Applications of Digital Image Processing VIII

Andrew G. Tescher
Chairman/Editor

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APPLICATIONS OF DIGITAL IMAGE PROCESSING VIII

Volume 575

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INTRODUCTION

Digital image processing has continued to demonstrate the vitality of this field through the contribution of many excellent papers at this conference. The technical program covered three days consisting of six sessions. Major topics included image transmission, restoration, enhancement, pattern recognition, display techniques and algorithm issues. The international community was well represented through the presentations by several authors from foreign countries.

The contributions and efforts of the cochairs and session chairs are gratefully acknowledged.

Andrew G. Tescher
The Aerospace Corporation

APPLICATIONS OF DIGITAL IMAGE PROCESSING VIII

Volume 575

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APPLICATIONS OF DIGITAL IMAGE PROCESSING VIII

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Session 1

Algorithms and Analysis

Chairman
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A Comparative Analysis of Digital Filters for Image Decimation and Interpolation

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ABSTRACT

It is sometimes necessary to increase or decrease the sampling rate of previously sampled images. Technically speaking, the process of increasing the sampling rate is called interpolation and the process of decreasing the sampling rate is called decimation. In this paper we present both linear and higher-order polynomial interpolation techniques in terms of linear filtering operations (convolutions) and discuss the associated impulse responses. Several simple filters for decimation are also discussed. All these filters are compared vis-a-vis their suitability for decimation and interpolation particularly the degree to which the balance between sharpness and prevention of aliasing is maintained. Images which have been decimated or interpolated using these filters are shown.

INTRODUCTION

This paper deals with the problem of image decimation and interpolation in general and a comparative analysis of certain filters for doing this in particular. Technically speaking, decimation is the process of decreasing the image sampling rate while interpolation is the process of increasing the sampling rate.

There are many practical systems in which decimation and/or interpolation is needed. Examples of these are: a) digital television systems in which, sometimes, conversion from one sampling rate to another is required to achieve system compatibility,[1,2] b) speech processing systems in which relevant speech parameters are computed at one sampling rate (chosen to save time) but the actual reconstruction of the speech is done at a higher sampling rate[3] and c) image processing systems[4] in which interpolation and/or decimation is almost always needed when going from one display medium to another e.g., interpolation may be needed for going from TV signal to hand print and decimation for the reverse operation. Also, going from one format to another requires decimation or interpolation.

This paper is devoted solely to decimation and interpolation in image processing. We first give a quick review of sampling theory as it applies to sampling rate conversion. The relationship between interpolation and linear filtering is given and the general characteristics of filters suitable for sample rate conversion are discussed. Several specific filters are compared vis-a-vis the degree to which they are able to maintain the balance between sharpness and prevention of aliasing. Finally, images which have been decimated or interpolated are shown. For simplicity all the analysis is done in one dimension since extension to two dimensions is simple and straightforward. The filtering of the actual images has, of course, been done in two dimensions using separable filters.

Therefore, consider the sequence s_n which consists of samples from the continuous function $\hat{s}(x)$ i.e.,

$$s_n = \hat{s}(nX_o), \quad -\infty < n < \infty$$

where X_o is the sampling period.

The Fourier Transform (FT) of $\hat{s}_n$ is

$$S(f) = \frac{1}{X_o} \ \mathrm{Rep}_{\frac{1}{X_o}} \ \hat{S}(f)$$

$$= \frac{1}{X_o} \sum_{k=-\infty}^{\infty} \hat{S}(f + \frac{k}{X_o}) \qquad (1)$$

where $\hat{S}(f)$ is the FT of $\hat{s}(x)$

If $\hat{s}(x)$ is band-limited to:

$$f_o \leq \frac{1}{2X_o} \quad \text{, then}$$

$$S(f) = \frac{1}{X_o} \hat{S}(f), \ -f_o \leq f \leq f_o$$

and

$$\hat{s}(x) = \sum_{n=-\infty}^{\infty} s_n \ \mathrm{sinc}\, f_o (x - \frac{n}{f_o});$$

A value of $f_o > \frac{1}{2}X_o$ will result in aliasing and $\hat{s}(x)$ cannot be reconstructed exactly. Cases of $f_o = \frac{1}{2}X_o$ and $f_o > \frac{1}{2}X_o$ are illustrated graphically in Figure 1.

Now consider the decimated sequence d_n which consists of taking every M^{th} sample of s_n: i.e.,

$$d_n = s_{Mn} \quad -\infty < n < \infty$$

Note that the sampling period for d_n is MX_o where M, the decimation factor, is an integer.

