

FAST TRANSFORMS

Algorithms, Analyses, Applications

Douglas F. Elliott

K. Ramamohan Rao

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Douglas F. Elliott

Electronics Research Center
Rockwell International
Anaheim, California

K. Ramamohan Rao

Department of Electrical Engineering
The University of Texas at Arlington
Arlington, Texas



1982

ACADEMIC PRESS

A Subsidiary of Harcourt Brace Jovanovich, Publishers

New York London

Paris San Diego San Francisco São Paulo Sydney Tokyo Toronto

5506584

34/10

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ACADEMIC PRESS, INC.
111 Fifth Avenue, New York, New York 10003

United Kingdom Edition published by
ACADEMIC PRESS, INC. (LONDON) LTD.
24/28 Oval Road, London NW1 7DX

Library of Congress Cataloging in Publication Data

Elliott, Douglas F.

Fast transforms: algorithms, analysis, applications.

Includes bibliographical references and index.

1. Fourier transformations--Data processing.

2. Algorithms. I. Rao, K. Ramamohan (Kamisetty Ramamohan)

II. Title. III. Series

QA403.5.E4

515.7'23

79-8852

ISBN 0-12-237080-5

AACR2

AMS (MOS) 1980 Subject Classifications: 68C25, 42C20, 68C05, 42C10

PRINTED IN THE UNITED STATES OF AMERICA

82 83 84 85 9 8 7 6 5 4 3 2 1

PREFACE

Fast transforms are playing an increasingly important role in applied engineering practices. Not only do they provide spectral analysis in speech, sonar, radar, and vibration detection, but also they provide bandwidth reduction in video transmission and signal filtering. Fast transforms are used directly to filter signals in the frequency domain and indirectly to design digital filters for time domain processing. They are also used for convolution evaluation and signal decomposition. Perhaps the reader can anticipate other applications, and as time passes the list of applications will doubtlessly grow.

At the present time to the authors' knowledge there is no single book that discusses the many fast transforms and their uses. The purpose of this book is to provide a single source that covers fast transform algorithms, analyses, and applications. It is the result of collaboration by an author in the aerospace industry with another in the university community. The authors hope that the collaboration has resulted in a suitable mix of theoretical development and practical uses of fast transforms.

This book has grown from notes used by the authors to instruct fast transform classes. One class was sponsored by the Training Department of Rockwell International, and another was sponsored by the Department of Electrical Engineering of The University of Texas at Arlington. Some of the material was also used in a short course sponsored by the University of Southern California. The authors are indebted to their students for motivating the writing of this book and for suggestions to improve it.

The development in this book is at a level suitable for advanced undergraduate or beginning graduate students and for practicing engineers and scientists. It is assumed that the reader has a knowledge of linear system theory and the applied mathematics that is part of a standard undergraduate engineering curriculum. The emphasis in this book is on material not directly covered in other books at the time it was written. Thus readers will find practical approaches not covered elsewhere for the design and development of spectral analysis systems.

The long list of references at the end of the book attests to the volume of literature on fast transforms and related digital signal processing. Since it is impractical to cover all of the information available, the authors have tried to list as many relevant references as possible under some of the topics discussed only briefly. The authors hope this will serve as a guide to those seeking additional material.

Digital computer programs for evaluation of the transforms are not listed, as these are readily available in the literature. Problems have been used to convey information by means of the format: If A is true, use B to show C. This format gives useful information both in the premise and in the conclusion. The format also gives an approach to the solution of the problem.

