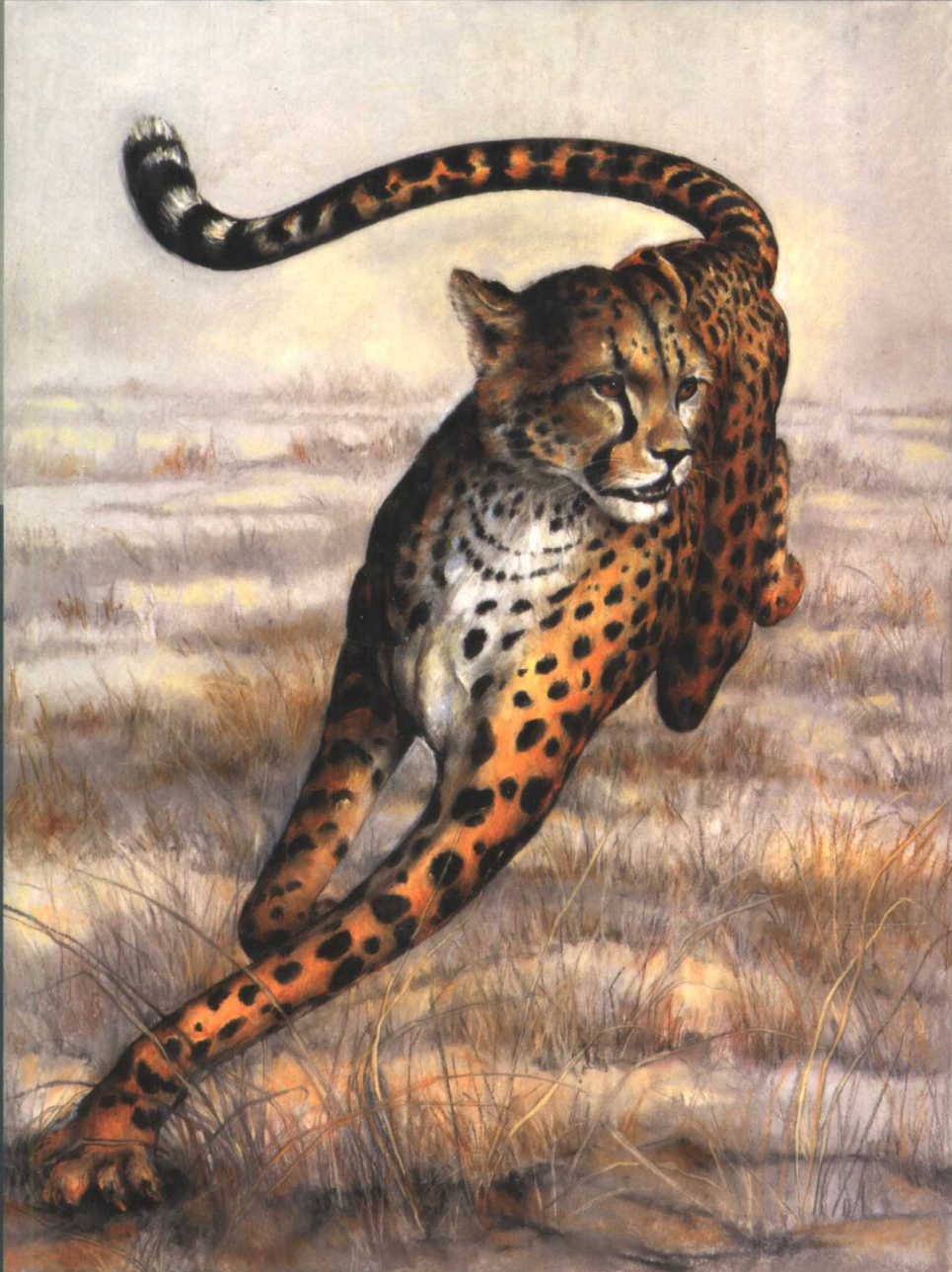


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# Continuous and Discrete Control Systems