As a consequence of Sampling theorem, the Fourier Transform of d_n is

$$D(f) = \frac{1}{MX_o} \sum_{k=-\infty}^{\infty} \hat{S}(f + \frac{k}{MX_o}) \qquad (2)$$

and d_n will be alias-free if

$$f_o \leq \frac{1}{2MX_o}$$

$$\text{i.e.,} \quad X_o \geq \frac{1}{2Mf_o}$$

This statement is illustrated in Figure 2[3]. The top diagram of this figure shows the case of $X_o < ½Mf_o$ while the central diagram illustrated the case of $X_o > ½Mf_o$. Note the aliasing in this diagram. If one passes the sequence d_n through an ideal low-pass filter whose cut-off frequency $f'_o = ½MX_o$, the FT of the resulting sequence is shown in the lower diagram. It is important to note the difference between this diagram and the one above it: in the lower diagram, the higher frequencies have been attenuated while the higher frequencies in the central diagram have been folded over into the lower frequencies (i.e., aliased). This illustrates the kind of trade-off that is involved in the process of decimation: sharpness versus prevention of aliasing. While the resulting image may not show any aliasing, it may also not be as sharp as the original image. This balance depends on several factors, particularly the frequency contents and sampling rate of the original image, the pass-band and transition zone characteristics of the low-pass filter and the decimation factor. Sharpness can be restored to some degree but there is very little that can be done to remove aliasing if its presence is objectionable. It is the function of the low-pass filter to maintain a desirable balance between these two parameters.

This then is the basis for decimation: In order to decimate a sequence by an interger factor M, pass the original sequence through a low-pass filter whose cut-off frequency is $½MX_o$ and take every M^{th} sample of the resulting sequence.

INTERPOLATION

Interpolation can be linear, in which case all the interpolated samples between two consecutive samples of the original sequence lie on the straight line defined by the consecutive pair or nonlinéar in which case two or more consecutive samples from the original sequence are used to generate the interpolated samples which lie generally on a curve. Both linear and nonlinear interpolation can be treated as either a linear filtering problem or a polynomial fitting problem.

In order to derive the general scheme for interpolation by an integer factor L, consider the sequence b_n which is constructed by adding L-1 zeros between each consecutive pair of samples of sequence s_n i.e.,

$$b_n = \begin{cases} s_{n/L}, & n = 0, \pm L, \pm 2L, - - - - \\ 0, & \text{otherwise} \end{cases}$$

The FT of b_n can be shown to be

$$B(f) = S(f)$$

This FT is shown at the top of Figure 3 for the case of L=2 and $X_o = ½f_o$. Under this is the FT I (f) of the actual interpolated sequence i_n. From this figure, it is clear that in order to obtain i_n from b_n, one must pass the latter through a lowpass filter whose cut-off frequency is half the sampling rate of s_n and whose stop-band is such that the pass bands located at non-zero integer multiples of the original sampling rate up to but not including the new sampling rate are filtered out.

Thus to interpolate a given sequence by an interger factor L, insert (L-1) zeros between each consecutive sample pair of the sequence and pass the resulting sequence through a low pass filter with the characteristics described above.

The convolution equation which describes this filtering in the spatial domain is

$$i_n = \sum_{k = n - \frac{N-1}{2}}^{n + \frac{N-1}{2}} h_{n-k} / s_{k/L} \quad ,k/L \text{ an integer}$$

$$= \sum_{k = \text{int}\left[\frac{n}{L} + \frac{N-1}{2L}\right]}^{\text{Int}\left[\frac{n}{L} + \frac{N-1}{2L}\right]} h_{n-kL}\, S_k$$

where h_k is the filter impulse response and N is the filter length.

In order to preserve the original sample values,

$$i_{jL} = \sum_{k = \text{int}\left[j + \frac{N-1}{2L}\right]}^{\text{int}\left[j + \frac{N-1}{2L}\right]} h_{jL-kL}\, S_k = S_j \qquad (3)$$

This implies that

$$h_o = 1$$

and $\quad h_{n/L} = 0$, n/L a non-zero integer

Also, for original sample value preservation, the filter length must be odd since an even filter length would result in a sequence with an odd number of half-sample delays. For a zero delay (zero phase) sequence, the filter must be symmetric. In general, filter length N depends on the number P of original samples used to obtain the interpolated samples and the interpolation factor L:

$$N = \begin{cases} P \cdot L, & \text{for P and L odd} \\ P \cdot L - 1, & \text{for P or L or both even} \end{cases}$$

A summary of the procedure for decimating or interpolating by integer factors is shown in Figure 4. An arbitrary sampling rate conversion ratio α can be expressed as a ratio of two integers i.e.,

$$\alpha = L/M$$

The scheme for this kind of sampling rate conversion is shown in Figure 5.