ACKNOWLEDGMENTS

It is a pleasure to acknowledge helpful discussions with our colleagues who contributed to our understanding of fast transforms. In particular, fruitful discussions were held with Thomas A. Becker, William S. Burdic, Tien-Lin Chang, Robert J. Doyle, Lloyd O. Krause, David A. Orton, David L. Hench, Stanley A. White, and Lee S. Young of Rockwell International; Fredric J. Harris of San Diego State University and the Naval Ocean Systems Center; I. Luis Ayala of Vitro Tec, Monterrey, Mexico; and Patrick Yip of McMaster University. It is also a pleasure to acknowledge support from Thomas A. Becker, Mauro J. Dentino, J. David Hirstein, Thomas H. Moore, Visvaldis A. Vitols, and Stanley A. White of Rockwell International and Floyd L. Cash, Charles W. Jiles, John W. Rouse, Jr., and Andrew E. Salis of The University of Texas at Arlington.

Portions of the manuscript were reviewed by a number of people who pointed out corrections or suggested clarifications. These people include Thomas A. Becker, William S. Burdic, Tien-Lin Chang, Paul J. Cuenin, David L. Hench, Lloyd O. Krause, James B. Larson, Lester Mintzer, Thomas H. Moore, David A. Orton, Ralph E. Smith, Jeffrey P. Strauss, and Stanley A. White of Rockwell International; Henry J. Nussbaumer of the École Polytechnique Fédérale de Lausanne; Ramesh C. Agarwal of the Indian Institute of Technology; Minsoo Suk of the Korea Advanced Institute of Science; Patrick Yip of McMaster University; Richard W. Hamming of the Naval Postgraduate School; G. Clifford Carter and Albert H. Nuttall of the Naval Underwater Systems Center; C. Sidney Burrus of Rice University; Fredric J. Harris of San Diego State University and the Naval Ocean Systems Center; Samuel D. Stearns of Sandia Laboratories; Philip A. Hallenborg of Northrup Corporation; I. Luis Ayala of Vitro Tec; George Szentirmai of CGIS, Palo Alto, California; and Roger Lighty of the Jet Propulsion Laboratory.

The authors wish to thank several hardworking people who contributed to the manuscript typing. The bulk of the manuscript was typed by Mrs.

Ruth E. Flanagan, Mrs. Verna E. Jones, and Mrs. Azalee Tatum. The authors especially appreciate their patience and willingness to help far beyond the call of duty.

The encouragement and understanding of our families during the preparation of this book is gratefully acknowledged. The time and effort spent on writing must certainly have been reflected in neglect of our families, whom we thank for their forbearance.

LIST OF ACRONYMS

ADC	Analog-to-digital converter	FOM	Figure of merit
AGC	Automatic gain control	FWT	Fast Walsh transform
BCM	Block circulant matrix	GCBC	Gray code to binary conversion
BIFORE	Binary Fourier representation	GCD	Greatest common divisor
BPF	Bandpass filter	GT	Generalized discrete transform
BR	Bit reversal	(GT),	r th-order generalized discrete transform
BRO	Bit-reversed order	HHT	Hadamard-Haar transform
CBT	Complex BIFORE transform	(HHT),	r th-order Hadamard-Haar transform
CCP	Circular convolution property	HT	Haar transform
CFNT	Complex Fermat number transform	IDCT	Inverse discrete cosine transform
CHT	Complex Haar transform	IDFT	Inverse discrete Fourier transform
CMNT	Complex Mersenne number transform	IF	Intermediate frequency
CMPY	Complex multiplications	IFFT	Inverse fast Fourier transform
CNTT	Complex number theoretic transform	IFNT	Inverse Fermat number transform
CPFNT	Complex pseudo-Fermat number transform	IGT	Inverse generalized transform
CPMNT	Complex pseudo-Mersenne number transform	(IGT),	r th-order inverse generalized transform
CRT	Chinese remainder theorem	IIR	Infinite impulse response
DAC	Digital-to-analog converter	IMNT	Inverse Mersenne number transform
DCT	Discrete cosine transform	KLT	Karhunen-Loève transform
DDT	Discrete D transform	LPF	Low pass filter
DFT	Discrete Fourier transform	lsb	Least significant bit
DIF	Decimation in frequency	lsd	Least significant digit
DIT	Decimation in time	MBT	Modified BIFORE transform
DM	Dyadic matrix	MCBT	Modified complex BIFORE transform
DST	Discrete sine transform	MGT	Modified generalized transform
ENBR	Effective noise bandwidth ratio	(MGT),	r th-order modified generalized discrete transform
ENBW	Equivalent noise bandwidth	MIR	Mixed radix integer representation
EPE	Energy packing efficiency	MNT	Mersenne number transform
FDCT	Fast discrete cosine transform	MPY	Multiplications
FFT	Fast Fourier transform	msb	Most significant bit
FGT	Fast generalized transform	msd	Most significant digit
FIR	Finite impulse response	mse	Mean-square error
FNT	Fermat number transform		