Modeling, Identification,



Design, and Implementation

John Dorsey

# **Continuous and Discrete Control Systems**

*Modeling, Identification, Design,  
and Implementation*

**John Dorsey**

*Georgia Institute of Technology  
Atlanta, Georgia*



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**CONTINUOUS AND DISCRETE CONTROL SYSTEMS: MODELING, IDENTIFICATION, DESIGN,  
AND IMPLEMENTATION**

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When I first began teaching the senior elective in control systems at Georgia Tech twenty years ago, I had no interest in writing a textbook on the subject. There were plenty of good texts available, as there are today. Over time, though, I began to typeset my own notes and hand them out to the students. Eventually, those notes became complete enough that the students seem to prefer them to the official text. Even at that point my attitude was simply that the notes offered the students an alternative point of view to that of the official text, thereby enhancing their understanding.

Around 1984 Abe Haddad joined the faculty and within a year or so had managed to pry some money out of the dean of engineering to create a control system laboratory. He asked me to build that lab and I agreed. It took a long time to get that laboratory to function the way I wanted, mainly because I had the students design all the equipment for the laboratory. I had no interest in off-the-shelf experiments or off-the-shelf equipment. The students designed power supplies, power amplifiers, and any number of test fixtures.

Meanwhile, I was trying to design a sequence of laboratory sessions that would take the students through the whole design cycle, beginning with the modeling and identification of the plant transfer function, proceeding through the analysis and design stages, and ending with the implementation of the control on the actual physical system. Eventually, I ended up with three basic devices or "systems" and sixteen laboratory sessions. During those sixteen sessions the students who took the two quarter sequence in control went through the complete design cycle on each of the three systems. Those three basic systems, and a some of the content of those sixteen laboratory sessions appear in this book as three case studies that also take the reader through the entire design cycle.

So it was that when Catherine Fields showed up in my office a couple of years ago, asking if I wanted to write a book, I was somewhat less reluctant than I had been sixteen years previously. Catherine is a very good listener, and very patient, and she let me ramble on about control system education, laboratories, and cheetahs for a long time. In the end, her patience was rewarded because she convinced me to send her a manuscript.

When the initial reviews came back and were favorable, I began to overcome my natural pessimism and started thinking about how I was going to turn a set of class notes into a book in less than six months. Along the way I decided to add an introductory chapter on modern robust control, and after the second round of reviews I made some major revisions to answer the legitimate complaints of the reviewers. Like most writers I would have liked more time, but I kept remembering the character in Albert Camus' novel *The Plague* who spent twenty years trying to get the first sentence of his novel "just right."

I think the distinguishing feature of this book is that I have tried to lay out for the reader all the steps in what I call the design cycle: modeling, identification, analysis, design, simulation, and implementation. Chapters 8, 13, and 16 are case studies that take the reader through the entire design cycle for three different systems. My intent is to bring the reader as close to a real laboratory experience as I can within the confines of a textbook. In the end there is no substitute for getting in the laboratory and getting your hands dirty, but I have tried to come as close to that as I could.

The first two steps in the design cycle, *modeling and identification*, are to my mind the most crucial. Once the plant has been determined with sufficient accuracy, the design, simulation, and implementation normally follow without too much trouble. In this book modeling means finding the structure of the transfer function, by which I mean deciding how many poles and zeros the transfer function has. Once that decision is made, the more difficult problem of parameter identification, deciding on the exact locations of the poles and zeros, has to be addressed.

Most books do a good job with the first step, but I don't know of many, at least at the undergraduate level, that put much emphasis on parameter identification, which, to me, is the most crucial step in the whole design cycle. Beginning in Chapter 3, and again in Chapters 8, 9, 13, and 16, I provide the student with a variety of techniques for parameter identification, *and* some experience in implementing these techniques.

Parameter identification is not easy. In describing it to students in the laboratory I often paraphrase the great running back Jim Brown, who was once asked by a sportswriter, who had probably never carried a football in his life, to describe the techniques and the finesse he had used to gain all those yards. Brown snorted and replied something like, "it's not technique, it's not finesse. I just go out there and scuffle and claw for every yard I can get." That to me is the perfect description of parameter identification, and in Chapters 8, 13, and 16 I have endeavored to show the reader how to scuffle and claw his way to a viable model of a physical system. Once the parameters of the transfer function are identified, the control turns out to be very straightforward, almost anticlimactic.