COMPARISON OF CERTAIN FILTERS

Let us now compare the effectiveness of certain digital filters that are sometimes used for decimation or interpolation.

A) The Binomial Weight Filter

The weights for this filter are chosen from the binomial distribution whose first few orders are shown below:

$$h_n = \begin{array}{c} 1\ \ 1 \\ 1\ \ 2\ \ 1 \\ 1\ \ 3\ \ 3\ \ 1 \\ 1\ \ 4\ \ 6\ \ 4\ \ 1 \\ 1\ \ 5\ \ 10\ \ 10\ \ 5\ \ 1 \\ 1\ \ 6\ \ 15\ \ 20\ \ 15\ \ 6\ \ 1 \\ ------------ \\ ------------ \end{array}$$

The order chosen depends on the filter length required and the weights must be normalized as discussed earlier for interpolation. This filter uses only a consecutive pair of the input samples to generate the interpolated values. One important fact about these weights is that they resemble the B-spline function 5. In fact if the distance between the knots of the spline is chosen to be two sampling intervals, then the two are identical. An appeal of the binomial weight filter is its simplicity.

B) The Raised Cosine Filter

The Raised Cosine Filter is a rectangular unit pulse response with a sinusoidal roll-off:

$$h_n = \begin{cases} 1 & , |n| \leq N \\ \tfrac{1}{2}\,[1 - \mathrm{Sin}\,(\alpha n + \beta)] & , N < |n| < L \\ 0 & , |n| \geq L \end{cases}$$

where $\alpha = \pi/(L-N)$

$$\beta = -\frac{\pi}{2}\,\frac{L+N}{L-N}$$

$h_n <=> H(f)$ where

$$H(f) = \frac{\sin^2\frac{\alpha}{2}\ \cos\frac{a}{2}}{2\sin\frac{a}{2}}\ \ \frac{\sin aL \qquad + \qquad \sin a N}{\sin^2\frac{\alpha}{2}\ \cos^2\frac{a}{2}\ \ -\cos^2\frac{\alpha}{2}\ \sin^2\frac{a}{2}}$$

with $a = 2\pi f X_o$

$$H(o) = \tfrac{1}{2}\,(L + N)$$

$$H(\alpha) = \tfrac{1}{2}\,(L - N)\cos\alpha N$$

This kind of filter can be used to evaluate the performance of various degrees of roll-off of the unit pulse response.

C) Pixel Replication Filter (Interpolation)

This filter is used only for interpolation and its weights are:

$$h_n = \begin{cases} 1\ , & |n| \leq (L-1)/2 \\ 0\ , & |n| > (L-1)/2\ ,\ L \text{ odd} \end{cases}$$

It is a special case of the raised cosine filter (N = L).

o o x o o o o x o o o o x o o (L = 5)

D) Pixel Skipping

This is used for decimation only and it is an all-pass filter:

$$h_n = \begin{cases} 1\ , & n = 0 \\ 0\ , & n \neq 0 \end{cases}$$

0 x x 0 x x 0 x x 0 x x 0 (M=3)

E) Lagrange Filter

The interpolation equation using Lagrange coefficients is:[3]

$$i_n = \sum_{K=-\frac{P-2}{2}}^{P/2} C(K,\ P;\ n/L)s_k,\ P \text{ even}$$

$$i_n = \sum_{K=-\frac{P-1}{2}}^{\frac{P-1}{2}} C(K,\ P;\ n/L)s_k,\ P \text{ odd}$$

where P = number of consecutive original samples used in the interpolation

and C(K, P; n/L) are the Lagrange Coefficients

Comparing this equation with the convolution equation [Eq (3)], it is obvious that one can obtain equivalent linear filters to the Lagrange equation. However, since the Lagrange equation uses the same set of P original samples to obtain the interpolated samples, it is also equally obvious that one cannot obtain the same results with just one linear filter[3] if one is to stay within the filter length restriction mentioned earlier. Indeed, a set of P-1 linear filters, one for each interval, is required to obtain the same results. Figure 6 illustrates this point for the cases L=4, P=3 and L=4, P=4. This figure also illustrates another point: the P-1 linear filters are all non-symmetric for P odd and for P even, only the filter for the central interval is symmetric. Thus one cannot approximate the Lagrange equation with one filter for odd P if original samples were to be preserved; on the other hand, one could achieve this for P odd by using the filter corresponding to the central interval. In this case the results will be the same in the central interval but approximate in the other intervals. In our filter comparison, we have considered the P even case and used the filter for the central interval to approximate the Lagrange interpolation scheme.