MWHT	Modified Walsh Hadamard transform	SHT	Slant-Haar transform
(MWHT) _h	Hadamard ordered modified Walsh Hadamard transform	(SHT) _r	rth-order slant-Haar transform
NPSD	Noise power spectral density	SIR	Second integer representation
NTT	Number theoretic transform	SNR	Signal-to-noise ratio
PSD	Power spectral density	ST	Slant transform
RF	Radio frequency	WFTA	Winograd Fourier transform algorithm
RHT	Rationalized Haar transform	WHT	Walsh Hadamard transform
RHHT	Rationalized Hadamard-Haar transform	(WHT) _{cs}	Cal-sal ordered Walsh-Hadamard transform
(RHHT) _r	rth-order rationalized Hadamard-Haar transform	(WHT) _h	Hadamard ordered Walsh-Hadamard transform
RMFFT	Reduced multiplications fast Fourier transform	(WHT) _p	Paley ordered Walsh-Hadamard transform
RMS	Root mean square	(WHT) _w	Walsh ordered Walsh-Hadamard transform
RNS	Residue number system	zps	Zero crossings per second
RT	Rapid transform		

NOTATION

<i>Symbol</i>	<i>Meaning</i>	<i>Symbol</i>	<i>Meaning</i>
A, B, \dots	Matrices are designated by capital letters	$[H_s(L)]$	Walsh Hadamard matrix of size $(2^L \times 2^L)$. The subscript s can be w, h, p, or cs, denoting Walsh, Hadamard, Paley or canonical ordering, respectively.
$A \otimes B$	The Kronecker product of A and B (see Appendix)	$[Ha(L)]$	Haar matrix of size $(2^L \times 2^L)$
A^T	The transpose of matrix A	$[Hh_r(L)]$	r th order (HHT) _{r} matrix of size $(2^L \times 2^L)$
A^{-1}	The inverse of matrix A	\bar{I}_m	Opposite diagonal matrix, e.g.,
$[A(L)]$	DCT matrix of size $(2^L \times 2^L)$		$\bar{I}_4 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$
$D(f)$	Periodic DFT filter frequency response, which for $P = 1$ s is given by	\bar{I}_N^{cm}	Columns of I_N are shifted circularly to the right by m places
	$\exp\left[-j\pi f\left(1 - \frac{1}{N}\right)\right] \frac{\sin(\pi f)}{N \sin(\pi f/N)}$	\bar{I}_N^{cm}	Columns of I_N are shifted circularly to the left by m places
$\hat{D}(f)$	Periodic frequency response of DFT with weighted input (windowed output)	I_N^{dl}	Columns of I_N are shifted dyadically by l places
$D'(f)$	Nonperiodic DFT filter frequency response which for $P = 1$ s is given by	I_R	Identity matrix of size $(R \times R)$
	$\exp[-j\pi f(1 - 1/N)] [\sin(\pi f)] (\pi f)$	$\text{Im}[\]$	The imaginary part of the quantity in the square brackets
$\hat{D}'(f)$	Nonperiodic frequency response of DFT with weighted input (windowed output)	IDFT $[X(k)]$	The inverse discrete Fourier transform of the sequence $\{X(0), X(1), \dots, X(N-1)\}$
DFT $[x(n)]$	The discrete Fourier transform of the sequence $\{x(0), x(1), \dots, x(N-1)\}$	$[K(L)]$	KLT matrix of size $(2^L \times 2^L)$
$[D_r^j(L)]$	j th matrix factor of $[G_r(L)]$	L	Integer such that $N = \alpha^L$
E	Expectation operator	M_p	Mersenne number, $M_p = 2^p - 1$, where P is a prime number
$[E_r^j(L)]$	j th matrix factor of $[M_r(L)]$		
F_t	t th Fermat number, $F_t = (2^{2^t} + 1)$, $t = 0, 1, 2, \dots$		
$[G_r(L)]$	(GT) _{r} matrix of size $(2^L \times 2^L)$		
$[H_{mh}(L)]$	MWHT matrix of size $(2^L \times 2^L)$		