Aside from Chapters 8, 13, and 16, the book contains the same material found in most undergraduate control books, although in a slightly different arrangement. My goal was to get the student designing control systems as quickly as possible. Thus, after discussing transfer functions and feedback in Chapters 3 and 4, I introduce root locus analysis in Chapter 5 and then follow it with a chapter on specifications. Most books go about it the other way around, which is fine, but my goal was to get the root locus technique in front of the student and then pose the question: now that we have a picture of all the possible *sets* of closed-loop poles, which set do we want to choose? Having answered that question in Chapter 6, the student begins designing systems in Chapter 7, using the root locus method. Thus, 164 pages into the book, the student is designing.

In Chapter 8, I build on Chapter 7 by taking the student through the whole design cycle. First two models, i.e., transfer functions, both based on the analysis of a dc motor from Chapter 3, are chosen. Then the parameters of these two transfer functions are identified using two identification techniques, one that was introduced in Chapter 3,

and a second developed in Chapter 8. Then a control is designed, implemented, and compared with simulation results for both systems. In Chapter 8, I try to give the student something close to an actual laboratory experience. Of course it isn't a real laboratory experience, but I hope the student can at least see the basic steps that have to be followed to implement the control.

Chapters 9 through 13 might be called the heart of the book. Chapter 9 begins with frequency response analysis and ends with the design of compensators that provide disturbance rejection for disturbances at the output. Along the way I spend some time discussing some of the other positive benefits of feedback, a discussion originally begun in Chapter 4. Chapter 10 covers the Nyquist criterion, in preparation for Bode design in Chapter 11 and robust control in Chapter 12. Chapter 13 is again a case study, taking the student through the entire design cycle for a simple robotic system, namely, a positioning table.

Chapters 10, 11, and 12 cover a good deal of the history of control, starting with Nyquist's contributions in Chapter 10; continuing with the contributions of Bode, Nichols, Chestnut, Axleby, Truxal, and many others in Chapter 11; and finishing with the contributions of Doyle, Francis, Tannenbaum, and others in Chapter 12. I think these chapters fit together pretty well, and the student should come away with a sense of the importance of the ideas first introduced in the 1930s and 1940s. Chapter 12 is really only an introduction to robust control, as I try to point out at the beginning of the chapter. But I think it does give the student an idea of how robust control builds on previous work, and hence some sense of the evolution of control design.

Chapter 14 is a review of discrete systems and is followed by Chapter 15 on the design of sampled data systems. Chapter 16 is the third, and final, case study, and is based on an apparatus that captures the essential pitch dynamics of an airplane. This time the modeling and parameter identification are done using frequency domain methods, and the control design and implementation are for a sampled data system.

Chapter 17 is devoted to systems that have a transportation lag. Both continuous and sampled data systems are discussed. The pure delay, or transportation lag, and the zero-order hold have similar effects on the stability of a system. Both introduce negative phase, which tends to destabilize the closed-loop system, and that was the motivation for having a separate chapter devoted to this subject. It is possible, however, to use only the portion of the chapter devoted to continuous systems, or that on sampled data systems.

Chapters 18, 19, and 20, cover continuous and discrete state models, and the controller/observer formulation of feedback control for the single input single output case. This is pretty standard fare. In Chapter 18 the continuous state model is derived from a transfer function. Second-order state models are then studied in depth using phase portraits to provide some intuition for stability. The extension to systems of higher dimension is quite easy. Chapter 19 begins with the discretization of the state model. This is followed by an informal discussion of pole placement. Controllability, observability, and the associated canonical forms are then introduced. Chapter 20 covers the controller/observer for both the regulator and tracking problems. Current as well as prediction estimators are discussed. At the end of Chapter 20 I tie the state

model and transfer functions methods together by showing that the controller/observer and plant can be turned back into a closed-loop transfer function.

It is not realistic to try to cover the entire book in one semester. There are, however, several “one-semester books” within the book. Which one you choose depends on the background of the students. Some of the different courses that could be taught from the book are now discussed.

## **INTRODUCTION TO DESIGN**

For students with no background in control, the first thirteen chapters represent a thorough treatment of continuous systems, beginning with an introduction to the Laplace transform, ending with modern robust control, and offering the student two case studies that cover the entire design cycle. This is not likely to be an option for electrical engineers (EEs), but it might be a good one for mechanical or civil engineers.