F) 'Designed' FIR Filter

This filter is obtained using well known design techniques for digital filters. FIR (Finite Impulse Response) filters have been chosen for their relative ease of design as compared with Infinite Impulse Response (IIR) filters. Parameters of interest in digital filter design are cut-off frequency, pass-band width, stop-band width, transition region, maximum ripples in pass-band and stop-band regions and filter length. A well designed digital filter represents the best compromise among these parameters.

The approach usually taken is to minimize a function of the difference between the desired frequency response and a parameterized frequency response of a linear phase filter over all frequencies from zero to the desired cut-off frequency. The function of the error typically chosen is either mean—squared or maximum, absolute error.

The minimization is done such that the ripples in both the pass-band and stop-band are within a certain tolerance and the parameters that correspond to the minimum error are used to approximate the desired frequency response.

For this work, the function of the error that has been minimized is the maximum absolute error and the parameterized frequency response function is of the form

$$H(f) = G(f) \exp[j(\frac{L\pi}{2} - (N-1)\pi f)]$$

where

$$G(f) = \sum_{n=o}^{N/2-1} b_n \cos(2\pi nf)$$

where

N = filter length ,

L determines the symmetry of the impulse response [even ($L=0$) or odd ($L=1$)]

and the b_n's are the parameters with respect to which the error function is minimized.

The desired frequency response is that of an ideal low pass filter whose cut-off frequency, pass-band/stop-band characteristics and filter length are those that are appropriate for decimation and interpolation as discussed previously.

FILTER COMPARISONS

The transfer functions over a period corresponding to these filters have been plotted in Figure 7 for comparison for $L = 5$; $P = 4$ for the 'equivalent' symmetric Lagrange filter and $N=0$ for the raised - cosine filter. On the frequency axis $\upsilon = 50$ corresponds to the Sampling rate for the interpolation factor of 5 and those bands which must be filtered out are located at $\upsilon = 10$, 20, 30 and 40.

From this figure we see that the raised cosine, FIR and Lagrange filters do a good job of removing the unwanted pass-bands; the binomial filter is also effective in filtering out the unwanted pass-bands except those at $\upsilon = 10$ and $\upsilon = 40$. Of these filters, the least sensitive to the Nyquist frequency of the original signal is the FIR filter. The worst filter is, as one would expect, the pixel replication filter. It is zero only at isolated points and therefore does a poor job of filtering out the unwanted pass-bands. The pixel skipping filter, of course, cannot be used for interpolation.

Any of these filters can be used for decimation depending on the decimation factor and the Nyquist frequency of the original signal. For example, one could make the cut-off frequency (i.e., $\frac{1}{2}MX_o$) equal to the frequency at which the transfer function is first equal to zero to obtain the decimation factor M for which that filter would be appropriate. The pixel skipping filter is, of course, never equal to zero but it can still be successfully used for decimation if the Nyquist frequency is less than $\frac{1}{2}MX_c$

PRINT RESULTS AND CONCLUSION

Two sets of prints have been made: one using a computer generated test patch and the other using a real scene. Factors of 2 and 3 have been used for both the decimation and interpolation. Differences in sharpness due to differences in pass-band characteristics can be seen among the filters. The filter that produces the sharpest image (both in decimation and interpolation applications) is the designed FIR filter followed by the binomial weight filter, the 100% rolled-off raised-cosine filter and the Lagrange filter with $P=6$. The least sharp images are produced by the replication filter (raised-cosine with no roll-off). All of the filters except the replication filter are effective in removing the unwanted frequency bands associated with interpolation. The filters that most effectively prevent aliasing are the designed FIR filter and the binomial weight filter. The worst aliasing is caused by the pixel skipping filter (as one would expect).

These observations lead to the following general conclusions:

a) The pass-band characteristics and cut-off frequency are the two most important characteristics of filters to be used for decimation and interpolation. The pass-band characteristics affect sharpness while the cut-off frequency determines how much aliasing will be present in the output image. Depending on the frequency contents of the input image and decimation or interpolation factor, one of the two characteristics can be emphasized at the expense of the other. The designed FIR filter gives the best compromise between these two characteristics. Since aliasing could be more objectionable (if it is present) and more difficult to remove, emphasis should be put on its prevention with possible loss of some sharpness. It is possible to restore this sharpness using one of the standard algorithms.

b) Filter length is important if one wishes to preserve the original samples after interpolation. Choice of filter length must be balanced against the amount of time required for the computation.

c) The sampling rate of the original image is the most important determining factor in the sense that if the original image is severely undersampled, not even the best filter can help; on the other hand, if the original sampling rate is much greater than the Nyquist frequency, just about any of the filters can be used. This parameter also affects the degree to which the balance between sharpness and aliasing can be maintained.