<i>Symbol</i>	<i>Meaning</i>	<i>Symbol</i>	<i>Meaning</i>
$[M_r(L)]$	(MGT), matrix of size $(2^L \times 2^L)$	$X(f)$ or $X_a(f)$	Spectrum defined by the Fourier (or generalized) transform of the (analog) function $x(t)$
N	Transform dimension	$ X(f) ^2$	Power spectral density with units of watts per hertz
N^{-1}	Multiplicative inverse of the integer N such that $N \times N^{-1} \equiv 1$ (modulo M)	$X(k)$	Coefficient number $k, k = 0, \pm 1, \pm 2, \dots$, in series expansion of periodic function $x(t)$
P	1. Period of periodic time function in seconds 2. In Chapter 11, prime number	$ X(k) ^2$	Power spectrum for a function with a series representation
$[P(L)]$	Diagonal matrix whose diagonal elements are negative integer powers of 2	X_c	DCT of x
$P_h(m)$	(WHT) _h circular shift-invariant power spectral point	X_{cf}	CFNT of x
$P_r(l)$	l th power spectral point of (GT) _r	$\tilde{X}^{(cm)}$	Transform of \tilde{x}^{cm}
$P_w(m)$	m th sequency power spectrum	$\tilde{X}^{(cm)}$	Transform of \tilde{x}^{cm}
Q	1. Ratio of the filter center frequency and the filter bandwidth (Chapter 6) 2. Least significant bit value (Chapter 7)	X_{cm}	CMNT of x
$Re[\]$	The real part of the quantity in the square brackets	X_{cpf}	CPFNT of x
$R(D)$	Rate distortion	X_{cpm}	CPMNT of x
$[Rh(L)]$	RHT matrix of size $(2^L \times 2^L)$	$X^{(dt)}$	Transform of x^{dt}
$[S(L)]$	ST matrix of size $(2^L \times 2^L)$	X_f	FNT of x
$[\tilde{S}^{(cm)}(L)]$	Shift matrix relating $\tilde{X}^{(cm)}$ and X	X_{ha}	HT of x
$[\tilde{S}^{(cm)'}(L)]$	Shift matrix relating $\tilde{X}^{(cm)}$ and X	X_{hhr}	(HHT), of x
$[S^{(dt)}(L)]$	Shift matrix relating $X^{(dt)}$ and X	X_k	KLT of x
$[Sh_r(L)]$	r th order (SHT), matrix of size $(2^L \times 2^L)$	X_m	MNT of x
T	Sampling interval	X_{mh}	MWHT of x
W	1. $\exp(-j2\pi/N)$ for FFT 2. $\exp(-j2\pi/\alpha^{l+1})$ for FGT	X_{mr}	(MGT), of x
$W^{(-)}$	The element \cdot in a matrix means $-j\infty$ so that $W^{(-)} = W^{-j\infty} = e^{-x} = 0$	X_{pf}	PFNT of x
W^{A+B}	Shorthand notation for matrix product $W^A W^B$, where A and B are $N \times N$ matrices	X_{pm}	PMNT of x
W^E	Matrix with entry $W^{E(k,n)}$ in row k and column n , where E is a matrix of size $(N \times N)$, $E(k,n)$ is the entry in row k and column n for $k, n = 0, 1, \dots, N-1$	X_r	(GT), of x
		$\tilde{X}_r^{(cm)}$	(GT), of \tilde{x}^{cm}
		X_{rh}	RHT of x
		X_s	ST of x
		$X_s(k)$	k th WHT coefficient. The subscript s is defined in $[H_s(L)]$
		X_{shr}	(SHT), of x
		Z_M	Ring of integers modulo M represented by the set $\{0, 1, 2, \dots, M-1\}$
		Z_M^c	Ring of complex integers. If $c = a + j\ell$, where $a = \text{Re}[c]$ and $\ell = \text{Im}[c]$, then c is represented in Z_M^c by $\hat{a} + j\hat{\ell}$, where $\hat{a} = a \text{ mod } M$ and $\hat{\ell} = \ell \text{ mod } M$
		$a \leftarrow b$	Give variable a the value of expression b (or replace a by b)
		$a \in B$	a is an element of the set B
		$a \in [c, d)$	$c \leq a < d$
		comb_T	The infinite series of impulse functions defined by

