft
ω	Angular velocity

A .... Cross-sectional area
a, b, c, d, e .. Distances
C .... Torsional rigidity
C_1 .... Warping rigidity
D .... Flexural rigidity
d .... Diameter
E, E_t, E_r .... Modulus of elasticity, tangent modulus, reduced modulus
f .... Shear flow
G .... Modulus of elasticity in shear
h .... Height, thickness
I_p, I_o ...... Polar moments of inertia of a plane area with respect to centroid and shear center
I_x, I_y, I_z .... Moments of inertia of a plane area with respect to x, y, and z axes
k .... Modulus of foundation, radius of gyration, stress concentration factor, numerical constant
l .... Length, span
M .... Bending moment
M_{ult} .... Ultimate bending moment
$M_{Y.P.}$ .... Bending moment at which yielding begins
M_t .... Torque
$(M_t)_{ult}$ ..... Ultimate torque
$(M_t)_{Y.P.}$ ..... Torque at which yielding begins
n .... Factor of safety
P, Q .... Concentrated forces
p .... Pressure, frequency of vibration
q .... Load per unit length, reduction in area, sensitivity factor
R .... Reaction, force, radius, range of stress
r .... Radius, radius of curvature
S .... Axial force, surface tension
s .... Length
T .... Axial force, absolute temperature
t .... Temperature, thickness
U .... Strain energy
u .... Rate of strain, displacement in x direction
V .... Volume, shearing force

v. . . . . Velocity, creep rate, displacement in y direction
W. . . . . Weight
w. . . . . Strain energy per unit volume, displacement in z direction
x, y, z. . . . . . . Rectangular coordinates
Z. . . . . Section modulus

CONTENTS