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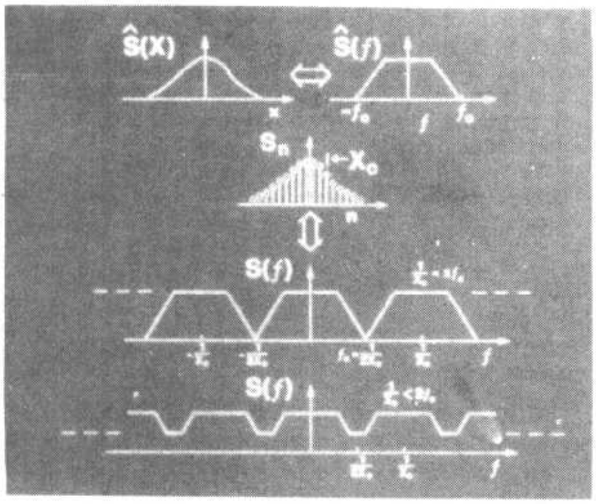

Figure 1. FT of S_n

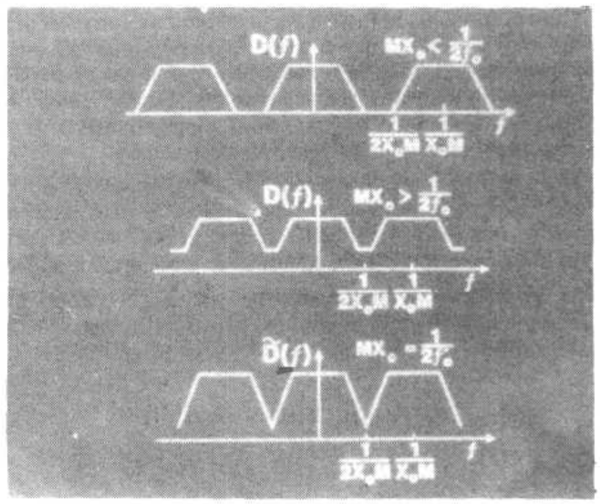

Figure 2. FT of decimated sequence d_n

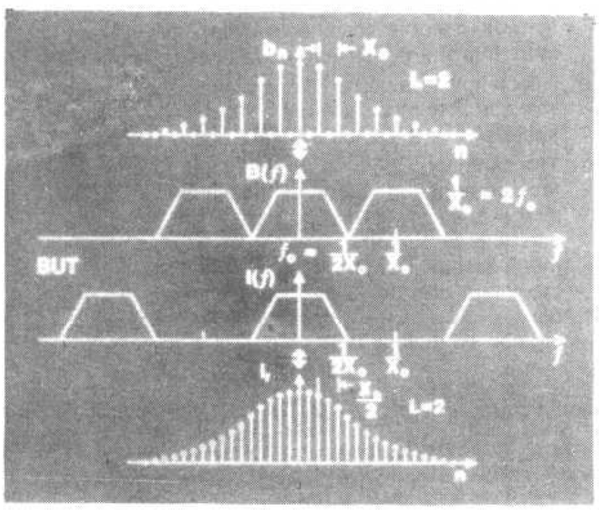

Figure 3.. FT of interpolated Sequence i_n

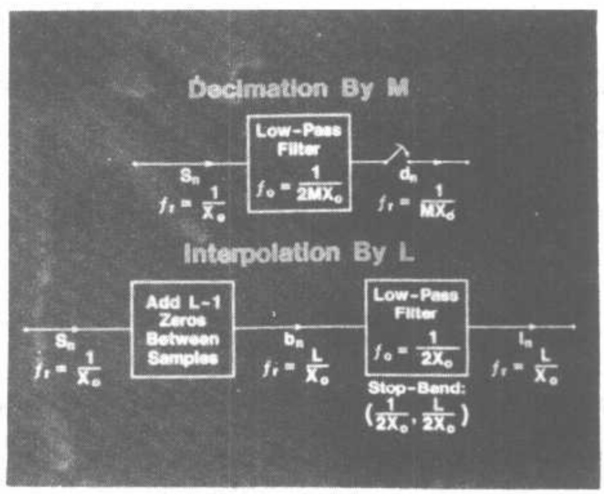

Figure 4. Scheme for Decimation and Interpolation by integer ratios

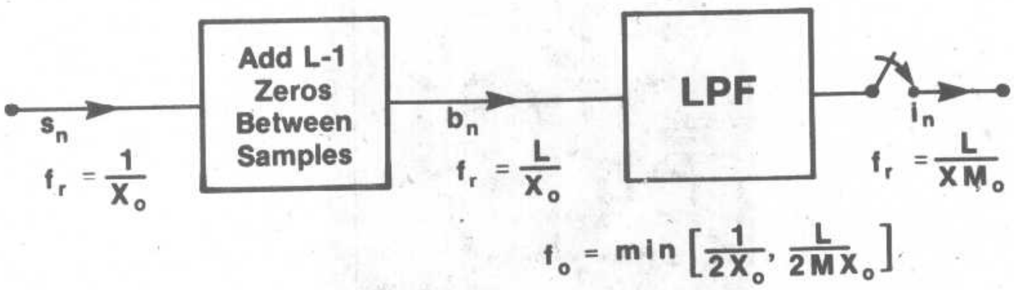

Figure 5. Scheme for decimation and interpolation by non-integer factor

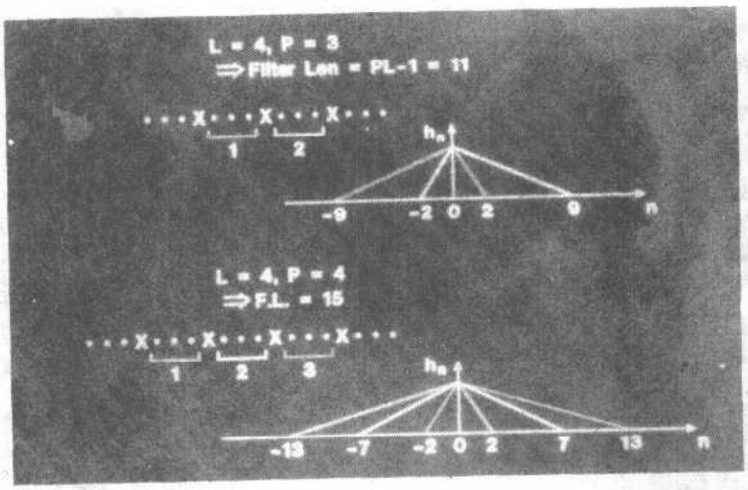

Figure 6. Illustrating the characteristics of the multiple filters for Lagrange interpolation