Symbol	Meaning	Symbol	Meaning
	$\sum_{k=-\infty}^{\infty} \delta(t - kT)$	r	Integer in the set $(0, 1, 2, \dots, L - 1)$
cube[t/p]	Cubic-shaped function defined by $\text{cube}\left[\frac{t}{p}\right] = \text{tri}\left[\frac{t}{P/2}\right] * \text{tri}\left[\frac{t}{P/2}\right]$	rad(m, t)	m th Rademacher function
deg[]	The degree of the polynomial in the square brackets	rect[t/P]	Rectangular-shaped function defined by $\text{rect}\left[\frac{t}{P}\right] = \begin{cases} 1, & t \leq P/2 \\ 0, & \text{otherwise} \end{cases}$
f	Frequency in hertz	rep $_{f_s}[X(f)]$	The repetition of $X(f)$ every f_s units as defined by the convolution $X(f) * \text{comb}_{f_s}$
f_k	Digit in expansion of $f = \sum_{k=l}^m f_k \alpha^k,$ where l is the least significant digit (lsd) and m is the most significant digit (msd)	s	Seconds
f_s	$f_s = 1/T$ is the sampling frequency	sinc(fQ)	$[\sin(\pi fQ)]/(\pi fQ)$
$\langle ft \rangle$	$\sum_k q_k t^{-k}$	t	Time in seconds
$h_s(k, n)$	Element of $[H_s(L)]$ in row k and column n . The subscript s is defined in $[H_s(L)]$	tr[]	Trace of a matrix
j	$\sqrt{-1}$	tri[t/P]	Triangular-shaped function defined by $\text{tri}\left[\frac{t}{P}\right] = \text{rect}\left[\frac{t}{P/2}\right] * \text{rect}\left[\frac{t}{P/2}\right]$
k	Transform coefficient number	$\mathcal{U}(t - t_0)$	Unit step function defined by $\mathcal{U}(t - t_0) = \begin{cases} 1, & t \geq t_0 \\ 0, & \text{otherwise} \end{cases}$
$\langle\langle k \rangle\rangle$	The decimal number obtained by the bit reversal of the L bit binary representation of k	wal $_s(k, t)$	k th Walsh function. The subscript s is defined in $[H_s(L)]$
$\bar{k} \cdot \bar{s}$	The integer defined by $\sum_{l=0}^{r+1} k_{r+1-l} 2^{s-l},$ where $s = r + 2, r + 3, \dots, L$, $k = 2^r, 2^{r+1}, \dots, (2^{r+1} - 1)$, and $k_l, l = 0, 1, \dots, r + 1$, is a bit in the binary representation of k	x^*	Complex conjugate of x
ln	Logarithm to the base e (natural logarithm)	\bar{x}^{*m}	x shifted circularly to the left by m places
log	Logarithm to the base 10	\bar{x}^{*m}	x shifted circularly to the right by m places
log $_2$	Logarithm to the base 2	x^{dl}	x is shifted dyadically by l places
n	Data sequence number	$x(n)$	Sampled-data value of x for sample number n
q_k	Integerization of frequency given by $q_k = \left\ \sum_{l=-\infty}^{\infty} f_{k-r-1+l} a_l \right\ $	$x(n) \leftrightarrow X(k)$	Both $x(n)$ and $X(k)$ exist
		$x(t)$	Time domain scalar-valued function at time t
		$x(t)$	Time domain vector-valued function at time t
		$x(t) \leftrightarrow X(f)$	Both $x(t)$ and $X(f)$ exist
		$x_s(t)$	Sampled function
		$x * y$	The convolution of x and y
		$x \circ y$	Element by element multiplication of the elements in x and y , e.g., if $a = x \circ y$, then $a(k) = x(k)y(k)$
		$(x)_\alpha$	Expression for x in number system with radix α , e.g., $(10.1)_2 = (2.5)_{10}$