## **SENIOR ELECTIVE FOR EEs—OPTION A**

Most electrical engineering students get a reasonably thorough treatment of transform theory and linear systems at the junior level. This means that a senior-level course could begin in Chapter 4, or even in Chapter 5. In that case it should be possible to cover the material up through Chapter 16 in one semester. By omitting some of the design methods in Chapter 15, Chapter 17 could also be included.

## **SENIOR ELECTIVE FOR EEs—OPTION B**

*If the students have seen the  $Z$  transform, then Chapter 14 will be mostly a review and won't take up much time. Then by being selective in Chapters 15, 17, and 18 it should be possible to cover Chapters 5–20. Some places where the instructor can be selective are:*

1. In Chapter 15 a variety of design techniques are presented. A couple of techniques can be omitted without much loss.
2. Chapter 17 is a very thorough study of transportation lags for both continuous and discrete systems. Some time could be saved by simply omitting transportation lags for discrete systems.
3. The material in Chapter 18 on the method of isoclines and generalized eigenvectors and the Jordan form does not impact the presentations in Chapters 19 and 20 and could be omitted.
4. Chapter 20 covers both prediction and current observers. Just covering the prediction observer provides the basics of controller/observer design. One could also omit the development on the tracking problem and concentrate on regulators.

## SENIOR ELECTIVE FOR EEs—OPTION C

It is also possible to teach a traditional senior level course by leaving out the case studies in Chapters 8, 13, and 16. In that case I think Chapters 5, 6, 7, 9, 10, 11, 12, 14, 15, 17, 18, 19, and 20 could be covered in one semester by omitting some topics in Chapters 15, 17, 18 and 20, as discussed under option B.

## DIGITAL CONTROL

Chapters 14–20 provide a thorough coverage of digital control and are suitable for a one *quarter* course on that subject. Chapter 18 covers state models for continuous systems, and can be included or omitted as the instructor sees fit. I would probably include it, but it is really a matter of taste.

I consider the book MATLAB-friendly but not MATLAB-crazy. I think the MATLAB control toolbox is of great use, *once* the students have done some hand calculations and know what they are doing. For me, the best thing about MATLAB is not the “canned” routines, but the fact that I can very quickly write a fifteen- or twenty-line m-file that lets me see the effect of varying a parameter, such as a pole or zero, over a range of values. This is something that would be hard to do with hand calculations, and is, I think, the most valuable feature of MATLAB. Many of the problems are designed to get the student to write this kind of program. The CD-ROM that comes with the book discusses the MATLAB commands relevant to each chapter and provides some examples of how to use these commands.

The CD-ROM also includes some actual test data for the three systems introduced in Chapters 8, 13, and 16. In the solution manual, I provide the results of the actual implementation of the control designs for the problems in these chapters.

In summary, I think the book offers some unique features, most notably attention to the parameter identification problem at a level that an undergraduate can understand, plus three complete trips through the design cycle. Not being much of a salesman I will stop short of saying your life will change if you adopt this book, and merely add that the students at Georgia Tech, who are about the same as the students at Nebraska, Penn State, Michigan State, or Kansas, or for that matter MIT, seem to like the notes on which this book is based.



## ACKNOWLEDGMENTS

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When I got out of the Navy at the end of 1967, having survived eleven months in-country in Viet Nam with the Marine special landing force, I wanted to get an engineering degree. My time in the service had made it clear that to really understand the twentieth century you needed a technical education. The trouble was I had exited the electrical engineering program at Purdue after my sophomore year to pursue a degree in the arts. It wasn't grades that drove me out of engineering but rather sheer boredom. The electrical engineering curriculum of the early sixties was not very exciting.

However, when I applied to Purdue for graduate school they wouldn't even answer my letter. My application to the University of Michigan received the same gracious treatment. So I got in my MGA, drove to East Lansing, and started barging into offices and asking questions. No one threw me out; in fact, they were quite cordial and I ended up in the chairman's office, talking to Harry Hedges. Harry was an ex-marine, and when he found out where I had just come back from, he and Herman Koenig found me an assistantship. So it was that with a bachelor's degree in creative writing I managed to acquire a master's degree in systems science. It is only fitting that the system science degree is no longer offered *anywhere*.