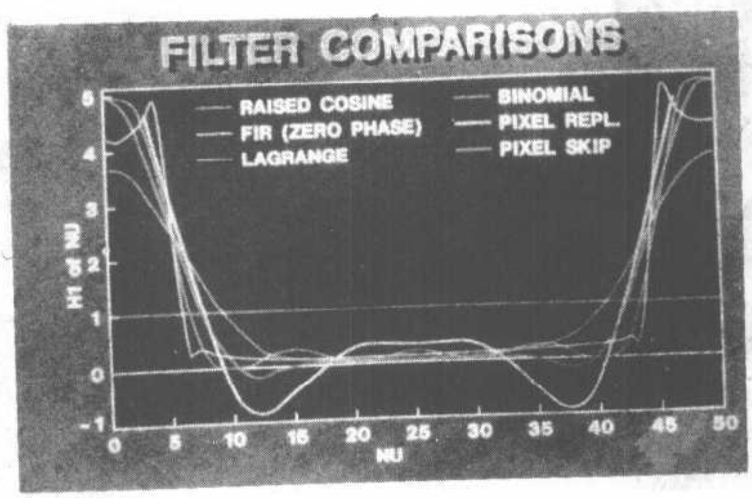

Figure 7. Comparison of filter Trasnfer Functions

INTERPOLATION: INTERPOLATION FACTOR L = 3

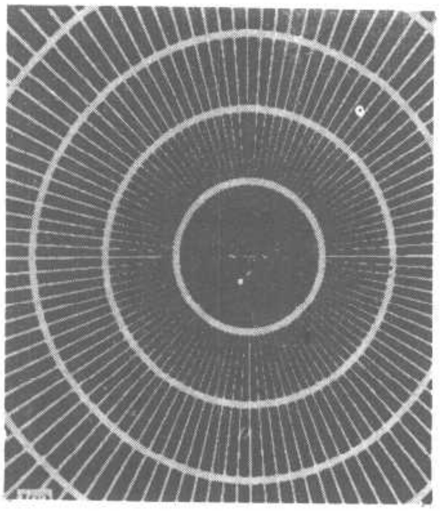

Figure 8. Original 500 x 500

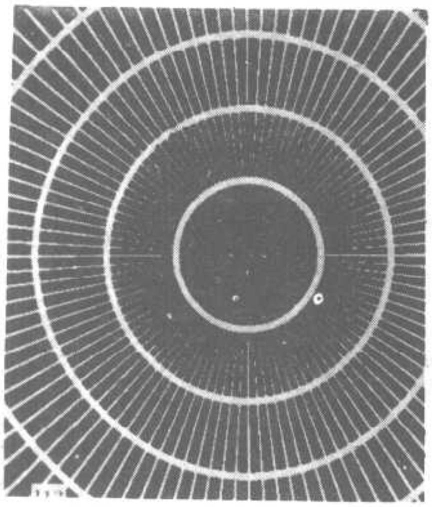

Figure 9. Lagrange Filter (P = 2)

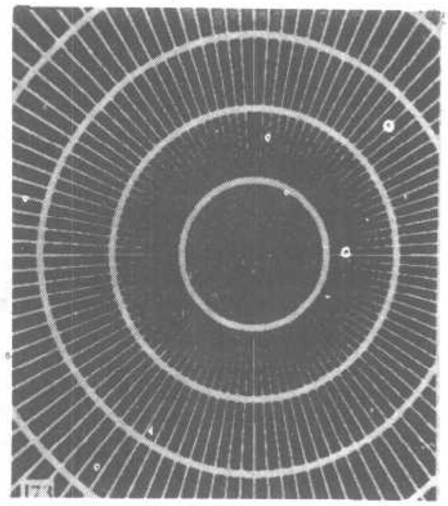

Fig. 10 Raised Cosine Filter (N = 0)