<i>Symbol</i>	<i>Meaning</i>	<i>Symbol</i>	<i>Meaning</i>
$\mathcal{F}[\delta(t - T)] = \exp(-j2\pi fT)$		δ_{kl}	Kronecker delta function with the property that
\mathcal{F}	Fourier transform operator		$\delta_{kl} = \begin{cases} 0, & k \neq l \\ 1, & k = l \end{cases}$
$\mathcal{R}(a/l)$	The remainder when a is divided by l	$\delta(t - t_0)$	Dirac delta function with the property that
\mathcal{F}	Generalized transform operator		$x(t_0) = \int_{-\infty}^{\infty} \delta(t - t_0)x(t) dt$
$\mathcal{H}(f)$	Fourier transform of $\omega(t)$	$\theta_r(l)$	l th phase spectral point of (GT) _r
a, l, \dots	Script lower case letters a, l, \dots and the italic letters i, k, l, m, n, p, q, r (Chapter 5 only), K, L, M , and N denote integers	λ_j	j th eigenvalue of $[\Sigma_x(L)]$
$a \equiv l \pmod{n}$	$\mathcal{H}(a/n) = \mathcal{H}(l/n)$, where a and l are either integers or polynomials	μ	$E[x]$
$a \bmod l$	$\mathcal{H}(a/l)$, where a and l are either integers or polynomials	ρ	Correlation coefficient
$l N$	l divides N , i.e., the ratio N/l is an integer and the set of such integers includes 1 and N	σ^2	$E[(x - \mu)^2]$
ω	Steps per second taken by the generalized transform basis functions	$\phi(N)$	The number of integers less than N and relatively prime to N
$\omega(t)$	Weighting function applied to modify DFT filter frequency response	$\phi_k(n)$	k th basis function $\phi_k(t)$ evaluated at $t = nT$
$[\Sigma_x(L)]$	Covariance matrix of x	$ \cdot $	Magnitude of (\cdot)
x	Number system radix or a primitive root of order N	$\ \cdot\ $	Integerize by truncation (or rounding)
x^{r+1}	Number of equal sectors on the unit circle in the complex plane with first sector starting on the positive real axis	$\lceil \cdot \rceil$	Smallest integer $\geq (\cdot)$, e.g., $\lceil 3.5 \rceil = 4$, $\lceil -2.5 \rceil = -2$
		$\lfloor \cdot \rfloor$	Largest integer $\leq (\cdot)$, e.g., $\lfloor 3.5 \rfloor = 3$, $\lfloor -2.5 \rfloor = -3$
		\oplus	Signed digit addition performed digit by digit modulo x

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