Never one to take advantage of my good luck, I left with my master's degree, and worked in the aerospace and automobile industries for a few years before throwing my skis in my Mustang and heading for the Rockies. Over the next few years you could have found me anywhere between Montana and Peru, ruminating on life in general and the perils of undeclared wars in particular. Finally, in 1976, when I decided I was too young to retire, I reapplied to Michigan State, and, unbelievably, they let me back in.

It goes without saying that I am deeply indebted to Michigan State University and to the faculty of the Electrical Engineering Department. Herman Koenig, Gerald Park, Roland Zapp, Don Reinhard, Jim Resh, Bob Barr, and Bob Schlueter were not only great teachers but good friends.

I am especially indebted to Jim Resh and Gerald Park. They are two of the best engineers I have ever known. Jim Resh took me out for a beer midway through the PhD experience when I was ready to go skiing again and showed me how to finish. Without him I probably would be working in a ski resort somewhere. Gerald Park is one of the most intelligent and literate people I have ever met. Broadly educated, tremendously perceptive, and extremely articulate, he contributed to my education in ways he never knew.

When I threw my earthly possessions in the back of my Truck (it was a great truck and deserves a capital letter) in 1980 and drove out of East Lansing to take a job at Georgia Tech, I didn't expect to be there long. Twenty years later I am still at Tech, but truth be known probably eighty percent of the faculty wish I had never

darkened the door of the place. Actually, I feel the same way, and it is only because of the efforts of a few individuals that I am still on the job. George "Pete" Rodrique, Bill Sayle, William "Russ" Callen, Cecil Alford, and Dan Fielder kept me going in the early years.

Then Abe Haddad showed up and became a lifelong friend. I owe him more than I can say. He pried the money loose from the dean of engineering in 1985 so I could build a controls laboratory. He patiently helped with the review of this book. And, most importantly, he went out of his way to help Zhihua Qu, one of the most gifted individuals I have ever met, who did me the great honor of letting me be his advisor. Abe was kind enough to help Zhihua in a number of ways that I could not, and for that I am forever in his debt.

After Abe left for Northwestern, Ed Kamen came back to Tech. He was a great supporter from the beginning. He helped me get promoted and underwrote the rejuvenation of the controls laboratory. I owe him a lot as well.

The days of asking a colleague to proof a book are long gone. I did the next best thing and got McGraw-Hill to pay Nathan "Scott" Clements, who will soon finish his PhD and become a colleague, a thousand bucks to go through the manuscript and check all the calculations. I think this worked out to a penny an hour, and I very much appreciate Scott's efforts. I have a tendency to jab the four inch brush in the can and start slinging paint around. Meticulous is not my middle name, and Scott's help was invaluable. Bruce McFarland made important contributions to Chapters 8, 13, and 16.

Of course, I would also like to thank the reviewers of this book, both those who have graciously allowed me to use their names, as well as those who didn't. The comments of Abraham Haddad, Naim A. Kheir of Oakland University, Augustus Morris of Central State University, and Gordon Parker of Michigan Technological University were extremely helpful. The enthusiasm of Augustus Morris for the book lifted my spirits at a time when I was realizing that the reviewers were on target and I had a lot of work yet to do. I took all the reviewer criticisms seriously and implemented about ninety percent of their suggestions.

Finally, Ann Sutherland did the cover. She and I share an admiration for the large cats, and especially for the endangered cheetah, one of nature's great control systems. If you ever visit the Atlanta Zoo you can admire her lifesize bronzes of lions and cheetahs. The little kids love them, and I have never visited the zoo when the kids were not climbing all over the cats. I am very grateful to Ann for her wonderful work. My feeling is that even if you don't like the book you can tear off the cover and frame it, and your money will still be well spent. On the other hand, if you like the book you can visit my Georgia Tech website and get Ann's address. She will send you a limited edition, quality print for a very fair price.