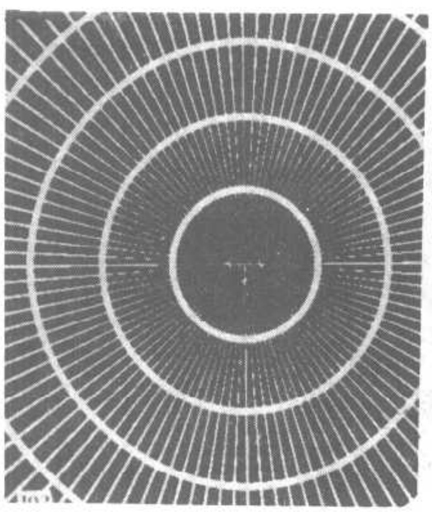

Figure 11. Pixel Replication

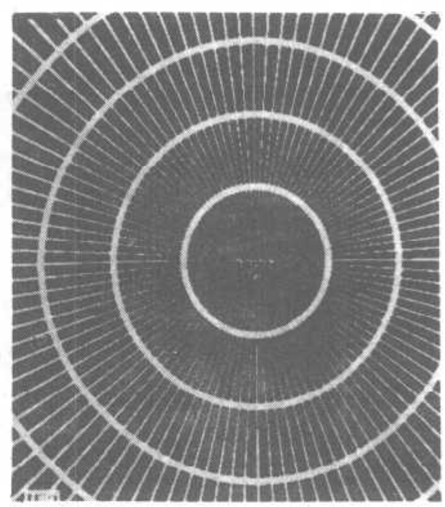

Figure 12. Designed FIR Filter

INTERPOLATION: INTERPOLATION FACTOR L = 3

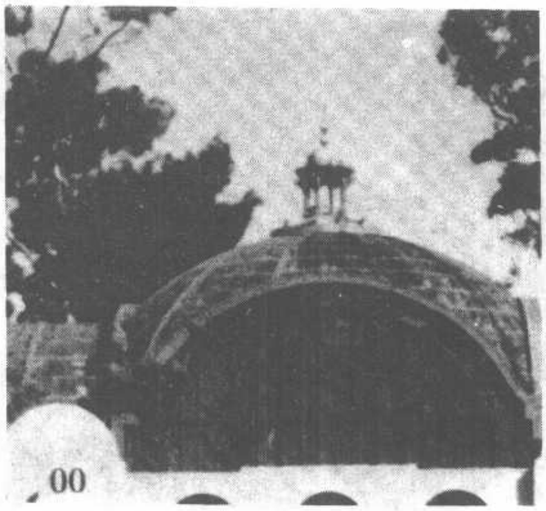

Figure 13. Original 500 x 500

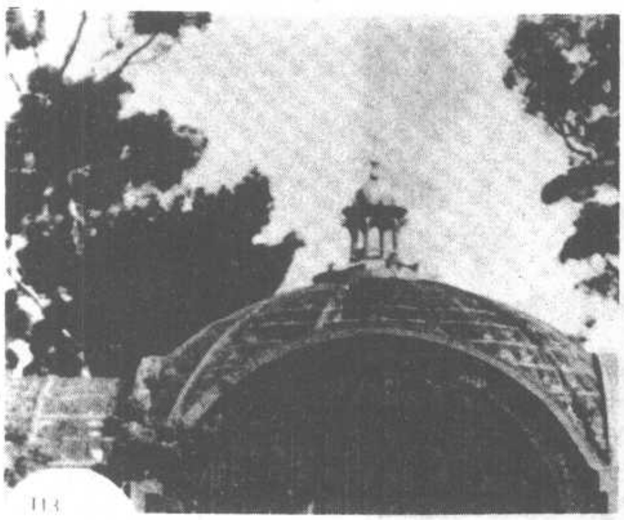

Figure 14. Lagrange Filter (P = 2)

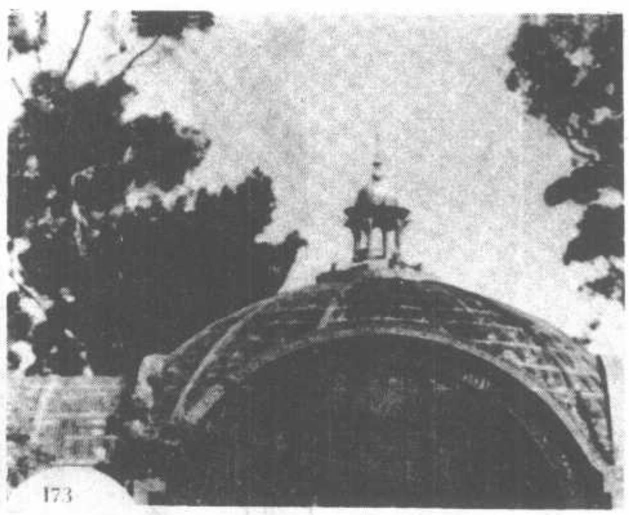

Figure 15. Raised Cosine Filter (N = 0)

INTERPOLATION: INTERPOLATION FACTOR L = 3

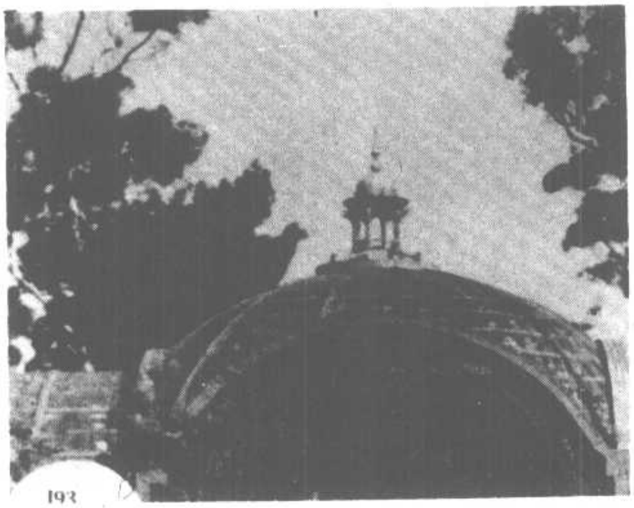

Figure 16. Pixel Replication

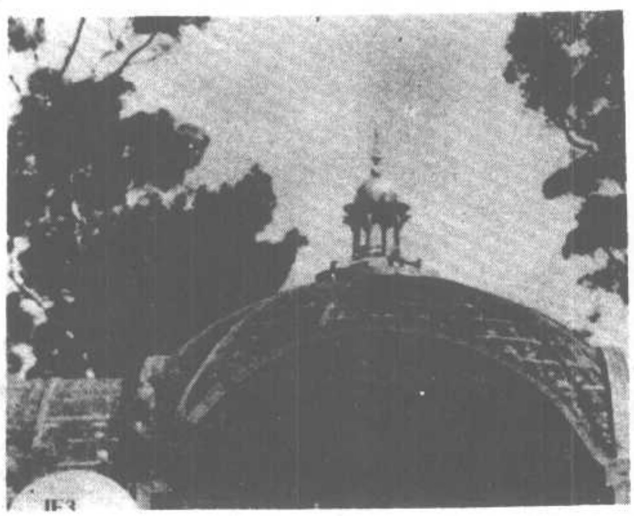

Figure 17. Designed FIR Filter