**j. dorsey**

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# Preliminaries

## 1.1 | WHY CONTROL?

From an aisle seat just behind the wings of a modern jet airliner it is possible to watch the control surfaces of both wings. The smaller outboard control surfaces are the ailerons. When one aileron pitches down, the other pitches up. Together, along with the rudder and elevator on the tail, they control the direction of flight. The inner, large control surfaces are commonly referred to as “flaps.” Passengers are more familiar with the action of these large control surfaces since they are active during take off and landing. Their purpose is to change the surface geometry of the wing, thereby giving the aircraft more, or less, lift.

The pilot moves the control surfaces via a linkage system consisting of electrical, mechanical, and hydraulic components. In the case of the ailerons, the pilot moves the “yoke.” The yoke also controls the rudder and elevator on the tail, acting very much like an automobile steering wheel. The pilot usually controls the flaps by moving a mechanical lever fore or aft.

In any case, sitting aft of the wings, watching the control surfaces during a descent into a modern airport, it becomes perfectly clear how these linkage systems that connect the pilot to the control surfaces should work if the plane is to land safely: these systems should move the control surfaces quickly and accurately to the position requested by the pilot. This is the heart of control: precise, responsive performance of a physical system such as a jetliner.

One might ask why control is even necessary, that is, why the system wasn’t designed to operate “better” in the first place. This is a valid question. In some cases, for instance in biological systems, the control schemes are quite minimal because the system has been superbly designed by the long process of evolution. The larger birds of prey, such as hawks and eagles, are a good example of this. These birds can hover nearly motionless in a head wind by making very slight adjustments in the feathers on the trailing edges of their wings.

In other cases, limitations to the basic design of a system require the addition of control. For instance, consider the cheetah whose prey are antelope and gazelle that

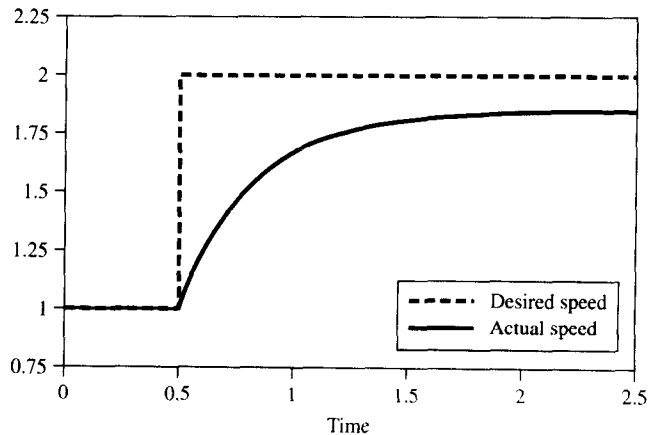


can run about 45 mph for several minutes. The cheetah, on the other hand, can run in short bursts at speeds up to 70 mph. But speed is not enough. A Thompson's gazelle can change direction on a dime, and to catch the gazelle, the cheetah must be able to react to its prey's maneuvers. To do so it has a superb control system consisting of excellent eyesight, a brain, and exceptional reflexes. The cheetah can overtake the gazelle in less than 5 s, but without a good control system it would never catch the gazelle.

An industrial analog of the cheetah versus the gazelle is a large motor driving a load. If the load presented a constant torque to the motor shaft, then no control would be required. Such a constant torque load would be like a gazelle that never changed direction. But the load does change, and as a result control is required if the motor is to achieve its purpose.

As an example, a typical industrial application may require the motor to achieve and maintain a sequence of different speeds while working against a load of varying torque. It is usually desirable for these speed changes to be accomplished as rapidly as possible. Figure 1.1 shows what the speed change might look like with no control. The speed rises gradually, so that a considerable length of time elapses before the speed gets close to its final value.

**Figure 1.1** | Response without control.



Thinking back to the descent of the airliner, we conclude that this type of response from the flaps might make the landing quite exciting. In the case of the airplane, however, there is an important additional control system present, namely the pilot. Skilled pilots have been able to land planes whose control systems are badly damaged. In one case, the plane could only fly in circles but the pilot managed to get the plane down onto a runway. Thus even if the control were inaccurate, the pilot would probably be able to land safely.

The figure also shows that the final speed is not exactly the requested speed. That is, there is an offset or steady state error. Thus, two improvements that could be expected from a control scheme are a faster rise in the speed, so that the motor speed